Strategic experimentation in patent races*

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Abstract

We study a patent race between two firms as a two-armed bandit model. The first firm that successfully completes two phases (R&D) acquires a patent license. Each firm can learn from its rival's actions and outcomes. We show that two possible inefficiencies can be observed in equilibrium. On the one hand, spill-overs of information reduce the expected profitability of the patent and therefore firms may invest inadequately in , in the fear of releasing good news to the market. On the other hand, an opposite, "tragedy-of-the-commons", effect may prevail, according to which investment is socially excessive.

1 Introduction

■ One of the main questions in "*patent races*" concerns the level of R&D activity. Ever since Kamien and Schwartz (1972, 1976) and Loury (1979), many studies have stressed the inefficiencies caused by competition in this type of races. A central result they establish is that aggregate investment in R&D is usually excessive, which relies on the well-known "tragedy-of-the-commons" effect. Nonetheless, one can imagine situations in which an opposite inefficiency prevails. For, if the competing firms can learn from each other's activities, information spill-overs could cause a "free-riding" effect. Our main contribution is to model a patent race in which both effects are possible equilibrium phenomena.

As an example, consider the development of a new medicine. If the pharmaceutical industry is highly concentrated, each company has almost perfect information with regards to the R&D activities undertaken by the other companies. Therefore, the release of good news by some company could spark excessive competition and therefore decrease the expected profitability of the patent for the medicine. Each company, rationally expecting that, would refrain from undertaking the risky venture.

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In this article, we attempt to model a simple multistage patent race in order to stress this type of information externalities. By building on the literature of "*strategic experimentation*", started by Bolton and Harris (1999), we analyse a dynamic race between two firms using two-armed bandit technologies. Our model is meant to capture possible inefficiencies that are caused by information externalities (spillovers) as well as by strategic externalities as those discussed in the already existing literature.

The race is between two firms. The winner of the race is the firm that completes two phases: Research & Development. The winner acquires a patent and therefore fixed monopoly profits for a long period of time. For instance in the development of a new medicine, the research phase may refer to the identification of the mechanisms that cause a certain disease, as well as the substances that may control its symptoms or help the immune system in the fight against it. On the other hand, the development phase may refer to the trials of the medicine in animal or human subjects before the new product is released to the market. Both firms start the race equipped with similar bandit technologies which are all armed in the same state (either good or bad). However, successes occur independently. In fact, one success is enough for each stage to be completed.

In the research phase, the technologies can be either armed in a good state, and generate a success with positive probability, or in a bad state, and never generate a success. Therefore, with some positive probability, the research phase can never be completed successfully, regardless of the efforts of the two firms. In the medicine example discussed above, this idea captures the uncertainty that pharmaceutical companies usually face when they start investigating the possible causes of a disease. On the contrary, and without loss of generality, we assume that in the development phase, the technologies are always armed in a good state (and therefore known to be able to generate successes) but successes occur only with some positive probability.

The central results we establish are the following: In the research phase, the two firms may *under-invest* relative to the socially efficient level of investment because of information spillovers. After the release of a success from one firm in the research phase, information becomes a public good. At that point, the lagging firm realises that advancement to the development phase is feasible and therefore has an incentive to get back into the race in order to catch up and pass the finish line first. This reduces the profitability of the leading firm and each firm, rationally expecting that, decides not to experiment at all in the research phase.

On the other hand, the *over-investment* (or duplication of efforts) effect that has been widely stressed in the R&D literature can also be uncovered in equilibrium. In our model, this is translated to duplication of efforts (investments) compared to social optimality. In fact, this effect can be realised in any phase but it is more detrimental towards the end of the race, or in other words, during the development phase. As we show, there are cases in which both firms invest, even though social optimality requires only one of them to invest. Thus, while the current R&D literature identifies the inefficiency involved in over-investment as a result of externalities ignored by the rival firms, and the experimentation literature identifies the fundamental inefficiency of information acquisition because of free-riding, in our model both of these forces are at play.

□ **Related Literature.** Loury (1979), Lee and Wilde (1980) and Dasgupta and

Stiglitz (1980) analyse static strategic patent races in which investment in R&D is irreversible and takes place only once. Each firm's investment determines its speed of innovation. Loury (1979) and Lee and Wilde (1980) do not explicitly model a market after the innovation occurs. They are mostly interested in the individual and aggregate levels of R&D investment as well as the degree of market concentration. On the contrary, Dasgupta and Stiglitz (1980) explicitly model the market after the innovation. They proceed to comparative static analysis in order to stress the effect market demand has in the innovation process. One of the central results of all these articles is that aggregate investment is socially excessive.

Reinganum (1981,1982), Fudenberg et al. (1983) and Harris and Vickers (1985,1987) extend the above static models to dynamic models in which firms have the right to change their level of investment during the race. Even though they make severe restrictions of how firms can alternate their investments, the analysis seems turns out to be complicated. Reinganum (1981,1982) analyse a race in which each firm accumulates knowledge through time by investing in R&D. Innovation is stochastic and the time of successful completion increases with the stock of relevant knowledge. Part of Reinganum's contribution is technical since this game is a representative differential game with many technical difficulties. When knowledge is a private good, i.e. it can only be used by the firm who accumulates it, then Reinganum shows that in equilibrium there is over-investment and early innovation. On the other hand, when knowledge accumulated by one firm can be used by other firms, then there is a free-riding problem and therefore under-investment. Fudenberg et al. (1983) analyse a multistage patent race. They provide conditions under which a race will be characterised by vigorous competition or will degenerate into monopoly. The key idea is whether the lagging firm in the race has time to "leapfrog" and catch up. In Harris and Vickers (1987), it is shown that the leader in the race exerts higher effort than the follower, and effort increases as the gap between competitors decreases.

Grossman and Shapiro (1987) is perhaps closer to our approach. They extend the model of Lee and Wilde (1980) by adding one more phase in the development of the project. Therefore, a firm, in order to win the race, must, as in our model, complete two phases R&D. Firms have complete information about each other's actions. In their model, there is over-investment in equilibrium relative to the socially efficient level of investment. The main difference from our model is that we introduce uncertainty in the bandit technology of the research phase and therefore informational externalities. This allows us to stress the fundamental under-investment in the research phase along with the over-investment in the development phase.

Lastly, there is by now a vast literature on strategic experimentation started from Bolton and Harris (1999). Bolton and Harris (1999) analyse a two-armed bandit model with several agents competing. Each player can learn from the actions and outcomes of the other players and therefore, similarly to our approach, information becomes a public good. They show that, in equilibrium, a free-riding effect arises. Similarly to our approach, experimentation in their article is below the socially efficient level. Keller, Rady and Krips (2005) analyse a similar model but with exponential bandits. The use of exponential bandits in contrast to Bolton and Harris (1999) allows for analytical tractability and therefore Keller, Rady and Krips (2005) uncover symmetric equilibria. Our model is way simpler than Bolton and Harris (1999) and Keller, Rady and Krips

(2005) and the focus is both on free-riding as well as over-investment. Bergemann and Hege (1998) and Bergemann and Hege (2005) introduce the two-armed bandit technologies into a dynamic moral hazard model.[1] In their model an entrepreneur seeks financing in order to run experiments needed to complete a project. The technologies are similar to our available technologies in the research phase. They show that because of moral hazard, and funds diversion, investment is relatively low compared to the socially efficient counterpart and financing stops prematurely. In contrast to our model, those models has only one firm experimenting, relying however to external financing to run the experiments. In our model there are at least two firms experimenting and they self-finance their experiments.

In Section 2 we present the model and notation. In Section 3 we analyse the equilibria of the game and we show the possible inefficiencies that may be cause in the level of R&D activity.

2 The Model

■ **Timing and Payoffs.** Time is discrete and infinite: $t \in N$. There are two firms, denoted by $i = \{1, 2\}$, that are engaged in an patent race. Both firms are risk-neutral, Bayesian expected utility maximisers. There is no discounting. There are two phases to be completed in order for a firm to be able to acquire the patent, which can be considered as Research and Development (R&D). Completion of every phase requires investment (or effort), and the fixed investment cost to be paid every period is c. The first firm that successfully completes both phases receives a patent that allows it to receive monopoly profits indefinitely equal to V (in expectation).[2,3] As soon as some firm succeeds in the research phase, it can proceed to the last (and final) phase of development. Even if one firm passes the research phase, the other firm has still time to invest and potentially qualify to the development phase as well. In case both firms finish at the same time, then each firm receives the patent with equal probability.[4] Note that for a firm to qualify to the development phase, it is a prerequisite to first pass the research phase.[5] Each firm observes the actions and outcomes of the rival firm, therefore the game is considered as one of complete information.[6]

[1] See also Horner and Samuelson (2012).

[2] We assume that V is not observable or verifiable by any outsider apart from the two firms. This simply means that no explicit contract can be written between the two firms contingent on the breakthrough since its value is not verifiable by a court of law.

[3] The cost c can be thought either as a monetary cost or a disutility cost of effort. This distinction is irrelevant in our model.

[4] Alternatively, we could assume that in case both firms succeed at the same time then no one receives the patent and therefore they have to "split the market". Obviously, we do not model a market for the good after the invention. However, all our results would remain the same in case the demand for the good is linear and firms engage in "Bertrand competition" after they finish at the same time or they just split the profits in half as we assume in the model.

[5] This assumption is easily justifiable if we consider the project as a product where both phases must be completed in order to be sold to the market. If one phase has not been completed then the product is considered as problematic and it has no market value.

[6] Given that the two firms are specialising on the same product, they have inside information of each other's actions.

□ **Notation and Technologies.** In every phase there is a technology that can be used to provide the breakthrough needed, and this is available to both firms. The technology in the research phase is uncertain and risky. When both firms are in the research phase, each one of them decides either to use the technology or not, but, as we mentioned above, this is costly and the fixed monetary amount to be paid is constant and denoted by c. On the other hand, there is always the possibility of no experimentation at zero cost. In this case, the probability of success is zero. There is uncertainty regarding the suitability of the technology to provide a success in the research phase when in use. Specifically, the technology can be either in a good state with probability α or in a bad state with probability $1-\alpha$. If in a good state, the technology, after experimentation in the research phase, can generate a success with probability θ or a failure with probability $1-\theta$. On the other hand, if in a bad state, the technology can only generate a failure with probability 1. If no success takes place after one round of experimentation, then the posterior is updated downwards. If a success occurs then the posterior is updated to one since success is a fully informative signal.

On the contrary, the technology in the development phase is risky but not uncertain. We assume that by investing c, each firm it can realise a success with probability η and a failure with probability $1-\eta$. Therefore, there is a fundamental difference between the available technologies in the two phases. In the development phase the probability of success in each stage is fixed every period. In the research phase the probability of success is not fixed but can vary depending on the outcome of experimentation. This captures the fundamental uncertainty firms face in the initial stages of R&D activity.

$W_X^n(Y,Z)$ denotes the individual value of the project for a firm in phase $X \in \{R,D\}$ if there are n firms active in some period, one firm is in phase $Y=\{R,D\}$ and the other one in phase $Z=\{R,D\}$.

3 Equilibrium in Research and Development

It is easy to see that, due to the assumption of no discounting, it can never be socially optimal to operate both technologies in any of the periods, in any of the phases. This is because the social cost of operating one technology, given that the other one is active is c, and the social benefit is zero (the payoff is zero if one succeeds given that the other succeeds). In fact, if both technologies, are operated by a benevolent social planner possessing the same information as the two firms, it is strictly dominant to operate one technology after the other instead of both together.[7] A we will show, this is not true when the two firms operate independently as there are equilibria at which they are both active. The assumption of no discounting is without loss of generality. All our results hold even if the discount factor is strictly less than one.

A. Both Firms in the Development Phase

Assume that both firms have qualified to the development phase. We will first examine the socially efficient investment level and we will compare this with the individual investment levels.

[7] We could equally assume that both firms merge and operate together.

First assume that both technologies, being in the development phase, are operated by a benevolent social planner possessing the same information as the two firms. As we claimed earlier, it is never socially optimal to operate both technologies in the same period. Therefore we will examine when it is optimal to operate one technology at a time. Note that because of the stationarity of the model, if it is ever optimal to operate one technology in some period, then it will be optimal in any period thereafter. Therefore, it is socially efficient to invest in one technology in the development phase if and only if:

$$\eta V \geq c$$

We can define a cutoff point in the profitability of the project, such that it is socially efficient to operate one technology, if and only if the "payoff-to-cost" ratio is weakly above this cutoff point, or:

$$\frac{V}{c} \geq \frac{1}{\eta}$$

This is a general necessary condition for any of the technologies to be active. In any other case, i.e. $\frac{V}{c} < \frac{1}{\eta}$, it is never socially optimal to operate any of the technologies in the development phase.

We can easily establish that the social value of the project in any period t if at least one technology is in the development phase is given by:[8]

$$V - \frac{c}{\eta}$$

Turning our interest to the individual behaviour of the two firms being both in the same phase, note that they will both be active if and only if:

$$[\eta(1-\eta) + 0.5\eta^2)]V - c \geq 0$$

Perhaps, as we did above, we could define a cutoff point in the profitability of the product, such that both firms will be certainly active in the development phase, as:

$$\frac{V}{c} \geq \frac{1}{\eta(1-0.5\eta)}$$

In case the profitability of the product is such that:

$$\frac{1}{\eta(1-0.5\eta)} > \frac{V}{c} \geq \frac{1}{\eta}$$

then only one firm will be active in equilibrium. Even though there are equilibria in which the identity of the firm investing is determined randomly, we will concentrate, without loss of generality, in equilibria in which the identity of the firm who invests remains the same.[9]

[8] The expected value is $\eta V - c + (1-\eta)(\eta V - c) + (1-\eta)^2(\eta V - c) + ... = \frac{\eta V - c}{1-(1-\eta)} = V - \frac{c}{\eta}$

[9] There are two symmetric SPNE: If each firm believes that the other one will invest, then it is best response for it not to invest. The subgame is similar to the "Battle-of-Sexes" game. All along we will not consider mixed-strategy equilibria. We only examine equilibria in which both firms play degenerate strategies.

The value of any firm, if both firms are active in the development phase is denoted by $W_D^2(D,D)$ and is given by:

$$W_D^2(D,D) = \eta(1-0.5\eta)V - c + (1-(1-\eta)^2)W_D^2(D,D)$$

or

$$W_D^2(D,D) = \frac{\eta(1-0.5\eta)V - c}{1-(1-\eta)^2}$$

On the other hand, the value of the only active firm in the development phase is denoted as $W_D^1(D,D)$ and is given by:

$$W_D^1(D,D) = V - \frac{c}{\eta}$$

Given that it is never socially optimal both technologies to be active, but as we saw there are cases where both firms acting independently are indeed active, we can easily deduce that there may be an inefficient allocation of resources in equilibrium. Note that both firms would be better off using only one technology, bear one cost and share the payoff with equal probability.[10]

Proposition 1. *If both firms have advanced to the development phase and* $\frac{V}{c} \geq \frac{1}{\eta(1-0.5\eta)}$, *then there is over-investment in equilibrium.*

The above result is relatively intuitive. Over-investment occurs in the last phase because in probability η^2 both firms make a breakthrough and one of the costs c is, in this event, a social waste which the firms do not internalise. The externalities caused by both firms to each other lead to socially inefficient levels of investment.

B. One Team in the Research and the Other in the Development Phase

In this section, we assume that one firm has already qualified to the development phase and the other one is still lagging in the race. Given the success of the the leading firm however, the firm that is now lagging knows that advancing to the development phase is possible or $\alpha = 1$.

First, given that one technology has qualified to the development phase, it is never socially optimal to invest in the technology that is still in the research phase. However,

[10] Of course, it is assumed that no explicit contracts can be written that allow cooperation, exactly as in the Cournot-Nash equilibrium or any game in which the Nash equilibrium is inefficient. We can further justify this assumption on the following “incomplete contracts reasoning": Assume that the payoff of successfully developing the product and selling it in the market is X, distributed in the interval $[0, A]$ with a cdf function $F(X)$. Moreover, $\mathbf{E}(X) = V$ where $\mathbf{E}$ is the mean expectational operator of the distribution. Therefore, both firms believe that the ex-ante expected payoff of selling the product in the market is V, but the true payoff can be anything from 0 to A. The true payoff can be only realised when the product is developed. Assuming that no outsider can observe or verify the true payoff, the firm that develops the product and promises to deliver half the payoff to the other firm will never do so by claiming that the realised payoff is zero. Given that this is not verifiable, no firm will agree to pay half the cost to the other firm and then expect to receive half the payoff after realisation of uncertainty in the fear of being “held-up". This ex-post opportunistic behaviour makes cooperation impossible and therefore leads to inefficient Nash equilibria.

from an individual point of view, there are SPNE depending on the parameter values, in which, in some cases, the firm that is still in the research phase invests.

Even though there may be equilibria in which the leading firm stops investing when the lagging also advances to the development phase, we find as more reasonable equilibria those at which the leading firm never does. Given this, it is easy to verify that if $\frac{1}{\eta(1-0.5\eta)} > \frac{V}{c} \geq \frac{1}{\eta}$, then it is not optimal for the lagging firm to invest. This is because in this case, only one firm will be active in the development phase. Therefore, a necessary condition for the lagging firm to be active is $\frac{V}{c} > \frac{1}{\eta(1-0.5\eta)}$. The expected value of the firm that is in the research phase, while the other firm is in the development phase and they are both active is denoted by $W_R^2(R,D)$. This value can be calculated as:

$$W_R^2(R,D) = (1-\theta)(1-\eta)W_R^2(R,D) + \theta(1-\eta)W_D^2(D,D) - c$$

or

$$W_R^2(R,D) = \frac{-c + \theta(1-\eta)\frac{\eta(1-0.5\eta)V-c}{1-(1-\eta)^2}}{1-(1-\theta)(1-\eta)}$$

We can see that as long as this value is positive, or $\theta(1-\eta)\frac{\eta(1-0.5\eta)V-c}{1-(1-\eta)^2} \geq c$, then in the unique SPNE the lagging firm always invests because it is always profitable to do so.

As previously, we can find a cutoff in the profitability of the project such that there is an inefficiency because the firm that has lagged in the race does not internalise the externality that imposes to the other firm. This cutoff is given by:

$$\frac{V}{c} \geq \frac{1-(1-\eta)^2 + \theta(1-\eta)}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$$

We conclude this section with the following proposition that highlights the inefficiencies caused in the Development phase.

Proposition 2. *If one firm has advanced to the development phase, the other is still in the research phase and* $\frac{V}{c} \geq \frac{1-(1-\eta)^2+\theta(1-\eta)}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$ *there is over-investment in equilibrium.*

For later results, it is useful to also characterise the value of the firm that is leading the race- i.e. the firm that is in the development phase given that the other firm is in the research phase- when both firms are active. This is denoted by $W_D^2(R,D)$ and is given by:

$$W_D^2(R,D) = \eta V + (1-\theta)(1-\eta)W_D^2(R,D) + \theta(1-\eta)W_D^2(D,D) - c$$

or

$$W_D^2(R,D) = \frac{-c + \eta V + (1-\eta)\,\theta\left(\frac{1}{2}V - c - \frac{c}{2\eta-\eta^2}\right)}{1-(1-\theta)\,(1-\eta)}$$

Vacuously, the value of the leading firm, when the lagging firm is not active any more is denoted by $W_D^1(R,D)$ and equals the social value or $W_D^1(R,D) = V - \frac{c}{\eta}$.

C. Research Phase

We are now ready to analyse the socially optimal behaviour as well as the equilibrium behaviour when both firms (or technologies) are in the research phase. We know that if one of the two firms advances to the development phase, then the equilibrium is given in Subsection B. On the other hand, in case both firms advance to the development phase, then the equilibrium is given in Subsection A. In the two previous cases, we saw that over-investment is possible, since there are parameter values according to which only one technology should be active, but both are, in equilibrium. In this section we would like to establish that if both firms are in the research phase, another fundamental inefficiency commonly observed in patent races is possible. This is the well known *under-investment*. Over-investment is possible in this model since success in the research phase by one firm releases good news to the rival firm. Information therefore becomes a public good and a free-riding effect arises in equilibrium. On the other hand, it is possible to have over-investment in this phase as well, for some parameter values. Thus, in the research phase, both over-investment as well under-investment are possible equilibrium phenomena.

Given the uncertainty about the quality of the technology in the research phase, one needs to specify the evolution of the belief after experimentation. There are three possible cases to consider: If no firm experimented, then it is apparent that no updating takes place and the posterior probability remains the same (as the prior). If, however, one firm experimented and succeeded in some period then $\alpha = 1$, since success is a fully informative signal. On the other hand, if some firm experimented and failed then this brings bad news regarding the suitability of the technology to complete the state and the posterior probability is downgraded, by Bayes' rule, to:

$$\alpha' = \frac{\alpha(1-\theta)}{(1-\alpha)+\alpha(1-\theta)} < \alpha$$

Note that the behaviour of the two firms in the research phase crucially depends on the continuation value of the project, as this has been determined in the previous two sections. It is common sense to note that the more lucrative the project is (α is higher ceteris paribus), the higher the experimentation will be in the research phase. Note that even in this phase, it is never socially optimal to experiment in both technologies for the same reason it was never optimal in the development phase. It seems rather complicated to analyse the equilibrium given that there are too many cases to be considered in the continuation of the game as these have been examined in the two previous sections. To stress the result of the article as clear as possible we will analyse only the two most interesting cases.

The first case is when $\frac{V}{c} < \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$. From Subsection B, we know that as long as one firm advances to the development phase, the firm that remained behind will not experiment anymore. For some parameter values, both firms would be active in the development phase, if both advanced to it. Thus, it is rather straightforward to see that the two firms compete in the research phase exactly as they do in the development phase in this range of parameters. In other words, the first firm that advances to the development phase is considered the "winner" of the phase. We should, therefore, expect that over-investment is possible in this phase for the exact same reason that it was possible in the development phase. The two firms do no internalise the externalities impose through their actions. Indeed, as we prove in the following lemma, over-investment is the only source of externality in this range of parameters.

Lemma 1. *If both firms are in the research phase, $\frac{V}{c} < \frac{1}{\eta(1-0.5\eta)}$ and $\alpha\theta(1-0.5\theta)W_D^1(D,D) \geq 0$ then there is over-investment in equilibrium.*

Proof. First, note that when $\frac{V}{c} < \frac{1}{\eta(1-0.5\eta)}$, from the analysis above, the lagging firm will not invest. Therefore, in this range of parameters the race in the research phase is similar to the race in the development phase. The winner is the first one that succeeds in the research phase. If both succeed simultaneously then we assume that each firm has equal probability to keep on investing in the development phase. The other firm becomes inactive.

Substituting for $W_D^1(D,D) = V - \frac{c}{\eta}$, the question boils down to whether

$$\frac{c}{\theta(1-0.5\theta)(V-\frac{c}{\eta})} \leq 1$$

Rearranging, we can find that over-investment occurs if and only if

$$\frac{V}{c} \geq \frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)}$$

To complete the proof, we have to check that for some parameter values

$$\frac{\theta(1-\eta)+1-(1-\eta)^2}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)} > \frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)} > \frac{1}{\eta(1-0.5\eta)}$$

For the first inequality, by rearranging we get that

$$(1-(1-\eta)^2+\theta(1-\eta))(1-0.5\theta) > (1-\eta)(1-0.5\eta)$$

which for each value of η is a quadratic equation with respect to θ. One can easily see that there are parameter values that this inequality is satisfied and moreover $\frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)} > \frac{1}{\eta(1-0.5\eta)}$ (for instance $\eta = 0.5$, $\theta = 0.9$).

□

The intuition behind the above result is that when $\frac{V}{c} < \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$, there is no room for under-investment because there is no informational externalities and therefore no free-riding effect. This is because as long as one firm remains behind in the race, the game is over because it is not profitable anymore to experiment in order to catch up.

The most interesting case, however, and one of the distinguishing features of this article, is when $\frac{V}{c} \geq \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$. Note that in this range of parameter values, it is never socially optimal to operate the technology that has lagged in the race, but it is individually optimal. However, we will show that it is possible, when both firms are in the beginning of the race to be socially optimal to operate one of the two technologies, but none to be active in equilibrium. This is indeed true as it is proved in the proposition below.

Proposition 3. *If both firms are in the research phase and:*

1. $\frac{V}{c} \geq \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$
2. $\alpha\ \theta\ (\eta V - c + (1-\eta)(V - \frac{c}{\eta})) - c \geq 0$
3. $\alpha\ \theta\ [(\eta V - c) + (1-\eta) W_D^2(R, D)] - c < 0$

then there is under-investment in equilibrium.

Proof. Assume that (1) holds. We will show that there are parameter values such that (2) and (3) hold. If both equations hold for some range of parameters, then it is necessarily the case that under-investment happens in equilibrium. The question boils down to whether there are parameter values such that $V - \frac{c}{\eta} > W_D^2(R, D)$. By manipulating $W_D^2(R, D)$, we can find that

$$\begin{aligned} W_D^2(R,D) &= \frac{-c + \eta V + (1-\eta)\,\theta\left(\frac{1}{2}V - c - \frac{c}{2\eta-\eta^2}\right)}{1-(1-\theta)\,(1-\eta)} \\ &= \frac{V\left(\eta + \frac{\theta(1-\eta)}{2}\right) - c\left(1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}\right)}{\eta + \theta - \theta\eta} \\ &= \frac{\eta + \frac{\theta - \eta\theta}{2}}{\eta + \theta - \theta\eta}\left(V - c\frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta(1-\eta)}{2}}\right) \end{aligned}$$

Let

$$\gamma = \frac{\eta + \frac{\theta-\eta\theta}{2}}{\eta + \theta - \theta\eta}, \quad \beta = \frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta-\eta\theta}{2}}$$

Thus, our goal is to show that

$$V - \frac{c}{\eta} > \gamma[V - \beta c]$$

Given that $\frac{\eta+\frac{\theta-\eta\theta}{2}}{\eta+\theta-\theta\eta} < 1$, for any $\eta \in (0,1)$, it is sufficient to show that

$$\frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta-\eta\theta}{2}} > \frac{1}{\eta}$$

which holds for all $\theta \in (0,1)$ and $\eta \in (0,1)$. □

The intuition behind the above result is the following: Assume that the prior probability α is small enough such that it is socially optimal for only one technology to be active in the research phase and none to be active after a failure. It may be that because the firm that decides to invest in some period in the research phase knows that the other firm will get into the race after a success refrains from doing so. Therefore, for α small enough, it may be socially optimal for one firm to be active in some period, but none is in equilibrium. This is a well-known free-riding effect commonly observed in practice, especially in patent races. When some research firm is planning to invest in the development of some product (drug, high-technology product, etc.) and cannot control the information to be released to the competitors, it may decide not to do so. Research firms refrain from developing new risky products, in the fear of vast competition after the announcement of good news that would decrease profitability of the product considerably.

4 Conclusion

In this article we studied a two-stage patent race as a two-armed bandit model. Two firms competed on the acquisition of a patent but they first had to pass two phases (Research & Development). We analysed the SPNE of this dynamic game and compared the levels of investment of the two firms with the socially optimal counterparts. We showed that, in equilibrium, two possible inefficiencies are likely to occur. On the one hand, in the initial phase (research), there were parameter values such that under-investment could occur in equilibrium. Even though it was socially optimal for at least one firm to be active, none was in equilibrium. On the other hand, another fundamental inefficiency, this of over-investment, that has been extensively examined in the R&D literature was uncovered in equilibrium. This is a phenomenon according to which both firms experiment, even though one of the costs of experimentation is a social waste. Over-investment could occur in any of the phases, but it was more frequent towards the end of the race.

The model we analysed was very elementary with many simplifying assumptions. In a previous draft of this article, we had analysed a more complicated model with discounting and both phases to be uncertain.[11] Even though this model was more general the analytical tractability was prohibitive. However, all our results go through even in this more general model and, hence, we traded-off generality for simplicity and tractability. In case of discounting, the planner would trade-off the time cost and wasting one cost every period (in case he experimented in both technologies). On the other hand, the individual teams would not take into account this trade-off and would only experiment as long as the expected payoff exceeded the cost (if the rival firm was active). Henceforth, it is clear that even there we could find parameter values such that over-investment could occur in equilibrium, at least in the development phase. On the other hand, in the research phase, under-investment would be possible in this model for the exact same reason under-investment can happen in the more basic model. Again we could find parameter values, such that the planner would not invest in the lagging technology after a success in the research phase but, acting individually, the lagging team would do. Therefore, for α low enough, no team would be active (even if it would be socially optimal one to be), because of the free-riding effect.

[11] By this we mean that in the development phase, the technology was either in a good or in a bad state.

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