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Nonparametric Estimation of the Spatial Connectivity Matrix Using Spatial Panel Data

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We use moments from the covariance matrix for spatial panel data to estimate the parameters of the spatial autoregression model, including the spatial connectivity matrix W. *In the unrestricted spatial autoregression model, the parameters are underidentified by one when* W *is symmetric. We show that a special case exists in which* W *is asymmetric and its parameters are exactly identified. If the panel data are stationary and ergodic, spatially and temporally, the estimates of* W *and the spatial autoregression coefficients are consistent. Spatial panel data for house prices in Israel are used to illustrate this methodology.*

> When observations are available over time as well as across space, these constraints (on W) can be relaxed. In the particular case where the time dimension is larger than the spatial dimension, a spatial weight matrix is no longer necessary . . . (Anselin 1988, p. 176)

Introduction

Ever since its inception in the 1970s, the spatial connectivity matrix, commonly denoted by its row-standardized version, W, has been imposed rather than estimated. Matrix W is imposed exogenously based on a general notion of how distance affects connectivity.[1] In principle, goodness-of-fit tests may be used to chose between rival definitions of W. In practice, however, most researchers impose W without empirically testing its restrictions. If W is misspecified, parameter estimates are likely to be biased and inconsistent in models containing spatial lags (Stakhovych and Bijmolt 2008). Cuaresma and Feldkircher (2012) show how estimates of income convergence across European regions may be biased up to 100%, depending on the specification of W.

In this article, we propose a methodology to estimate W from spatial panel data. Our estimator is based on the method of moments and is entirely nonparametric. Specifically, our estimator is designed for the spatial autoregression model in which W is estimated rather than imposed and in which the spatial autoregression model is heterogeneous across spatial units.

Attempts to estimate W empirically fall into two main groups. The first infers W from the data using various geostatistical modeling techniques.[2] The second, originally proposed by Meen

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(1996), involves the estimation of spatial autocorrelation (SAC) coefficients by regressing residuals for region i at time t on the residuals for all other regions. This method is feasible if the panel is sufficiently long, so that the number of data points exceeds the number of spatial units. Bhattacharjee and Jensen-Butler (2006) extend this idea to the case in which the SAC coefficients are inferred from the estimated covariance matrix of the spatial errors. They use this method to study the diffusion of housing demand across U.K. regions.

Our proposed methodology is similar to that of Bhattacharjee and Jensen-Butler (2006), except that we are concerned with the estimation of spatial lag coefficients, whereas they are concerned with SAC. Specifically, we hypothesize a spatial autoregression model to be estimated from spatial panel data. We infer W directly from the covariance matrix for the data, from which we also infer heterogeneous spatial autoregression coefficients. We show that W and the spatial autoregression coefficients frequently are not identified because insufficient moment conditions exist. To address this issue, we suggest a special case in which W is asymmetric and the spatial autoregression coefficients are exactly identified and may be estimated consistently. Moreover, we show that this special case turns out to be numerically and computationally tractable. We present an empirical application for this special case using spatial panel data for house prices in Israel.

Methodology

In this section, we show how spatial weights and SAC coefficients may be estimated from spatial panel data. Our identification procedure is based on restrictions applied to these weights and coefficients.

The data generating process

Spatial units are labeled by $i = 1, 2, \ldots, n$ and time periods by $t = 1, 2, \ldots, T$. Let y_t denote an n-vector of outcomes in period t for each spatial unit. The panel spatial autoregression model may be written as

$$y_t = \alpha + BWy_t + \varepsilon_t \tag{1}$$

where α is an n-vector of common or fixed effects, and B is an n-by-n diagonal matrix of spatial autoregression coefficients with diagonal elements β_i. If these spatial autoregression coefficients are homogeneous so that $\beta_i = \beta$, B is replaced by a scalar β in equation (1). The variance-covariance matrix $\Sigma = E(\varepsilon\varepsilon')$ is assumed to be time invariant (temporal homoscedasticity) and diagonal (no SAC between ε_i and ε_j), but may be spatially heteroscedastic so that the variance of ε_i (σ_i^2) may vary between spatial units. The spatial Wold (1938) representation of equation (1) is

$$\begin{aligned} y_t &= A(\alpha + \varepsilon_t) \\ A &= (I_N - BW)^{-1} \end{aligned} \tag{2}$$

Invertibility requires that the determinant of $I_n - BW$ be nonzero: that is, the rank of $I_N - BW$ equals n. If, for example, $\beta_i = \beta = 1$ and the row elements of W sum to 1, $I_N - BW$ is not invertible. In this case, $I_n - BW$ is invertible provided at least one of the spatial autoregression coefficients is less than one. Let $V = yy'$ denote the population covariance matrix of the ys. Substituting equation (2) for y gives

$$V = A\Sigma A' = H \tag{3}$$

Note that if A is symmetric, then $A = A'$. Because V is symmetric, it contains $^1/_2 n(n+1)$ independent elements. If W is symmetric and $\Sigma_j w_{ij} = 1$ (row sum equals unity), W contains $^1/_2(n-2)(n-1)$ unknown w_{ij} elements. There are also n unknown spatial autoregression coefficients and n unknown variances (diagonal elements of Σ), resulting in $^1/_2 n(n+1) + 1$ unknown parameters altogether. Therefore, an identification deficit of one exists; the number of population moments in V is one less than the number of unknown parameters. If W is asymmetric, the identification deficit increases to $^1/_2 n(n+1)$. It increases yet further if ε happens to be spatially autocorrelated.

Even if W is symmetric, BW and therefore A generally are asymmetric. If the spatial autoregression coefficients are homogeneous, that is, $\beta_i = \beta$, A is symmetric if W is symmetric. Nevertheless, H in equation (3) is symmetric. As mentioned, the identification deficit is smallest and equal to one when W is symmetric. In this case, a tempting change might be to exogenize one element of W or B in order to exactly identify all of the parameters. However, testing the validity of such arbitrary identifying restrictions is impossible.

A special case: heterogeneous mutuality

The identification deficit disappears in a special case in which $BW = G$ happens to be a symmetric matrix, which implies that W is asymmetric,[3] but the direct spillover effect of shocks between spatial units[4] are mutual. In this special case, the direct spillover effects of shocks in unit j on unit i are assumed to equal the direct effects of shocks in unit i on unit j. This mutuality is heterogeneous because it varies between spatial units. The leading diagonal of G is zero; hence, $g_{ii} = 0$. The off-diagonal elements have the property $g_{ij} = -\beta_i w_{ij} = g_{ji} = -\beta_j w_{ji}$, which implies that

$$\frac{\beta_i}{\beta_j} = \frac{w_{ji}}{w_{ij}} \tag{4}$$

In this case, matrices B, W, and Σ are exactly identified. H comprises $^1/_2(n-1)(n-2)$ unknown elements of G and n diagonal elements of Σ, which are exactly equal to the $^1/_2 n(n+1)$ independent elements of V.

Using the row sum constraints, identification of the spatial autoregression coefficients becomes possible because

$$\sum_{j\neq i}^{N} g_{ij} = \beta_i \sum_{j\neq i}^{N} w_{ij} = \beta_i \tag{5}$$

which, in turn, may be used to solve for the spatial weight as follows:

$$w_{ij} = -\frac{g_{ij}}{\beta_i} = -\frac{g_{ij}}{\sum_{j\neq i}^{N} g_{ij}} \tag{6}$$

This special case is solvable because the solution for $V = H$ conveniently has a hierarchical mathematical structure.[5] First, G and Σ are solved by equation (3). Then B is solved by equation (5). Finally, W is solved by equation (6).

The first step is facilitated by using $V^{-1} = (I_n - G)\Sigma^{-1}(I_N - G)$. If, for illustrative purposes, $n = 3$, then

$$V^{-1} = \begin{bmatrix} \left(\frac{1}{\sigma_1^2}+\frac{g_{12}^2}{\sigma_2^2}+\frac{g_{13}^2}{\sigma_3^2}\right) & \left(-\frac{g_{12}}{\sigma_1^2}-\frac{g_{12}}{\sigma_2^2}+\frac{g_{13}g_{23}}{\sigma_3^2}\right) & \left(-\frac{g_{13}}{\sigma_1^2}+\frac{g_{12}g_{23}}{\sigma_2^2}-\frac{g_{13}}{\sigma_3^2}\right) \\ \left(-\frac{g_{12}}{\sigma_1^2}-\frac{g_{12}}{\sigma_2^2}+\frac{g_{13}g_{23}}{\sigma_3^2}\right) & \left(\frac{g_{12}^2}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{g_{23}^2}{\sigma_3^2}\right) & \left(\frac{g_{12}g_{13}}{\sigma_1^2}-\frac{g_{23}}{\sigma_2^2}-\frac{g_{23}}{\sigma_3^2}\right) \\ \left(-\frac{g_{13}}{\sigma_1^2}+\frac{g_{12}g_{23}}{\sigma_2^2}-\frac{g_{13}}{\sigma_3^2}\right) & \left(\frac{g_{12}g_{13}}{\sigma_1^2}-\frac{g_{23}}{\sigma_2^2}-\frac{g_{23}}{\sigma_3^2}\right) & \left(\frac{g_{13}^2}{\sigma_1^2}+\frac{g_{23}^2}{\sigma_2^2}+\frac{1}{\sigma_3^2}\right) \end{bmatrix} \quad (7)$$

The six independent elements of V^{-1} solve for the six unknown parameters: g_{12}, g_{13}, g_{23}, σ_1, σ_2, and σ_3. The six independent elements in equation (7) are nonlinear quadratic equations in the unknown parameters. The elements of V^{-1} involve products of the variances and covariances of the data. Therefore, the determinant of V^{-1} involves products of the fourth moments of the data. Subsequently, the variances of the estimates of G and Σ involve the eighth moments of the data. Consequently, the eighth moments of the data are assumed to be finite. If the data are normally distributed, then their eighth moment equals $105\sigma^8$.

Consistent estimates of *B*, *W*, and Σ

In this section, we discuss how the population parameters (B, W, and Σ) can be estimated for the special case from sample panel data of T observations on n spatial units. In contrast to nonspatial panel data, n is naturally fixed in spatial panel data because it usually comprises all the spatial units in a country or region. The sample size varies with T. Given n, the sample covariance matrix estimated from T observations, $\hat{V}_T$, equals the estimate of $A\Sigma A'$, which requires that $T > n$. Because the probability limit of a product is equal to the product of the individual probability limits, equation (3) implies

$$p\lim\left(\hat{V}_T\right) = p\lim\left(\hat{A}_T\right)p\lim\left(\hat{\Sigma}_T\right)p\lim\left(\hat{A}_T\right)' \quad (8)$$

Therefore, if

$$p\lim\left(\hat{V}_T\right) = V \quad (9)$$

then $p\lim(\hat{A}_T) = A$ and $p\lim\left(\hat{\Sigma}_T\right) = \Sigma$. The main parameters of interest are B and W, which according to equation (2) are related to A nonlinearly. According to the Slutzky theorem,[6] the probability limit of a nonlinear function of x equals the nonlinear function of the probability limit of x. Therefore, because $p\lim(\hat{A}_T) = A$, the Slutzky theorem states that $p\lim\left(\hat{B}_T\right) = B$ and $p\lim(\hat{W}_T) = W$. In short, consistency requires that equation (9) be valid.[7]

If the panel data are independent, equation (9) is obviously valid. However, they are dependent for two reasons. First, the units in the panel are spatially dependent. Second, the data may be temporally dependent. For example, y_{it} might be temporally autocorrelated within and perhaps between spatial units.[8] Because n is fixed, the former dependence is not important for consistency of $\hat{V}$. However, the latter dependence is obviously important. The conditions for consistency[9] due to the latter are as follows:

(1) the panel data are temporally stationary; that is, the unconditional sample moments are independent of t; and
(2) the panel data are ergodic; that is, events that are separated far enough in time are asymptotically independent.

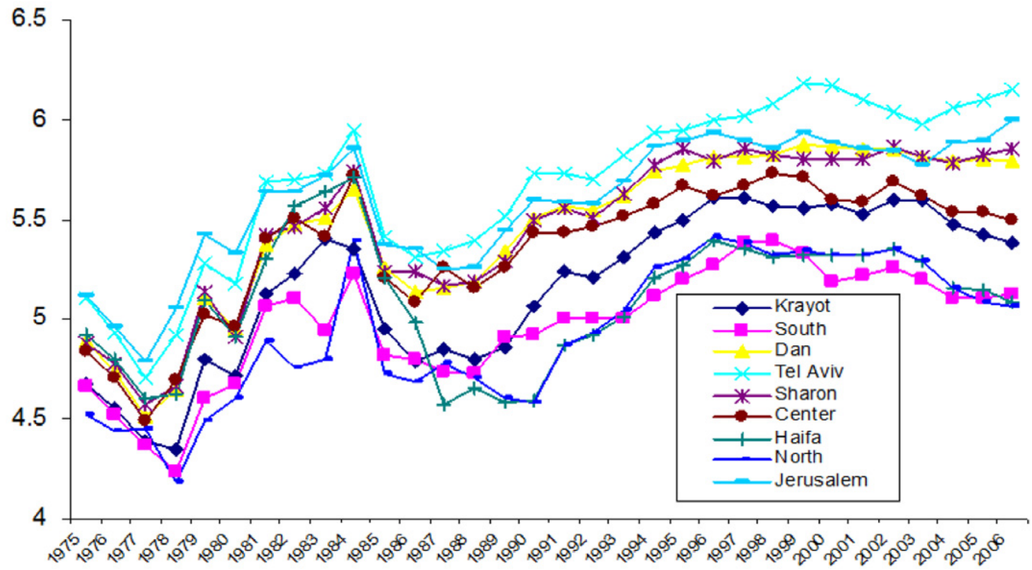

Figure 1. The logarithm of 1991 regional house prices in Israel.

In the next section, we evaluate these conditions using panel unit root tests. If the data happen to be temporally autocorrelated, then the long-term covariance matrix (V_{LT}) rather than V should be used because consistent estimates of B and W refer to long-term covariances, which are asymptotic, rather than to sample covariances. If the data are not autocorrelated, then $V_{LT} = V$.

In conventional spatial autoregressive models in which W is imposed, the spatial autoregression coefficients cannot be estimated by ordinary least squares because Wy_t in equation (1) is endogenous; it is not independent of ε_t. Instead, the spatial autoregressive model coefficients have to be estimated by the methods of maximum likelihood or instrumental variables. This endogeneity problem does not arise[10] with our proposed estimator because equation (3) is not affected by the endogeneity of Wy_t. This convenient property results from equation (2) being the spatially reduced form of the spatial autoregression model. Therefore, the population moments of V exactly identify B, W, and Σ in the special case. Estimation error is induced in the usual way because, in practice, these parameters are estimated from sample moments. Analytical solutions for the variances of the estimates of W and B are not available because these estimates are nonlinear functions of the sample moments. However, they may be obtained numerically by panel bootstrapping $\hat{V}_T$. As mentioned, the eighth moments of the data are assumed to be finite; otherwise, the variances of the estimates of B and W do not exist.

Empirical application

This section illustrates the methodology. We use spatial panel data for the logarithm of regional house prices (measured in constant prices; see Fig. 1) in Israel observed annually between 1975 and 2006 for the nine regions mapped in Fig. 2. We report estimates of the spatial connectivity matrix (W) and the spatial autoregressive model coefficients (B) for the special case given in the section "A special case: heterogeneous mutuality."

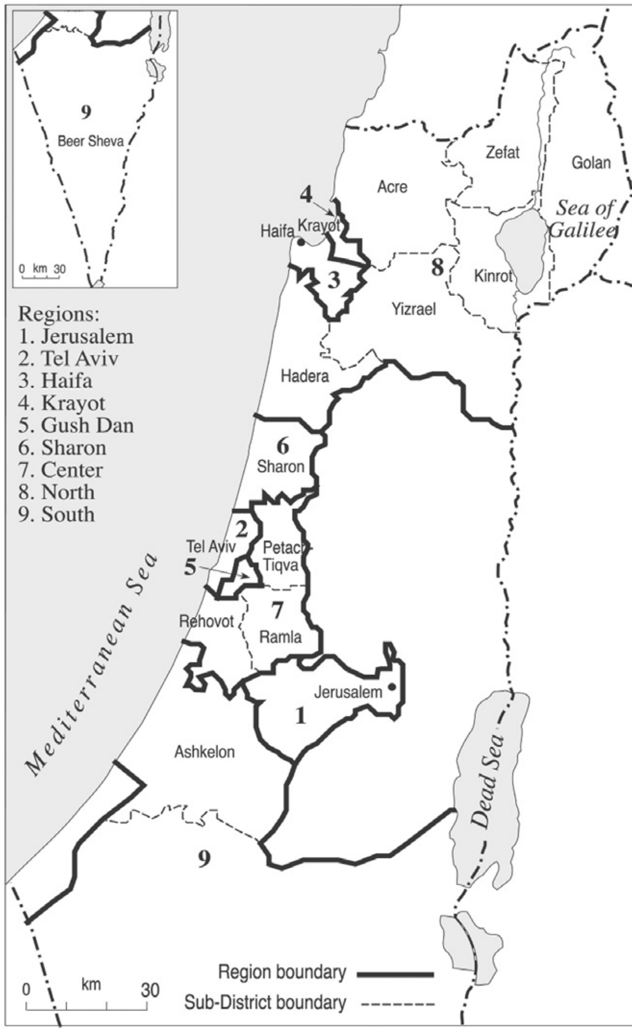

Figure 2. Geographic regions of Israel.

The data

The panel unit root test statistic (named IPS because of its formulation by Im, Pesaran, and Shin (2003)) is −1.74, and its common factor counterpart (CIPS, so named by Pesaran (2007)) is −1.90. Therefore, the data are nonstationary.[11] However, the data are stationary in first differences because the IPS and CIPS are −6.03 and −4.22, respectively.

The correlation matrices for levels ($d = 0$) and first differences ($d = 1$) of the logarithm of real house prices are given in Table 1. Not surprisingly, the correlations are larger in levels ($d = 0$) than in first differences ($d = 1$). However, even in this latter case, the correlations are large and positive, and range between 0.22 and 0.95. Because the data are stationary in first differences, the correlation matrix in Table 1 is a consistent estimate for the case when $d = 1$.

Table 1 presents the correlation matrix rather than the covariance matrix because correlations are easier to interpret. The long-term covariance matrix ($d = 1$) differs from its (unreported) counterpart in Table 1 if the change in the logarithm of house prices is autocorrelated within and

Table 1 Correlation Matrix for Regional House Prices

	d	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	0	0.959	0.978	0.967	0.977	0.953	0.733	0.770	0.956
	1	0.711	0.888	0.797	0.865	0.755	0.824	0.225	0.748
South	0		0.954	0.943	0.953	0.959	0.709	0.918	0.028
	1		0.797	0.751	0.800	0.806	0.703	0.670	0.698
Dan	0			0.991	0.994	0.972	0.638	0.793	0.971
	1			0.950	0.947	0.891	0.817	0.391	0.907
Tel Aviv	0				0.987	0.961	0.648	0.774	0.980
	1				0.914	0.886	0.715	0.413	0.940
Sharon	0					0.971	0.662	0.779	0.975
	1					0.855	0.910	0.395	0.888
Center	0						0.667	0.765	0.948
	1						0.638	0.549	0.802
Haifa	0							0.469	0.708
	1							0.322	0.730
North	0								0.737
	1								0.326

between spatial units. We estimated a first-order panel vector autoregression (VAR) model in which the change in the logarithm of house prices in each region is hypothesized to depend on its own lag and on the lags of other regions. The *P*-value of the estimated model is 0.541, which indicates that the 81 VAR parameters are not significantly different from zero. We also estimated simple autoregressive (AR[1]) models for each region. Seven of the nine AR parameters are not statistically significant; Haifa and Dan have *P*-values of 0.015 and 0.022, respectively. A joint test of these nine AR coefficients shows that they are not statistically significant from zero. Therefore, we conclude that the logarithms of house prices are indistinguishable from a random walk. This means (conveniently) that the long-term covariance matrix (for $d = 1$) is equal to its counterpart in Table 1.

We also calculate the eighth sample moments to ensure that they are finite. These eighth moments are equal to $105\sigma^8$. Since the data are in logarithms, their standard deviation is a fraction, hence, $0 < \sigma < 1$. Since the eighth moments involve raising the standard deviation to the power of eight, these moments are likely to be small. The eighth moments are considerably smaller than $105\sigma^8$ in all regions except the North, where the eighth moment is 5.32×10^{-7} and $105\sigma^8 = 4.28 \times 10^{-7}$. Therefore, we are satisfied that the eighth moments are finite.

Basic results

This section summarizes results for the special case estimates of *W* and the spatial autoregression coefficients using equation (3) for the covariance matrix of the log-differences of regional house prices. We use differences rather than levels because the levels are nonstationary. Therefore, estimates of $G = BW$ are consistent because the covariance matrix is consistent. As noted in the methodology section, the solution to the special case has a three-step recursive structure that greatly simplifies the computational burden. We have no reason to believe that our proposed methodology for the special case would not be feasible if n happens to be larger than nine.

Table 2 Spatial Connectivity Matrix ($d = 1$)

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot		–0.058	0.279	0.037	0.249	0.111	0.365	0.123	–0.096
South	–0.059		0.127	–0.056	0.158	0.294	0.175	0.362	–0.002
Dan	0.214	0.097		0.295	0.119	0.176	0.120	–0.093	0.074
Tel Aviv	0.026	–0.039	0.273		0.125	0.199	–0.046	0.058	0.404
Sharon	0.177	0.111	0.110	0.126		0.125	0.146	0.026	0.179
Center	0.095	0.249	0.196	0.241	0.150		–0.099	0.130	0.038
Haifa	0.343	0.163	0.147	–0.061	0.193	–0.109		0.188	0.166
North	0.148	0.470	–0.159	0.108	0.048	0.198	0.210		–0.032
Jerusalem	–0.094	–0.002	0.095	0.561	0.248	0.044	0.173	–0.024	

Table 3 Estimates of Spatial Autoregressive Model Coefficients and $\sigma(\varepsilon)$

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Coefficient	0.726	0.721	0.949	1.027	1.023	0.848	0.772	0.554	0.739
$\sigma(\varepsilon)$	0.072	0.075	0.027	0.043	0.056	0.077	0.117	0.159	0.057

However, if n is relatively large, T must be correspondingly larger because the moment estimator requires that $T > n$. This means that the moment estimator is most probably not feasible if n is large.

The estimated W matrix for the special case is reported in Table 2 for when V is the covariance matrix for first differences ($d = 1$) of the data. The elements of the estimated W matrix range between –0.16 and 0.56, and the estimated W is asymmetric. Although some cases have near symmetry, for example, between South and Krayot, the elements of W are symmetric for most cases. Take, for instance, the weight linking Jerusalem with Tel Aviv: it is 0.40 and the weight linking Tel Aviv with Jerusalem is 0.56. Therefore, Jerusalem is more connected to Tel Aviv than Tel Aviv is to Jerusalem. Notice that these asymmetric weights are positively correlated. Note also that some elements are negative.[12] We see no reason why spatial weights must be positive, because spatial units may have "good" neighbors and "bad" neighbors.[13] However, no pair has w_{ij} with the opposite sign to w_{ji}, although no inherent reason exists why this should not arise.

Neither here nor elsewhere do we try to interpret the relative orders of magnitude of the estimated elements of W because our main purpose is to demonstrate the methodology. According to Table 2, the strongest spatial connectivity occurs between Jerusalem and Tel Aviv, between Haifa and Krayot, and between North and South. These connections suggest that imposing W a priori in terms of, for example, distance and contiguity would be quite inappropriate and misleading.

Table 3 reports the spatial autoregression coefficients and the variances of ε for each of the nine regions. The spatial autoregression coefficients range between 0.55 and 1.03. Because spatial weights are row summed to 1, spatial autoregression coefficients must be less than one for stationarity and the invertibility of $I - BW$. As mentioned in the "Data" section, invertibility does not require that all the spatial autoregression coefficients be less than one. Nevertheless, the panel unit root tests clearly indicate that the data are stationary in log first differences. The standard errors range between 0.03 and 0.16, which when expressed as percentages correspond to 2.74% and 17.23%, respectively. The spatial autoregression model fits best in Dan and worst in the North. Substantial heterogeneity exists in the spatial autoregression coefficients, and some of these exceed one. Unit spatial autoregression coefficients induce spatial unit roots within spatial units.[14] However, the majority of the spatial autoregression coefficients being less than one ensures that the data as a whole are stationary.

Letting the data "speak for themselves" seems to lead to quite different estimates of W than distance alone might suggest. Indeed, some of the spatial weights are negative, which cannot arise in conventional models. Therefore, these empirical estimates of W are very different from conventional definitions of spatial connectivity. Although the spatial weights are asymmetric, spatial impulse responses or spillover effects are symmetric by definition in the special case. The spatial spillover effect of unit j on unit i is defined as $g_{ij} = \frac{\partial y_i}{\partial \varepsilon_j} = \beta_i w_{ij}$. The assumption of heterogeneous mutuality means that $g_{ji} = g_{ij}$. For example, the spillover effect from Tel Aviv on Jerusalem is $0.74 \times 0.56 = 0.41$, which equals the spillover effect from Jerusalem to Tel Aviv. The spatial spillover effect will be negative if w_{ij} is negative because the spatial autoregressive model coefficient is positive. For example, in the case of the spatial spillover from the Dan region on the North, and vice versa, the spatial spillover is –0.15.

Bootstrapping

Had the population covariance matrix (V) been known, its $\frac{1}{2}n(n-1)$ elements would solve the unknown population parameters in the special case (BW and Σ). The population variances of BW may be calculated in the normal way using Σ. The sample covariance matrix is finite because T is given (36 in our case). However, because n is fixed at nine in our case, the sample covers all spatial units in Israel. Therefore, we have $nT = 324$ data points to estimate 90 parameters (72 elements of W, and 9 elements each of B and Σ), which leaves sufficient degrees of freedom. We use the panel bootstrap[15] to compute the standard errors of the estimated components of W and the spatial autoregression coefficients for the case when $d = 1$. This procedure draws samples from the residuals of the estimated spatial autoregression model (ε). Because these residuals are spatially uncorrelated, we do not have to take direct account of spatial dependence in the bootstrap; this dependency is taken into consideration by the spatial autoregression model itself. Therefore, the spatial dependence in the sample data is appropriately incorporated into the bootstrapping exercise. We used 1,000 replications.[16] Because the bootstrapped means differ slightly from the estimates reported in Tables 2 and 3, we report in Tables 4 and 5 the means as well as the standard deviations of the bootstrapped parameters.

Table 4 indicates that the elements of W are estimated imprecisely. Indeed, most of the elements of W are not statistically significant. We highlight in bold the eight elements of W that have means at least twice as large as their corresponding standard deviations. Notice that none of the negative spatial weights is statistically significant. By contrast, all the spatial autoregression coefficients reported in Table 5 are statistically significant.

Conclusion

We propose a nonparametric moment estimator, designed for spatial panel data, to estimate the spatial connectivity matrix W in heterogeneous spatial autoregression models. Normally, W is

Table 4 Bootstrapped Means and Standard Deviations of W

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot		–0.098	0.267	0.019	0.264	0.119	**0.378**	0.118	–0.069
		0.188	*0.138*	*0.120*	*0.157*	*0.193*	*0.173*	*0.168*	*0.161*
South	–0.102		0.140	–0.057	0.149	**0.361**	0.148	**0.369**	–0.007
	0.193		*0.143*	*0.128*	*0.180*	*0.162*	*0.179*	*0.131*	*0.204*
Dan	0.212	0.107		**0.332**	0.106	0.133	0.152	–0.109	0.068
	0.114	*0.105*		*0.124*	*0.110*	*0.102*	*0.082*	*0.072*	*0.118*
Tel Aviv	0.009	–0.045	**0.308**		0.119	0.186	–0.048	0.053	**0.416**
	0.089	*0.089*	*0.106*		*0.096*	*0.101*	*0.067*	*0.054*	*0.091*
Sharon	0.191	0.103	0.101	0.118		0.155	0.130	0.014	0.188
	0.105	*0.124*	*0.104*	*0.095*		*0.111*	*0.113*	*0.076*	*0.103*
Center	0.110	**0.305**	0.152	0.224	0.182		–0.075	0.075	0.026
	0.177	*0.129*	*0.118*	*0.120*	0.128		*0.189*	*0.144*	*0.177*
Haifa	**0.353**	0.150	0.182	–0.068	0.177	–0.097		0.172	0.130
	0.123	*0.172*	*0.083*	*0.092*	*0.149*	*0.210*		*0.142*	*0.150*
North	0.195	**0.515**	–0.191	0.101	0.028	0.120	0.251		–0.019
	0.321	*0.210*	*0.124*	*0.106*	*0.146*	*0.245*	*0.209*		*0.185*
Jerusalem	–0.076	–0.010	0.096	**0.576**	0.252	0.033	0.135	–0.005	
	0.159	*0.198*	*0.161*	*0.131*	*0.130*	*0.203*	*0.161*	*0.125*	

Note: Special case, $d = 1$. Italicized numbers indicate standard errors. Bold items exceed two standard errors.

Table 5 Bootstrapped Means and Standard Deviations of B

Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
0.763	0.727	0.957	1.022	1.001	0.847	0.778	0.540	0.740
0.138	*0.087*	*0.069*	*0.081*	*0.096*	*0.099*	*0.133*	*0.124*	*0.103*

Note: See Table 4.

imposed rather than estimated by an investigator. Our proposal joins recent suggestions to estimate W rather than to impose it. However, our proposal differs. The basic insight is that the variance-covariance matrix of the panel data contains information about latent spatial dependence.

If W is symmetric, then an identification deficit of one exists because the data have insufficient moment conditions to identify all the parameters in a spatial autoregression model. However, in a special case of mutual heterogeneity, the parameters are exactly identified. Therefore, we present results for the special case. Solving the parameters from the moment conditions involves finding solutions to a relatively large number of simultaneous equations that happen to be nonlinear polynomials. Thanks to the hierarchical structure of the special case; however, no difficulty exists in obtaining solutions.

We used panel data for regional house prices in Israel in nine regions to illustrate the methodology. The special case involves inversion of an n-by-n matrix and the solution to $\frac{1}{2}n(1 + n)$ nonlinear second-degree polynomials. Of course, matrix inversion is feasible for n considerably greater than nine. Our experience with MATLAB indicates that the solution to nonlinear polynomials with n considerably greater than nine also is feasible.

Notes

1 This approach is driven by a gravity-type notion of how proximity affects interaction using differing metrics for distance decay such as inverse distance raised to a power (Getis and Aldstadt 2004) and bandwidth distance decline (Fotheringham, Brunsdon, and Charlton 2002).
2 Aldstadt and Getis (2006) use an algorithm that searches for spatial clustering in the vicinity of selected seeds and constructs a data-driven empirical representation of W. Mur and Paelinck (2011) group correlation coefficients using an optimization algorithm so that correlated observations are also related spatially. Griffith (1996) suggests spatial filtering. The principal eigenvector of W provides a measure of the relative positioning of each spatial unit and expresses the general degree of connectivity between spatial units. Several studies present tests of the conditions under which spatial filters are statistically significant (Getis and Griffith 2002; Dray, Legendre, and Peres-Neto 2006; Tiefelsdorf and Griffith 2007).
3 This is the case to which Anselin (1988) refers in the opening quotation.
4 The direct spillover effect of unit j on unit i is defined as $g_{ij} = \partial y_i/\partial \varepsilon_j$.
5 The spatial autoregression model is not recursive because no unconnected spatial units are present. However, the solutions for B and W are recursive because they are derived from the solution for G.
6 The Slutzky theorem states that $p\lim[f(x)] = f[p\lim(x)]$. Also $\mathrm{p}\lim(A') = \mathrm{p}\lim(\mathrm{A})'$.
7 Because the Slutzky theorem implies $p\lim\left(\hat{V}^{-1}\right) = V^{-1}$, equation (7) implies $p\lim\left(\hat{G}\right) = G$, and $p\lim\left(\hat{\Sigma}\right) = \Sigma$.
8 In equation (1), ε_{it} might be correlated with ε_{it-1} and ε_{kt-1}.
9 See, for example, Spanos (1986) and Davidson and MacKinnon (1993).
10 The estimates of B and W depend upon the data. This is true of any estimate. However, this does not mean that the estimates are affected by simultaneous equations bias.
11 These unit root tests ignore spatial dependence. However, provided the SAC coefficient is not too large, the size of these tests is not seriously affected. See Baltagi, Bresson, and Pirotte (2007).
12 Meen (1996) and Bhattacharjee and Jensen-Butler (2006) also report negative elements in W.
13 In epidemiological models, contagion may be negative and positive (Aron 1983). Neighbors may be contagious as well as immunizing.
14 Unit root tests for spatial autoregressive model coefficients are discussed by Beenstock, Feldman, and Felsenstein (2012).
15 Bootstrapping is simpler here than in Bhattacharjee and Jensen-Butler (2006) because, apart from W, they estimate other structural model parameters. However, they do not use the panel bootstrap.
16 See Andrews and Buchinsky (2000) regarding the desirable number of bootstrap replications.

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