How to share it out: The value of information in teams*

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Abstract

We study the role of information exchange, leadership, and coordination in team and partnership structures. For this purpose, we view individuals jointly engaging in productive processes—a "team"—as endowed with individual and privately held information on the joint production process. Once each team member decides on whether or not to share his private information truthfully, he chooses which effort to exert in the joint production process. This effort, however, is not contractible: only the realized output (or profit) of the team can be observed. Our central question is whether or not incentives can be provided to a team in this environment such that team members communicate their private information and exert efficient productive efforts on the basis of this communication. Our main result shows that there exists a simple ranking-based contract that implements both desiderata in a wide set of situations.

JEL: *C7, D7, D8, L2.*
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"Forming a business partnership is the next best thing to getting married."
The Manufacturing Jeweler, March 1897

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1 Introduction

This paper analyzes combined moral hazard and adverse selection problems in teams and, synonymously throughout the paper, partnerships. Team members exert unobservable effort and the generated output is entirely allocated among the members of the team. Created output depends not only on the chosen efforts but also on an underlying productivity parameter. The source of adverse selection is that the information about this parameter may be privately known to some team members. Therefore, an efficient mechanism must generate the right incentives for both information revelation and the appropriate effort exertion. We ask the following question: is there a mechanism that can accomplish both goals if some additional information on performance (e.g., a noisy ranking of the team members' efforts) is available?[1]

The presence of asymmetric, private information enables informed team members to hide potentially inefficient effort choices with their private information. Intuitively, a privately informed player can pair a biased report of private information with a suitably chosen effort to manipulate the outcome.[2] In order for such behavior to be preventable, remuneration must be based on both the realized output and all privately informed team members' reports. Since the extended model also allows players to be ex-ante asymmetric, an efficient incentive scheme must be capable of personalizing incentives for every team member according to their role in the team. This applies to both effort exertion and reporting incentives but, again, the main problem is the added possibility of a concerted, combined deviation from efficiency along both dimensions.

The principal elements of our model are private information, unobservable efforts, and team structure.[3] As organizations and teams can be seen to exist precisely in order to resolve or process informational problems (Coase, 1937; Radner, 1962; Marschak & Radner, 1972), the introduction of asymmetric information into what is otherwise a standard team production problem seems to be natural. To fix ideas, consider a situation in which only one of the otherwise identical team members—whom we call the "team leader"—receives a private signal that affects the outcome of team production. This informed team member may not find it in her best interest to reveal this information truthfully to others. Our main result—which may be surprising given the classic inefficiency results of Holmström (1982) for moral hazard in teams

[1] In the words of Groves (1973, p. 618), the problem is to induce players "to behave as a team, i.e., to send optimal information and make optimal decisions from the point of view of the organization objective." It is well known that the answer to our research question in the absence of additional information on team member efforts is negative. In particular, Holmström (1982) shows in a complete information framework that moral hazard is incompatible with efficiency if efforts cannot be observed.

[2] A privately informed player may, for example, be able to misrepresent her private information about joint productivity in order to deceive other team members into providing inefficiently high (or low) efforts while planning to capitalize on this response through a low (or high) effort herself.

[3] In team or partnership structures, partners share the profit among themselves. Thus, any incentive mechanism is subject to the constraint to balance the team's budget. Many other bilateral or multilateral contractual situations are also subject to similar implicit budget restrictions (Spulber, 2009, p. 57, p. 97).

and Hermalin (1998) for the adverse selection leadership case—shows that a team remuneration scheme exists that can overcome this "communications dilemma" and implement both efficient information sharing and subsequent efficient efforts even though efforts are not contractible. Apart from observed output and the leader's report, the derived sharing rule also depends on some statistic of exerted efforts, for instance, a contractible noisy ranking of partners' efforts interpreted as a contest among team members. This statistic is the main element that our analysis adds to the literature and it allows for a first-best solution.[4] The profit-sharing rule subdivides realized team output unevenly among all team members in symmetric equilibrium and therefore balances its budget.

An example of the applicability of the model's formal structure is a situation in which a patient receives the attention of a medical team consisting of a doctor and several nurses. An initial diagnosis yields private, asymmetric information to the doctor that can be shared before a procedure is performed. During this treatment, the team members' efforts are not necessarily verifiable. Moreover, moral hazard arises if the doctor does not pay proper attention during the medical procedure and then blames the outcome on the initial condition of the patient on whom she is privately informed. Hence, incentives may exist to provide sub-optimal efforts, especially if they are coordinated with an initially misstated opinion on the patient's needs.[5]

Our proposed sharing rule can dissuade the leader from misreporting by ensuring that, even for misrepresented private information, efficient effort provision given the leader's report remains a best reply of both the leader and the other team members. Since our profit-sharing rule is explicitly constructed to guarantee this, a combination of a misleading report with a subsequent inefficient effort is not profitable. Consequently, as the leader has incentives to report her information truthfully, the other team members may base their reply on this report, which allows for a jointly efficient set of efforts.

We generalize our efficiency result in several directions, including the cases of *i*) ex-ante asymmetric players, *ii*) private information dispersed among several players, *iii*) an information structure that generalizes over relative rankings, and *iv*) "leading by example" in such a way that the leader can exert either contractible or non-contractible upfront efforts. In all four extensions the precise formulation of the required sharing rule changes but our principal result that full efficiency is implementable is robust to these model variations. The main additional

[4] Such a noisy ranking of efforts seems to be regularly collected in the form of relative performance information and is naturally available as part of incentive schemes in many organizations (Lazear & Shaw, 2007). Moreover, since the required effort information is not necessarily cardinal, collecting these statistics represents a weaker informational requirement than what is usually embodied in standard piece-rate-based contracts.

[5] Recent evaluations of (relative) performance pay in the medical professions include Bardach et al. (2013) and Himmelstein & Woolhandler (2014). Other applications can be found in the professional services industries in which partnership structures are the dominant organizational forms. Examples include law, accounting, hedge funds, and, until recently, investment banking (Greenwood & Empson, 2003). Individual performance evaluation is used by many partnerships for promotion decisions or the allocation of bonus payments. We provide a discussion of further applications and examples in our concluding section.

insight drawn from these generalizations is that, if agents are asymmetric with respect to their impact on production, efficiency can be obtained only if private information is restricted to a single agent. In the case of multiple informed partners, their asymmetry generally does not allow them to simultaneously align their incentives.

The plan for the remainder of the paper is as follows. After a short overview of the two literatures unified by our analysis, we define our model in Section 2. Section 3 presents our symmetric efficiency result. Section 4 illustrates several extensions of the main model. In the concluding section, we discuss of a further set of applications and examples of leadership and coordination, including rival project selection and leading by example. A detailed analysis of leading by example is provided in Appendix A. Proofs of all results in the main text can be found in Appendix B. A proof ensuring the existence of the equilibrium that we derive under a broad class of specifications is given in Appendix C, together with an example illustrating equilibrium existence in further cases.

Related literature

The present paper combines two distinct literatures on team production and information-based leadership into a unified contracting framework. The classic contributions to the literature on moral hazard in teams are Alchian & Demsetz (1972) and Holmström (1982) who establish the impossibility of efficiency in team production under a budget-balancing constraint.[6] When efforts are unobservable, players have an incentive to free ride because they share their marginal contributions with other players but bear the marginal costs on their own. Following these papers, Legros & Matthews (1993) analyze when approximate efficiency in team production can be achieved if one player chooses inefficient effort with a small probability in order to "monitor" other players. Strausz (1999) shows that efficiency can be achieved if sequential instead of simultaneous effort exertion is considered and players can observe their predecessors' input. Battaglini (2006) analyzes multi-dimensional team production problems and shows that efficiency can be achieved if the dimensionality of team output is sufficiently large. These contributions typically consider pure moral hazard problems and focus on how to mitigate players' free-riding incentives. We contribute to this literature by extending the standard team production setup by the addition of asymmetric information on team productivity. This extension is nontrivial because a mechanism that resolves free-riding incentives may very well fail to ensure truthful information transmission, or create incentives for combined deviation in both information revelation and effort exertion. For example, the solution in Legros & Matthews (1993) that implements approximate efficiency in pure moral hazard environments leads to an exaggeration of reported productivity if it is the private information of some player. The same

[6] This team production literature is distinct from the principal-agent framework because of the absence of a principal and the implied requirement for the budget to balance among team members.

is true for the sequential mechanism proposed by Strausz (1999). We show that a ranking-based sharing rule can be used to achieve efficiency in this nested problem of moral hazard and adverse selection. Gershkov et al. (2009) introduce the idea of adding a verifiable ranking of players' efforts in order to overcome pure moral hazard in team problems. The present paper improves on this analysis by allowing for the much more general setting of private information. As a result, the ideas in this paper focus on issues of information transmission that were entirely absent from previous analyzes. Consequently, we can show in which environments it is possible (and impossible) to balance free riding with information transmission incentives in order to obtain efficiency.

Interest in information-based leadership problems was initiated by Hermalin (1998) who studies a privately informed player who communicates her information to others and participates in team production. Hermalin (1998) defines a leader as a team member who induces a voluntary following by credibly transmitting private information. He shows that an observable sacrifice or tangible upfront investment by the leader ("leading-by-example") can mitigate the adverse selection problem. Following this paper, Komai et al. (2007) analyze whether it is better to concentrate information in a single player or make it transparent to all players. Komai & Stegeman (2010) broaden the study of leading-by-example games to binary participation choices and nonlinear utility functions. Zhou (2011) extends this information-based leadership framework to the study of organizational hierarchies. Hermalin (2014) examines the pros and cons of charismatic leadership. The contributions in this literature (on which recent and comprehensive surveys have been provided by Ahlquist & Levi (2011) and Hermalin (2012)) typically focus on information transmission between the players and how to mitigate the impact of adverse selection. These contributions show that the signalling incentives of the informed players can partially resolve the moral hazard (or free-riding) problem. By contrast, we develop a ranking-based compensation scheme in which the prize structure depends on the level of team output and the leader's announcement of the state of the world. This mechanism encourages the leader to truthfully reveal her private information while at the same time eliminating the free-riding incentives of all team members.

Both the psychology and management literatures contribute frequently to the study of information sharing in teams.[7] In a meta-study of "social loafing" (free-riding) experiments spanning more than a decade of observation of team information sharing decisions, Mesmer-Magnus & DeChurch (2009) report that information-sharing in teams is often problematic in the sense that individuals who together possess all relevant information do not share that information and thus keep the team sub-optimally informed. Similarly, Pearsall et al. (2010) report that "the interdependent nature of these teams presents a unique challenge to organizations, as

[7] The applied psychology literature takes Ringelmann's rope-pulling experiments (1882–87) as a starting point (Ringelmann, 1913; Ingham et al., 1974). This literature significantly predates and complements the economics literature on free-riding.

facilitating the effective performance of team members requires managers to find ways to motivate both individual effort and the coordination of diverse expertise." This paper provides an incentive system for members of such teams to share their information efficiently and truthfully and shows when such systems do not exist.

There has been intense interest in combined adverse selection and moral hazard problems in principal-agent settings. See Guesnerie et al. (1989) for a comprehensive review of the early literature. In a repeated setting, Rahman (2012) characterizes an optimal contract if the monitor's observations are private and costly. Gershkov & Perry (2012) characterize optimal contracts in a dynamic principal-agent setting with moral hazard and adverse selection (persistent as well as repeated). In a more general dynamic environment, Garrett & Pavan (2012) allow for the possibility of agent turnover and characterize the optimal retention policy. Garrett & Pavan (2015) analyze how the dynamic power of incentives optimally varies with the relationship tenure in a dynamic environment with moral hazard and adverse selection.

2 The model

There is a set $\mathcal{N}$ of $n \geq 2$ symmetric, risk-neutral players. Each player $i \in \mathcal{N}$ exerts an effort $e_i \in [0, \infty)$ that need not, in principle, be verifiable. Effort cost $c(e_i)$ is assumed to be strictly convex with $c(0) = 0$ and $c'(0) = 0$. Efforts generate increasing and concave team output of $y(\alpha; e_1, \dots, e_n)$, which depends on the players' efforts and the realization of some random variable α, interpreted as a productivity parameter, that is distributed according to function F on interval $[a, b]$, with[8] $0 \leq a < b \leq \infty$. We assume that y is symmetric with respect to its last n arguments, i.e., output is invariant with respect to permutations of the last n arguments. Denote

$$\boldsymbol{e}_{-i} = (e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n) \quad \text{and} \quad \boldsymbol{e} = (e_i, \boldsymbol{e}_{-i}). \tag{1}$$

Output $y(\alpha; e_i, \boldsymbol{e}_{-i})$ is twice continuously differentiable with $y_2(\alpha; 0, \boldsymbol{e}_{-i}) > 0$ for any $\alpha \in [a, b]$ and any[9] $\boldsymbol{e}_{-i}$. We assume that the team output or production function $y(\alpha; e_i, \boldsymbol{e}_{-i})$ exhibits positive cross-derivatives between e_i and α for any i and $\boldsymbol{e}_{-i}$. The signal α is privately observed by player 1, the team leader, while all other team members know only the distribution of α. Throughout, we denote the reported signal by α' and the actual realization of output by y^*.

We assume that, in addition to the observation of realized output, there is a contest that specifies a ranking of the agents according to their exerted efforts. The ranking is noisy and

[8] Although our analysis assumes that the production function is deterministic, our main result extends to the case of stochastic production, for instance, $y(\alpha; e_1, \dots, e_n) + \varepsilon$ with $\varepsilon \sim G[\underline{\varepsilon}, \bar{\varepsilon}]$ and $\mathbb{E}(\varepsilon) = 0$. The error term washes out in expectation and exactly the same sharing rule as in Proposition 1 implements full efficiency.

[9] Throughout the paper, h_i, $h \in \{y, f, s\}$ denotes the partial derivative of h with respect to the i^{th} argument. The second derivative with respect to the same i^{th} argument is denoted $h_{i,i}$ and the second-order mixed partial derivative with respect to the i^{th} and j^{th} arguments is written $h_{i,j}$. As usual, h' denotes the first derivative of a function with a single argument.

depends on the agents' exerted efforts. The outcome of the contest is observable and verifiable. We denote by $f^J(e_i, \boldsymbol{e}_{-i})$ the probability that player i is ranked[10] jth. Observe that for any $i \in \mathcal{N}$, e_i and $\boldsymbol{e}_{-i}$,

$$\sum_{J=1}^{n} f^J(e_i, \boldsymbol{e}_{-i}) = 1. \tag{2}$$

We assume that these functions f are symmetric with respect to the identity of the players. In addition to the differentiability of $f(e_i, \boldsymbol{e}_{-i})$ with respect to all arguments we assume that, for any $\boldsymbol{e}_{-i}$, $f^1(e_i, \boldsymbol{e}_{-i})$ increases with e_i; that is, the probability of being ranked first increases with own effort. A team contract specifies the shares of a team's output for each player. Budget balancing requires that these shares sum to one across players. We look for a team contract that is budget-balanced and implements the socially efficient efforts as defined in the next subsection.

2.1 Efficiency benchmark

Socially efficient team efforts are defined as the set of efforts that maximize social welfare as chosen by a benevolent planner (who knows α and can dictate agents' efforts), i.e.,

$$\max_{\boldsymbol{e}} \; y(\alpha; e_1, \dots, e_n) - \sum_{i=1}^{n} c(e_i). \tag{3}$$

Hence, first-best efforts, which are symmetric under the given assumptions on output $y(\cdot)$ and costs $c(\cdot)$, are denoted by $e^*(\alpha) = e_1^*(\alpha) = \cdots = e_n^*(\alpha)$ and are defined as

$$y_k(\alpha; e(\alpha), \dots, e(\alpha)) = c'(e(\alpha)) \tag{4}$$

for $k \in \{2, \dots, n+1\}$. Positivity of the cross-derivatives of the output function implies that $e^*(\alpha)$ is increasing in α. Note that the contest does not play any role in the efficient outcome but can be used as an information device for implementing the efficient effort choice.

In our basic setup, the single leader has private information on the value of the group's productivity parameter α. This opens a door to strategic manipulations by the leader, because although this information is valuable to everyone, a problem arises if the players share team output in some fixed way because the leader may then have an incentive to lie: intuitively, the leader may find it individually beneficial to claim that the group is in a "high-productivity" state by some misreport $\alpha' > \alpha$ to induce all other team members to exert high efforts, even

[10] A noisy ranking is neither fully informative nor fully uninformative about efforts. In the fully informative case, the ranking technology is not differentiable (which violates our assumptions), resulting in an all-pay auction environment. The polar case of a fully uninformative ranking (i.e., no additional information over the standard case) brings us back to the Holmström environment in which it is well known that efficiency cannot be implemented. Section 4.3 generalizes the available information on the contest structure assumed here.

if she plans to put in less. The other team members, anticipating this, may then disregard the leader's report. Thus, in this framework, an efficient team contract, while keeping the budget balanced, has to solve a dual incentive problem: *i*) eliciting true information from the leader and *ii*) encouraging efficient efforts from both the leader and the other team members.

3 Results

In this section we present the incentive mechanism and our results for the case in which players are ex-ante symmetric and information is isolated in the sense that only the team leader, called player 1, has private information. This setup is later generalized to dispersed information where each player receives a private signal on team productivity and to ex-ante asymmetries among team members.

The designer suggests the following mechanism consisting of both a ranking-based sharing rule that divides the total generated output y^* and a dynamic game structure. At the first stage, after the leader learns her private information and all players observe the proposed sharing rule, players either accept this proposal or disagree to participate in the mechanism. If the contract is rejected by at least one agent the game ends. Conditional on acceptance of the contract by all agents, the privately informed player reports her information publicly. At the second stage, all players exert efforts and, after the realization of both the output and the contest ranking, the generated team output is shared according to the proposed sharing rule.[11]

The efficient compensation scheme must deter the leader from any combined deviation consisting of misreporting the productivity parameter and exerting inefficient efforts by aligning her private utility with the social preferences. Therefore, the leader's private information makes off-equilibrium behavior (after a deviation by the leader at the reporting stage) crucial. Hence, one of the implications of adding private information to pure free-riding models is that a separate analysis of the effort exertion stage is not possible.

The efficient sharing rule depends on the report of the team leader α' and the realized output y^*. We denote by $s^\ell(y^*, \alpha')$ the share of the agent who is ranked ℓ^{th} according to the contest, when the realized output is y^* and the report of the leader is α'. Budget balancedness implies that, for any y^* and α',

$$\sum_{\ell=1}^{n} s^\ell(y^*, \alpha') = 1. \tag{5}$$

We now show that a leader's report of $\alpha' = \alpha$ is part of an ex-post equilibrium strategy of the game defined by the above mechanism and that, subsequently, exerting the efficient

[11] This is not the only mechanism that implements efficient efforts. In particular, the direct mechanism, in which the team leader reports her signal privately to the designer and the designer sends effort recommendations to all agents using a similar sharing rule, implements efficiency as well.

effort choices $e^*(\alpha)$ constitutes the counterpart strategies for all players. The expected utility of player $i \in \mathcal{N}$ from choosing effort level e_i after observing report α' in the true state of the world α, while the other players choose their equilibrium effort given the reported state $e^*(\alpha')$, is

$$u_i(e_i, \boldsymbol{e}^*_{-i}(\alpha'), \alpha) = \mathbb{E}_\alpha \left[y(\alpha; e_i, \boldsymbol{e}^*_{-i}(\alpha')) \left(\sum_{\ell=1}^{n} f^\ell(e_i, \boldsymbol{e}^*_{-i}(\alpha')) s^\ell(y^*, \alpha') \right) \middle| \alpha' \right] - c(e_i), \quad (6)$$

where $(s^1(\cdot,\cdot), \ldots, s^n(\cdot,\cdot))$ is the output- and report-dependent sharing rule. Competitors' report-contingent efficient efforts are

$$\boldsymbol{e}^*_{-i}(\alpha') = (\underbrace{e^*(\alpha'), \ldots, e^*(\alpha')}_{n-1 \text{ times}}), \quad (7)$$

and expectations in (6) are over α conditional on the reported α'.

In Appendix C we derive sufficient conditions for a sizable class of symmetric model specifications under which the solution obtained in the following Proposition implies a global maximum.[12] This class consists of linear production, convex (monomial) effort cost, and generalized Tullock ranking. Intuitively, the existence condition stated in Proposition 6 relies on the cost function being sufficiently convex relative to the slope of the Tullock ranking in equilibrium. Moreover, the set of admissible types needs to be strictly positive and bounded. These bounds on possible types are needed because—especially for few players—an excessive under-report on a high true type would otherwise induce overexertion of effort for the informed player in order to secure the winner's share. The main technical difficulty the proof encounters is that the generalized Tullock technology we employ is neither globally concave nor convex and, therefore, the implied optimization problems are not generally well-behaved.

Our first result states that, subject to equilibrium existence, ex-post efficient efforts by all players, $e^*(\alpha)$, can always be obtained from the first-order necessary conditions for an equilibrium of the specified game.

Proposition 1. *Efficient symmetric efforts for all players defined in* (4) *can be implemented through the winner's share*

$$s^1(y^*, \alpha') = \frac{1}{n} + \frac{n-1}{ny^* f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))} \left[c'(e^*(\alpha')) - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')))}{n} \right] \quad (8)$$

where $\alpha(y^*, \alpha')$ *is the solution to* $y^* = y(\alpha; \boldsymbol{e}^*(\alpha'))$ *and the losers' share* $s^j(y^*, \alpha') = (1 - s^1(y^*, \alpha'))/(n-1)$ *for all*[13] $j \neq 1$.

[12] For the (asymmetric) model extensions analyzed in Section 4, we further provide specific examples illustrating that the set of equilibria is nonempty.

[13] All losers are treated equally under this sharing rule. Alternatively, due to risk-neutrality, a sharing rule consisting of the winning share (8) and appropriately designed multiple losing shares would also implement full efficiency.

In Proposition 1 and the subsequent results of the main part of this paper we derive only first-order necessary conditions for the existence of equilibria (which we verify to imply sufficiency for a subclass in Appendix C). The idea of the proof of Proposition 1 is to construct a sharing rule that encourages the team leader to exert the efficient effort $e^*(\alpha')$ given her own report α' even if the report does not correspond to the true state of the world $\alpha' \neq \alpha$. This sharing rule, in addition to solving the moral hazard problem between all agents, provides appropriate incentives to the team leader to report the correct state of the world at the first stage. Given report-contingent equilibrium effort choices by all players, the function $\alpha(y^*, \alpha')$ can be interpreted as the productivity parameter that an outsider can deduce from observing realized output together with the leader's report.

Notice that a loser's share $(1 - s^1(y^*, \alpha'))/(n-1)$ is given by

$$\frac{1}{n} - \frac{1}{ny^* f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))} \left[c'(e^*(\alpha')) - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))}{n} \right]. \tag{9}$$

Therefore, the difference between the winner's and the loser's compensations is

$$\left[c'(e^*(\alpha')) - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))}{n} \right] \frac{1}{f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))} \tag{10}$$

where the expression in square brackets is the private marginal disutility from effort exertion. This element aligns agent incentives with the socially efficient objective, which, in turn, generates the correct incentives for the agents to report information and to exert socially efficient efforts.[14] To see this, note that we can rewrite the leader's utility, given sharing rule (8) and a report α', as $u_1(e, \boldsymbol{e}^*_{-1}(\alpha'), \alpha) =$

$$\begin{aligned} & \frac{y^*}{n} - \frac{1}{nf_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))} \left[c'(e^*(\alpha')) - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))}{n} \right] + \\ & f^1(e, \boldsymbol{e}^*_{-1}(\alpha')) \left[c'(e^*(\alpha')) - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))}{n} \right] \frac{1}{f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))} - c(e) \\ = & \frac{y^*}{n} - c(e) + \left[\frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))}{n} - c'(e^*(\alpha')) \right] \frac{1}{f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'))} \left[\frac{1}{n} - f^1(e, \boldsymbol{e}^*_{-1}(\alpha')) \right]. \end{aligned}$$

The last line illustrates how the leader's dual incentive problem is satisfied: in the third element, only $f^1(e, \boldsymbol{e}^*_{-1}(\alpha'))$ and y^* depend on the leader's effort. *i)* Even after a misreported state of the world $\alpha' \neq \alpha$, small deviations in the leader's effort from symmetric efforts, $e^*(\alpha')$, are neutralized because these only first-order affect the derivative of the averaged social surplus, $y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))/n - c'(e^*(\alpha'))$, through $f^1(e, \boldsymbol{e}^*_{-1}(\alpha'))$; recall that $f^1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')) = 1/n$. But this effect is exactly offset by the marginal change in the first part of the utility $y^*/n - c(e)$, even after a non-truthful report $\alpha' \neq \alpha$. *ii)* This anticipated

[14] This basic idea is similar to the construction of Vickrey–Clarke–Groves mechanisms in standard mechanism design but, instead of monetary transfers, we use output shares to align incentives.

symmetric effort at stage two aligns the leader's first-stage incentives for reporting the state of the world with those of the planner because she expects to get the averaged social surplus $y^*/n - c(e)$, and so, in effect, the leader seeks to maximize social surplus.

This illustration shows the effect of the leader's private information. In the case of moral hazard only there is no reporting stage and the symmetric sharing rule is not necessary. In fact, in such a case an identity-dependent and asymmetric sharing rule will generate the correct incentives to exert the efficient efforts even in a symmetric environment. In the present environment, the symmetry of the sharing rules is crucial as it is used to align the leader's preferences with the social preferences, since symmetric expected utility guarantees that at the first stage the leader will face a fixed share of the social surplus and, hence, choose the truthful report to maximize social surplus. Roughly speaking, given equilibrium behavior at stage two, the designed mechanism ensures that the privately informed player 1 finds it disadvantageous to choose a combination consisting of a misreport α' at stage one and an inefficient, signal-contingent effort choice at the second stage of the game. A pair consisting of a low misreport (inducing low efforts $e^*(\alpha')$ of the uninformed players in equilibrium) and a higher than efficient effort is undesirable under reward structure (8) because the individual convex effort cost is too high relative to the appropriately chosen winner's share of (lower) total output. Similarly, a high misreport (inducing high efforts $e^*(\alpha')$ of the uninformed players in equilibrium) and low own efforts (and costs) to win a larger prize is discouraged because a well-designed losing prize decreases in realized output. Since the informed player 1 has appropriate incentives to truthfully report her signal, the uninformed team members can rely on a truthful report in equilibrium.

Remark 1. *We can separate the effects of moral hazard and adverse selection on our sharing rule. In Gershkov et al. (2009), for the case of a commonly known state of the world α, the report-independent winner's share that implements efficiency is*

$$s^1(y^*, \alpha) = \frac{1}{n} + \frac{(n-1)^2 y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{n^2 f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) y(\alpha; \boldsymbol{e}^*(\alpha))} \tag{11}$$

with each of the losers receiving $s^{j\neq 1}(y^*, \alpha) = (1 - s^1(y^*, \alpha))/(n-1)$.

Our new sharing rule (8) *provides efficient incentives to the uninformed agents when they believe that the informed agent reported the correct state of the world. Inserting $\alpha' = \alpha$ into sharing rule* (8) *and recalling that efficiency implies both $y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) = c'(e^*(\alpha))$ and $\alpha(y^*, \alpha) = \alpha$, we get $s^1(y^*, \alpha) =$*

$$\begin{aligned} &\frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) y(\alpha; \boldsymbol{e}^*(\alpha))} - \frac{y_2(\alpha(y^*, \alpha); e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) y(\alpha; \boldsymbol{e}^*(\alpha)) n}\right] \\ &= \frac{1}{n} + \frac{n-1}{n y(\alpha; \boldsymbol{e}^*(\alpha))}\left[\frac{y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))} - \frac{y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) n}\right] \end{aligned} \tag{12}$$

which immediately yields (11)*. Therefore, the agents' shares along the equilibrium path are the same as in Gershkov et al. (2009). Hence, the "correction" of the sharing rule to take care of adverse selection can be expressed as*

$$\frac{n-1}{n}\left[\frac{c'(e^*(\alpha'))}{y^* f_1^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))} - \frac{y_2(\alpha(y^*,\alpha');e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))}{ny^* f_1^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))} - \frac{(n-1)y_2(\alpha;e^*(\alpha),\boldsymbol{e}^*_{-i}(\alpha))}{nf_1^1(e^*(\alpha),\boldsymbol{e}^*_{-i}(\alpha))y(\alpha;\boldsymbol{e}^*(\alpha))}\right] \tag{13}$$

which we interpret as the (off-equilibrium) value of information to the winner.[15]

As discussed in remark 1, sharing rule (8) corresponds in equilibrium to Gershkov et al. (2009) for the pure moral hazard case. As a result, the uninformed players' incentives also correspond to those in a pure moral hazard setting: by increasing own efforts, a player enlarges team output and own winning probability while at the same time increasing effort cost. The derived sharing rule exactly balances the positive and negative incentives so that the uninformed players choose efficient efforts.

Note that the derivative of the success function $f^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))$ in equilibrium with respect to e_i can be interpreted as the responsiveness (or precision) of the ranking to a deviation from equilibrium by player i. In other words, this derivative expresses the extent to which winning probabilities change if player i changes efforts. From (8), we get an immediate comparative statics result with respect to the precision of the success function $f_1^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))$.

Corollary 1. *The share of the winner $s^1(y^*,\alpha')$ decreases on the equilibrium path with the precision of the ranking $f_1^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))$.*

This is intuitive (and proved in the Appendix), since higher ranking precision increases the incentives to the agents. Therefore, if $f_1^1(e^*(\alpha'),\boldsymbol{e}^*_{-i}(\alpha'))$ increases, agents want to exert higher efforts. To restore their incentives, the share of the winner is adjusted/decreased.

Example 1. We illustrate our efficiency result from Proposition 1 in a simple example with n players, Tullock ranking technology[16] $f^1(e_i,\boldsymbol{e}_{-i}) = e_i^r/\sum_j e_j^r$ with $r>0$, linear production $y(\alpha;\boldsymbol{e}) = \alpha\sum_i e_i$, and quadratic effort cost $e_i^2/2$. Note that in this example the efficient effort level is $e^*(\alpha)=\alpha$. This is implemented through the following ranking-based sharing rule:

$$\begin{aligned} s^1(y^*,\alpha') &= \frac{1}{n} - \frac{n-1}{n^3 f_1^1(\alpha',\boldsymbol{\alpha}')\alpha'} + \frac{(n-1)\alpha'}{ny^* f_1^1(\alpha',\boldsymbol{\alpha}')} = \frac{1}{n} - \frac{1}{nr} + \frac{n\alpha'^2}{y^* r}, \\ s^{j\neq 1}(y^*,\alpha') &= \frac{1}{n} + \frac{1}{n^3 f_1^1(\alpha',\boldsymbol{\alpha}')\alpha'} - \frac{\alpha'}{ny^* f_1^1(\alpha',\boldsymbol{\alpha}')} = \frac{1}{n} + \frac{1}{n(n-1)r} - \frac{n\alpha'^2}{(n-1)y^* r} \end{aligned} \tag{14}$$

[15] This is different from the notion of value of information in Hermalin (1998, footnote 12) which shows that second-best team welfare under the true signal exceeds team welfare under the expected signal. (The same would be true in our model.)

[16] For completeness, we define $f^1(0,\ldots,0) = 1/n$; the implied discontinuity at point $(0,\ldots,0)$ plays no role in this example.

where $s^1(\cdot)$ of the final team output is awarded to the first-ranked player while $s^{j\neq 1}(\cdot)$ is given to all other players.

In Gershkov et al. (2009), the ranking-based sharing rule needed to implement full efficiency in a pure moral hazard setting for the same production function and cost function is given by $s^1 = (r-1+n)/(nr)$ and $s^{j\neq 1} = (r-1)/(nr)$. To provide the leader with incentives to truthfully report her private information, the new sharing rule has to depend upon the realized output y^* and the report α'. Contrasting with (14), the sharing rule obtained for the pure moral hazard case in Gershkov et al. (2009) for the same production function and costs does not depend on realized output y^* and the report.$\triangleleft$

An advantage of the contest approach is that it requires only ordinal noisy information on agents' efforts, which is arguably easier to collect than information on the precise effort realizations. Nevertheless, the fact that we are able to implement efficient efforts implies that a noisy ordinal ranking of efforts is a sufficient statistic in the sense of Holmström (1982, Section 3) for the cardinal effort information employed in standard contracts.

Remark 2. *In addition to efforts, the ranking we employ in the analysis can be based also on state-dependent variables such as marginal contributions. For this purpose, we define player i's marginal contribution from effort* e_i *as* $y^i(\alpha; e_i, \boldsymbol{e}_{-i}) = y(\alpha; e_i, \boldsymbol{e}_{-i}) - y(\alpha; 0, \boldsymbol{e}_{-i})$. *Suppose that we can express the resulting ranking technology by* $f(\alpha; e_1, \ldots, e_n)$ *which is symmetric with respect to the last n arguments and* $f^1(\alpha; e_i, \boldsymbol{e}_{-i})$ *is increasing in* e_i*; for instance, the probability* $f^1(\alpha; e_1, \ldots, e_n) = y^i(\alpha; e_i, \boldsymbol{e}_{-i}) / \sum_{j\in\mathcal{N}} y^j(\alpha; e_j, \boldsymbol{e}_{-j})$. *Then we can achieve efficiency with exactly the same sharing rule as in (8), except that* $f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))$ *is replaced by*[17] $f_2^1(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))$.

4 Extensions and robustness

4.1 Dispersed information

In this section we consider the case in which information about team productivity is dispersed among multiple, ex-ante symmetric players and thus information has to be revealed and aggregated before the players choose production efforts. Suppose that each individual receives some signal α_i and the production function is given by $y(\alpha_1, \ldots, \alpha_n; e_1, \ldots, e_n)$, which is symmetric with respect to both the first n arguments and the last n arguments.[18] Examples of such production functions are $y(\alpha_1, \ldots, \alpha_n; e_1, \ldots, e_n) = g(\alpha_1, \ldots, \alpha_n)\, h(e_1, \ldots, e_n)$ with symmetric functions g and h and, in particular, $y(\alpha_1, \ldots, \alpha_n; e_1, \ldots, e_n) = \left(\sum_{i=1}^n \alpha_i\right)\left(\sum_{i=1}^n e_i\right)$. We

[17] The proof for the case of $f^1(\alpha; e_i, \boldsymbol{e}_{-i})$ is virtually identical to that of Proposition 1 and is therefore omitted.
[18] That is, output is invariant under any permutation of the first n arguments and the last n arguments.

consider a game in which each player reports his private signal publicly and simultaneously. After observing all the reports, agents decide simultaneously on their efforts. We show that first-best can be implemented using a sharing rule similar to (8).

Proposition 2. *There exists a sharing rule that induces truthful reports by all the agents at the first stage and implements efficient effort choices at the second stage.*

In equilibrium, the players choose symmetric efforts at the second stage. At the reporting stage, every player expects to get $1/n$ of the social surplus, which aligns the players' preferences with the social preferences, as in the main model. Consequently, and in contrast to our analysis of ex-ante asymmetries in the following section, all players face an identical equal sharing problem at the reporting stage and we do not have to deal with ex-ante asymmetric reporting incentives.

One important question in organization theory is whether a team with information dispersed among multiple players performs better than a team in which only a single team member has private information and thus whether the optimal team structure should allow information to be dispersed among more than one player. Our Propositions 1 and 2 suggest that the allocation of information does not affect the optimal team structure if an appropriately-designed ranking-based sharing rule is used to reward the players because the first-best outcome can be achieved in both cases. However, as we will show next, the answer to this question changes if players are ex-ante asymmetric.

4.2 Asymmetric players

In this subsection, we study cases in which players are ex-ante asymmetric.[19] We first examine the one-dimensional signal case in which a team consists of two heterogeneous groups of players and only one player is informed about the productivity of the team. Second, we analyze the case in which more than one player receives private signals that affect the team's productivity. We then explore the intuition when and under what circumstances efficiency is (not) implementable in either case.

In contrast to the general specification we have analyzed so far for the symmetric case, we have to resort to specific production functions, contest success functions, and cost functions for the asymmetric case. The reason is that we could not otherwise express the efficient asymmetric efforts, $e_i^*(\boldsymbol{\alpha}')$, based on the vector of the informed players' reports $\boldsymbol{\alpha}'$, which enter the players' first-stage information revelation problem. If these efficient effort functions cannot be

[19] There are two dimensions along which players can be ex-ante asymmetric in this section. First, we allow for cost asymmetries using commonly known but idiosyncratic constants γ_i. Second, production itself may be asymmetric, with the asymmetry potentially depending on both ex-ante technological differences and the players' private information.

determined explicitly, we cannot solve the informed players' first-stage problem. Hence we have to resort to cases where the planner's problem can be solved explicitly. Our solution strategy is to first solve for asymmetric identity-dependent winning shares si that ensure report-contingent efficient efforts $e_i^*(\boldsymbol{\alpha}')$ for every player i at the second stage.[20] These shares have a degree of freedom captured by the term $C(\boldsymbol{\alpha}')$, which depends only upon the reported $\boldsymbol{\alpha}'$. We call this term "constant" $C(\boldsymbol{\alpha}')$ because it is the result of the solution of differential equations in output $y(\boldsymbol{e})$ that determine the shares guaranteeing report-contingent efficient efforts $e_i^*(\boldsymbol{\alpha}')$ at the second stage. We then insert the report-contingent efficient efforts $e_i^*(\boldsymbol{\alpha}')$ into the informed players' first-stage problems to pin down the term $C(\boldsymbol{\alpha}')$ in order to adjust the players' incentives to report truthfully.[21] This last step of using $C(\boldsymbol{\alpha}')$ to adjust incentives is irrelevant in a pure moral hazard problem. In the present setting with adverse selection, the new sharing rule again needs to depend appropriately on $\boldsymbol{\alpha}'$ in order to give the informed players the incentive to report truthfully.

4.2.1 A single informed player

Suppose that there are two groups of agents in a team.[22] In Group A, there are n_1 agents, each with cost function $\gamma e^2/2$, $\gamma > 0$, while in Group B there are n_2 agents, each with cost function $e^2/2$. The team production function is $y = \alpha \sum_{i=1}^{n} e_i$, with $n = n_1 + n_2$, $n_1 \geq 1$ and $n_2 \geq 1$. The efficient efforts are $e_A^*(\alpha) = \alpha/\gamma$ for players $i \in A$ and $e_B^*(\alpha) = \alpha$ for $i \in B$. The ranking technology is the standard Tullock contest success function with $f^1(e_i, \boldsymbol{e}_{-i}) = e_i / \sum_j e_j$. Among the n players, player 1 from group A is informed about the realization of α and she reports α publicly before any of the players choose their efforts.

We show that a group-dependent sharing rule can be used to implement full efficiency: the winning share of realized team output when a player from group A wins is different from the winning share when a player from group B wins the contest.

Proposition 3. *Efficiency is implementable using a group-dependent sharing rule.*

We prove Proposition 3 by showing that the following sharing rule implements efficiency if either set A or B has at least two members (a slightly simpler sharing rule is stated in the

[20] As $si(\cdot)$ denotes player i's winning share in the asymmetric mechanism this notation differs from the symmetric share $s^\ell(\cdot)$ introduced previously.

[21] We ignore the players' participation incentives in our analysis. If participation is of concern, there is another degree of freedom (i.e., another constant) in the solved term $C(\boldsymbol{\alpha}')$, which can be used to adjust the share of the team's output among the players to ascertain the players' participation incentives.

[22] Having a more heterogeneous team with a single informed agent merely complicates the notation without adding further insights.

proof of the case of $n_1 = n_2 = 1$):

$$\begin{aligned} s^A(y^*, \alpha') &= \frac{\alpha'^2 (n_1 + \gamma n_2)\left(n_1^2 + \gamma n_1 (n_2 - 1) - \gamma n_2\right)}{\gamma n_1 y^* (n_1 + \gamma (n_2 - 1) - 1)} + C(\alpha')((\gamma+1) y^*)^{-\frac{n_1 + \gamma n_2}{\gamma + 1}}, \\ s^B(y^*, \alpha') &= \frac{\alpha'^2 (n_1 + \gamma n_2)(n_1 (n_2 - 1) + n_2 (\gamma n_2 - 1))}{\gamma y^* (n_1 + \gamma (n_2 - 1) - 1) n_2} - \frac{n_1}{n_2} C(\alpha')((\gamma+1) y^*)^{-\frac{n_1 + \gamma n_2}{\gamma + 1}} \end{aligned} \tag{15}$$

with

$$C(\alpha') = \frac{(\gamma n_2 - n_1)\left(\frac{\alpha'^2 (\gamma+1)(n_1 + \gamma n_2)}{\gamma}\right)^{\frac{n_1 + \gamma n_2}{\gamma + 1}}}{n_1 (n_1 + \gamma (n_2 - 1) - 1)} \tag{16}$$

where $s^K(y^*, \alpha')$ is the winner's share of the team's output if the winner is from group $K \in \{A, B\}$, the losers in group K receive a zero share of the team's output, and players in group $J \neq K \in \{A, B\}$ receive a share $(1 - s^K(y^*, \alpha'))/n_J$ of the team's output.

The shares $s^A(y^*, \alpha')$ and $s^B(y^*, \alpha')$ that are necessary to ensure that $e^*(\alpha')$ are the mutually best replies of the players at the effort stage have a degree of freedom captured by the term $C(\alpha')$, which is independent of the realized output. Thus, one can choose $C(\alpha')$ so as to provide the correct incentive to the leader to report truthfully at the first stage. Put differently, $C(\alpha')$ can be used to transform the informed player's first-stage objective into any desired shape.

Figure 1 shows an example of the maximization problems of the two-player case of $n_1 = n_2 = 1$, $\gamma = 4/5$ and $\alpha = 2$ with respect to e_1, e_2 in the left and center panels and the choice of report α' in the right panel in which player 2's implied utility is the curve shown in gold. The right panel illustrates the way the mechanism provides the leader's incentives at the first stage: the "local flattening" of the uninformed player's expected utility (as a function of the leader's report) makes the leader the residual claimant and aligns her incentives with those of the planner. That is, the mechanism makes the uninformed player's utility, given the equilibrium effort at the second stage, "locally independent" of the leader's report, which in turn "corrects" the leader's incentives to report her signal truthfully.

4.2.2 More than one informed player

Consider now the case in which more than one player has private information that affects team productivity and the players are asymmetric. In particular, we assume that there are two players, each with a private signal α_i, $i \in \{1, 2\}$. The team production function takes the form $y = (\alpha_1 + \beta \alpha_2)(e_1 + e_2)$ and the two players' effort costs are $c_1(e) = \gamma e^2/2$, $c_2(e) = e^2/2$, in which β and γ are some commonly known parameters.[23] Player i's probability of winning the

[23] Efficiency can also be achieved if production is not a function of the aggregate effort. Examples include $y = (\alpha_1/\alpha_2)e_1 + \beta e_2$, $y = \alpha_1 \alpha_2 \sqrt{e_1 e_2}$, and $y = (\alpha_1 + \alpha_2 + \beta_1 \alpha_1 \alpha_2)(e_1 + e_2 - \beta_2 e_1 e_2)$, in which β, β_1, and β_2 are commonly known exogenous parameters. Likewise, allowing for cost functions of different curvatures is possible.

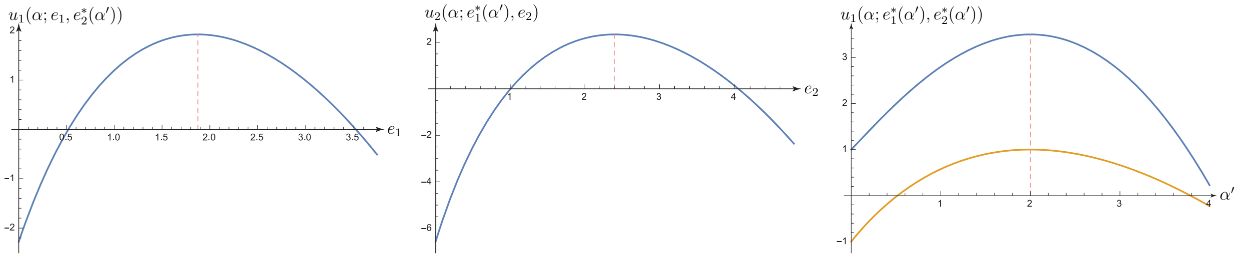

Figure 1: Potential unilateral deviations from efficient effort choice $e_i(\alpha')$ in the left and center panels for players 1 and 2, respectively. Player 1's reporting problem α' is shown in the right panel; the blue curve is player 1's utility $u_1(\alpha; e_1^*(\alpha'), e_2^*(\alpha'))$ from reporting α' and the gold curve is player 2's implied utility from this choice $u_2(\alpha; e_1^*(\alpha'), e_2^*(\alpha'))$. Parameters: two players with $n_1 = n_2 = 1$, production $y = \alpha(e_1 + e_2)$, $\alpha = 2$, $c_1(e_1) = \gamma e_1^2/2$, $c_2(e_2) = e_2^2/2$, $\gamma = 4/5$, and $f^1(e_1, e_2) = e_1/(e_1 + e_2)$.

first place is $f^1(e_i, e_{-i}) = e_i / \sum_j e_j$ for $i \in \{1, 2\}$. The first-best efforts are

$$e_1^*(\alpha_1, \alpha_2) = \frac{\alpha_1 + \beta\alpha_2}{\gamma}, \text{ and } e_2^*(\alpha_1, \alpha_2) = \alpha_1 + \beta\alpha_2. \tag{17}$$

These first-best efforts can be achieved with a ranking-based sharing rule, as we show next.

Example 2. With production function $y = (\alpha_1 + \beta\alpha_2)(e_1 + e_2)$, first-best efforts are implementable.$\lhd$

The details of the derivation can be found in the Appendix.

In the presence of multiple, privately informed players, the identity-dependent shares ensuring report-contingent efficient efforts have a degree of freedom captured by the two-dimensional constant $C(\alpha_1', \alpha_2') = C(\boldsymbol{\alpha}')$. At the reporting stage, both players know that subsequent equilibrium stage-two efforts will follow the reports. The sum of the players' utilities as a two-dimensional function of the players' reports is maximized at the true signals (α_1, α_2) by definition of efficiency. The two-dimensional constant $C(\alpha_1', \alpha_2')$ ties the two players' individual two-dimensional utilities together such that, at the point of truth telling, *both* players' utility functions must have a slope of zero at their partial derivative with respect to their own report.

Figure 2 shows an example of the involved maximization problems for the two-player case with $\gamma = 1/2$, $\alpha_1 = 3/2$, $\alpha_2 = 2$, and $\beta = 3/2$. The first row shows the two players' second-stage effort choice problems, player 1 on the left and player 2 on the right, given reports α_1', α_2'. The second row shows player 1's first-stage report choice problem of α_1' on the left and player 2's choice of α_2' on the right. The second row of the figure exhibits the same "local flattening" of the expected utility of an informed player, which makes each player a residual claimant given the other player's equilibrium report (similarly to the single-dimensional case in Figure 1 of the previous subsection). Note that the slope of player 2's "implied" utility at $\alpha_1' = \alpha_1$ (shown in gold in the left panel of the second row) is now as critical as its counterpart, the slope of player 1's "implied" utility at $\alpha_2' = \alpha_2$ (shown in gold in the right panel of the second row). This illustrates that the construction of the two-dimensional $C(\alpha_1', \alpha_2')$ needs to ensure that truthful reporting is optimal for both players.

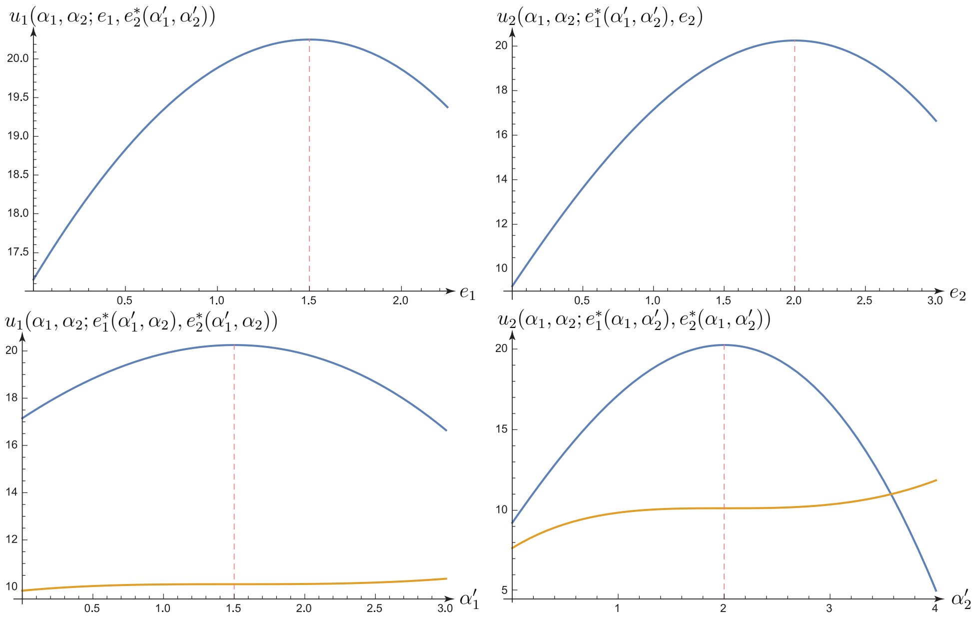

Figure 2: Potential unilateral deviations from efficient effort choice $e_i^*(\alpha_1', \alpha_2')$ in the left and right panels of the top row for players 1 and 2, respectively. Unilateral deviations from truthful reports α_i are shown in the bottom row: player 1's utility $u_1(\alpha_1, \alpha_2; e_1^*(\alpha_1', \alpha_2), e_2^*(\alpha_1', \alpha_2))$ is in blue (and the implied utility for player 2 in gold) on the left and player 2's utility $u_2(\alpha_1, \alpha_2; e_1^*(\alpha_1, \alpha_2'), e_2^*(\alpha_1, \alpha_2'))$ in blue (and the implied utility for player 1 in gold) on the right. Parameters: two privately informed players, production $y = (\alpha_1 + \beta\alpha_2)(e_1 + e_2)$, $c_1(e_1) = \gamma e_1^2/2$, $c_2(e_2) = e_2^2/2$, $\gamma = 1/2$, $\alpha_1 = 3/2$, $\alpha_2 = 2$, $\beta = 3/2$, and $f^1(e_1, e_2) = e_1/(e_1 + e_2)$.

Next we develop an example in which it is not possible to achieve efficiency using our ex-post construction. It provides us with an intuition about when it is not possible to simultaneously satisfy the requirements of *i*) generating correct incentives for effort exertion, *ii*) truth telling, and *iii*) balancing the budget. Consider a team with two players, production function $y = \alpha_1 e_1 + \alpha_2 e_2$, and symmetric cost functions $c(e_i) = e_i^2/2$. The efficient effort levels are $e_i^*(\alpha_i) = \alpha_i$. Suppose that the usual Tullock success function is employed as ranking technology.

Example 3. With production function $y = \alpha_1 e_1 + \alpha_2 e_2$, the players' first-stage incentives for truth telling are incompatible with their incentives for efficient effort provision at the second stage.$\triangleleft$

Again, the details of the derivation can be found in the Appendix.

To understand why efficiency fails with this production function but not with those we analyzed previously, take player 1's objective function at the effort stage given the true state $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$, reported signals $\boldsymbol{\alpha}' = (\alpha_1', \alpha_2')$, and the other player's effort $e_2^*(\boldsymbol{\alpha}')$:

$$f^1(e_1, e_2^*(\boldsymbol{\alpha}'))s1(y^*, \boldsymbol{\alpha}')y^* + (1 - f^1(e_1, e_2^*(\boldsymbol{\alpha}')))(1 - s2(y^*, \boldsymbol{\alpha}'))y^* - c(e_1). \tag{18}$$

The first-order condition evaluated at $e_1 = e_1^*(\boldsymbol{\alpha}')$ can be written as

$$\begin{aligned}
&f_1^1(e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}'))(s1(y^*, \boldsymbol{\alpha}') + s2(y^*, \boldsymbol{\alpha}') - 1)y^* \\
&\quad + y_2(\boldsymbol{\alpha}; e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}')) \left(f^1(e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}'))s1(y^*, \boldsymbol{\alpha}')\right. \\
&\quad + (1 - f^1(e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}'))(1 - s2(y^*, \boldsymbol{\alpha}')) \\
&\quad \left. + y^* \left(f^1(e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}'))s1_1(y^*, \boldsymbol{\alpha}') - (1 - f^1(e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}')))s2_1(y^*, \boldsymbol{\alpha}')\right)\right) = c(e_1^*(\boldsymbol{\alpha}')).
\end{aligned} \tag{19}$$

Note that if the production function has the feature that the agents' private information can be aggregated such that it is possible to represent $y_2(\boldsymbol{\alpha}; e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}'))$ by the observed output, y^*, and the reported signals $\boldsymbol{\alpha}'$, then the first-order condition depends only upon realized output and the reported signals (and is therefore independent of the true signals) and we could in principle solve the system of simultaneous equations for shares that implement the report-contingent efficient efforts.

In Example 2, for production function $y = (\alpha_1 + \beta\alpha_2)(e_1 + e_2)$ it is the case that

$$y_2(\boldsymbol{\alpha}; e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}')) = (\alpha_1 + \beta\alpha_2) = \frac{y^*}{e_1^*(\boldsymbol{\alpha}') + e_2^*(\boldsymbol{\alpha}')} = \frac{y^*\gamma}{(\alpha_1' + \beta\alpha_2')(1 + \gamma)} \tag{20}$$

while in the case of $y = \alpha_1 e_1 + \alpha_2 e_2$ in Example 3 we have that

$$y_2(\boldsymbol{\alpha}; e_1^*(\boldsymbol{\alpha}'), e_2^*(\boldsymbol{\alpha}')) = \alpha_1, \tag{21}$$

which cannot be represented by the realized output and the reported signals. As a result, one cannot solve for sharing rules that are independent of the true signals. In the cases analyzed previously, including all symmetric and asymmetric production functions we analyzed in Proposition 3 and Example 2, we could always represent a player's marginal contribution to the team's output by only the realized output and the reported signals. Thus it is always possible to express the first-order condition at the second stage for player i after a deviation of that player at the first stage as a function of the contractible variables only (realized output and reports), independently of the true signals observed by the players.

In other words, we can provide the correct incentives whenever we can aggregate the agents' private information such that it allows us to express the marginal production as a function of contractible parameters, i.e., reports and the observable output for any true signals, given the obedient behavior of all the players at the effort stage.[24]

Note that the reason for the failure of efficiency in the case of dispersed information when agents have asymmetric impacts on team production does not follow from the standard mechanism design argument on interdependent values. In fact, as we know from Mezzetti (2004), since the total output is observable and contractible, the adverse selection problem is solvable in this case. It is a combination of budget balancedness and effort exertion in and out of equilibrium that makes the efficient allocation non-implementable.

[24] This finding again shows the impact of adverse selection: in a pure moral hazard setting, no aggregation of private information is necessary and it is always possible to find a group of identity-dependent ranking-based sharing rules that ensure efficient efforts from every player.

4.3 Generalized information structure

The rest of this paper derives contracts implementing efficient efforts on the basis of the availability of some contractible ranking information on players' efforts (in addition to realized output and the privately informed players' reports). This section generalizes this available additional information over the relative rankings $f(e_i, \boldsymbol{e}_{-i})$ used elsewhere in the paper.

For this section, we assume that there are $n \geq 2$ symmetric players only one of whom is privately informed about α. The production function is $y(\alpha; e_1, \ldots, e_n)$, which we assume to be symmetric with respect to the last n arguments. The convex cost function is the same for all players $c(e_i)$, $i \in \mathcal{N}$. Denote by $e^*(\alpha)$ the efficient symmetric effort for all players when the state of the world is α. We denote by A, $|A|= k$, the set of all possible distinct k signals that may be observed by all players with generic element $a \in A$. Assume that $k \geq 2$. Denote by $P_a(\boldsymbol{e})$ the probability of observing signal $a \in A$ when agents exert efforts $\boldsymbol{e} = \{e_1, \ldots, e_n\}$. For any $\{e_1, \ldots, e_n\}$ we have

$$\sum_{a \in A} P_a(e_i, \boldsymbol{e}_{-i}) = 1. \tag{22}$$

Assume that $P_a(\boldsymbol{e})$ is differentiable with respect to every effort. Since the signal of the information device is contractible, the share of player i, $si(y^*, \alpha', a)$, may depend on the observed output y^*, the reported type α', and the observed signal from the information device a.

The utility of agent i in true state α, reported state α', and all other agents exerting efficient, report-contingent efforts $e^*(\alpha')$ while agent i exerts effort e_i is

$$u_i(\alpha; e_i, \boldsymbol{e}^*_{-i}(\alpha')) = y(\alpha; e_i, \boldsymbol{e}^*_{-i}(\alpha')) \left[\sum_{a \in A} P_a\left(e_i, \boldsymbol{e}^*_{-i}(\alpha')\right) si(y^*, \alpha', a)\right] - c(e_i). \tag{23}$$

We impose the balanced budget restriction that, for each event $a \in A$, the rewards across players sum to one, i.e.,

$$\sum_{i \in \mathcal{N}} si(y^*, \alpha', a) = 1. \tag{24}$$

The first-order condition of (23) with respect to effort choice at $e^*(\alpha')$ gives

$$\begin{aligned} & y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')) \sum_{a \in A} P_a\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')\right) si(y^*, \alpha', a) \\ & \quad + y^* \sum_{a \in A} P_{a,1}\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')\right) si(y^*, \alpha', a) \\ & \quad + y^* \left(\sum_{a \in A} P_a\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')\right) si_1(y^*, \alpha', a)\right) y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')) \\ & \quad - c'(e^*(\alpha')) = 0, \end{aligned} \tag{25}$$

where $P_{a,i}(e_1,\ldots,e_n)$ is the derivative with respect to i-th argument. From symmetry, $\sum_{a\in A} P_a\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}\right.$
$1/n$, and the fact that $\sum_{a\in A} P_a(\boldsymbol{e})\, si_1(y^*,\alpha',a)=0$, we get[25]

$$y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))\frac{1}{n} + y^* \sum_{a\in A} P_{a,1}\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')\right) si(y^*,\alpha',a) - c'(e^*(\alpha')) = 0. \quad (26)$$

Therefore, it is possible to implement efficiency if P_a are such that there exist shares $si(\cdot)$ that satisfy:

1. Symmetry: for any $i\in N$, y^*, α' and α when all the agents exert the same effort,

$$\sum_{a\in A} P_a(e,e,\ldots,e)\, si(y^*,\alpha',a) = \frac{1}{n}. \quad (27)$$

2. Budget balancedness: for any y^*, α' and a,

$$\sum_{i\in N} si(y^*,\alpha',a) = 1. \quad (28)$$

3. Incentive provision: for any $i\in N$, y^* and α',

$$y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))\frac{1}{n} + y^* \sum_{a\in A} P_{a,1}\left(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha')\right) si(y^*,\alpha',a) - c'(e^*(\alpha')) = 0. \quad (29)$$

It is always trivially possible to satisfy the first two conditions; complications are generated by adding condition 3. Therefore, to get a better understanding of the last condition, let us look at the two-player case. We can rewrite the last condition as follows. For agent 1

$$y_2(\alpha; e^*(\alpha'), e^*(\alpha'))\frac{1}{2} + y^* \sum_{a\in A} P_{a,1}(e^*(\alpha'), e^*(\alpha'))\, s1(y^*,\alpha',a) - c'(e^*(\alpha')) = 0; \quad (30)$$

for player 2,

$$y_3(\alpha; e^*(\alpha'), e^*(\alpha'))\frac{1}{2} + y^* \sum_{a\in A} P_{a,2}(e^*(\alpha'), e^*(\alpha'))\,(1 - s1(y^*,\alpha',a)) - c'(e^*(\alpha')) = 0. \quad (31)$$

Symmetry of the production function implies that $y_2(\alpha; e^*(\alpha'), e^*(\alpha')) = y_3(\alpha; e^*(\alpha'), e^*(\alpha'))$. Summing up the last two conditions gives us

$$y_2(\alpha; e^*(\alpha'), e^*(\alpha')) + y^* \sum_{a\in A} \left[P_{a,1}(e^*(\alpha'), e^*(\alpha')) - P_{a,2}(e^*(\alpha'), e^*(\alpha'))\right] s1(y^*,\alpha',a) - 2c'(e^*(\alpha')) = 0, \quad (32)$$

[25] Recall that efficiency requires that, with $\alpha'=\alpha$, it must be the case that $y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)) = c'(e^*(\alpha))$.

where $\sum_{a\in A} P_{a,2}\left(e^*\left(\alpha'\right), e^*\left(\alpha'\right)\right) = 0$. Note that at $\alpha = \alpha'$, efficiency requires that $y_2(\alpha; e^*(\alpha'), e^*(\alpha')) = c'\left(e^*\left(\alpha'\right)\right)$. Thus, for efficiency to obtain, it is necessary to have

$$y^* \sum_{a\,\in A} \left[P_{a,1}\left(e^*\left(\alpha'\right), e^*\left(\alpha'\right)\right) - P_{a,2}\left(e^*\left(\alpha'\right), e^*\left(\alpha'\right)\right)\right] s1\left(y^*, \alpha', a\right) - c'\left(e^*\left(\alpha'\right)\right) = 0. \quad (33)$$

That is, we need the information structure to satisfy a certain separation requirement: when agent 1 increases her effort, she increases the chances of observing some set of signals. This set must be different from the set of signals that are more likely to be observed after an effort increase of agent 2. Take, for example, $|A|= 2$ with

$$a = \begin{cases} 1 & \text{if } e_1 + e_2 + \epsilon > K \\ 2 & \quad\text{otherwise,} \end{cases} \quad (34)$$

where ϵ is a random variable that distributes according to some continuous distribution H. That is, the signal equals 1 if the noisy sum of efforts exceeds some constant K and equals 2 otherwise. In this case, for any $a \in A$, it is true that $P_{a,1}\left(e^*\left(\alpha'\right), e^*\left(\alpha'\right)\right) = P_{a,2}\left(e^*\left(\alpha'\right), e^*\left(\alpha'\right)\right)$ and hence efficient efforts are not implementable. In this environment, increasing efforts by any player increases the chances of receiving the same signal.

Example 4. A relative-ranking-based contest is the special case where $|A|= n$. For the Tullock success function, for any $i \in \{1, 2, \ldots, n\}$, $P_{a_i} = e_i^r/(e_1^r + \cdots + e_n^r)$ for $r > 0$. When $a = a_i$, the share

$$si(y^*, \alpha', a_i) = \frac{1}{n} + \frac{ne^*(\alpha')\left(nc'(e^*(\alpha')) - y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))\right)}{nry^*} \quad (35)$$

and $sj(y^*, \alpha', a_i) = (1 - si(y^*, \alpha', a_i))/(n - 1)$ for $j \neq i$, where $\alpha(y^*, \alpha')$ is as defined in Proposition 1, satisfy all of the above conditions 1–3 and implement the efficient outcome.$\lhd$

Example 5. Consider an even number of players $n \geq 2$. Assume that a "coarse ranking" exists which differentiates between the top half and the bottom half of the agents. That is, $|A|= \frac{(n)!}{(n/2)!(n/2)!}$, where signal a is a list of the top-half-ranked players. Denote by $P^B(e_i, \boldsymbol{e}_{-i})$ the probability for player i to be in the top-half group. Assume (similarly to the Tullock success function) that

$$P^B(e_i, \boldsymbol{e}_{-i}(\alpha')) \ = \frac{e_i^r + (n/2 - 1)e(\alpha')^r}{e_i^r + (n-1)e(\alpha')^r}, r > 0. \quad (36)$$

This results in a symmetric player's objective

$$\begin{aligned} u_i(\alpha; e_i, \boldsymbol{e}_{-i}(\alpha')) = y(\alpha; e_i, \boldsymbol{e}_{-i}(\alpha')) \Bigg(& P^B(e_i, \boldsymbol{e}_{-i}(\alpha'))\frac{s(y^*, \alpha', a)}{n/2} \\ & + (1 - P^B(e_i, \boldsymbol{e}_{-i}(\alpha')))\frac{1 - s(y^*, \alpha', a)}{n/2}\Bigg) - c(e_i). \end{aligned} \quad (37)$$

One can verify that a share of

$$s(y^*, \alpha', a) = \frac{ne^*(\alpha')\left(nc'(e^*(\alpha')) - y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))\right) + ry^*}{2ry^*} \tag{38}$$

given to all the top-half agents (with the remaining output equally spread among the bottom-half agents) satisfies all the above conditions 1–3.◁

In the following example we explore a different direction of generalization of the underlying information structure. We ask whether and under what circumstances efficient efforts can be implemented if the relative ranking that enters our construction is available for only a subset of the team members. Therefore, in the next example, we assume that agents are ex-ante asymmetric with respect to the available information on the players.

Example 6. In this example we consider the case of an incomplete ranking. Among the n team members in $\mathcal{N} = \mathcal{A} \cup \mathcal{B}$, the efforts of $n_1 \geq 2$ players in set $\mathcal{A}$ can be ranked using a Tullock ranking technology and the efforts of $n_2 \geq 1$ players in set $\mathcal{B}$ cannot be ranked. Suppose that the informed player is from set $\mathcal{B}$. The production function is $y(\alpha, \boldsymbol{e}) = \alpha \sum_i e_i$ and the effort cost is $c(e_i) = e_i^2/2$. The efficient efforts are $e^*(\alpha) = \alpha$.

The following ranking-based sharing rule with $(s1(y^*, \alpha'), s2(y^*, \alpha'))$ can be used to achieve efficiency. The share of output $s1(y^*, \alpha')$ is awarded to the winner of the contest, $s2(y^*, \alpha')$ is the share awarded to the losers of the contest, and each of the players whose efforts cannot be ranked receives an equal share $(1 - s1(y^*, \alpha') - (n_1 - 1)s2(y^*, \alpha'))/n_2$:

$$\begin{aligned} s1(y^*, \alpha') &= \frac{1}{n_1} - \frac{1}{rn} + \frac{nn_1\alpha'^2}{y^* r(n_1 - 1)} - \frac{nn_2\alpha'^2 \log(y^*)}{n_1 y^*} + \frac{C(\alpha')}{y^*}, \\ s2(y^*, \alpha') &= \frac{1}{n_1} + \frac{1}{nr(n_1 - 1)} - \frac{nn_2\alpha'^2 \log(y^*)}{n_1 y^*} + \frac{C(\alpha')}{y^*}. \end{aligned} \tag{39}$$

where

$$C(\alpha') = \frac{nn_2\alpha'^2 \log(n\alpha'^2)}{n_1} - \frac{(n+1)n_2\alpha'^2}{2n_1} - \frac{n\alpha'^2}{r(n_1 - 1)}. \tag{40}$$

The details of the derivation of these shares are relegated to the Appendix. How can efficiency be obtained in this setting? Consider first the second stage. Every unranked player who considers slightly varying his effort provision gets the full social return of his variation; i.e., for local deviations, every deviating unranked player is fully compensated.[26] Hence, out of equilibrium,

[26] This can be seen from the derived shares: the payoff of the unranked uninformed player is given by

$$n\alpha'^2 \log y^* - \frac{n_1 C(\alpha')}{n_2} - \frac{nn_1\alpha'^2}{n_2 r(n_1 - 1)} - \frac{e_i^2}{2}. \tag{41}$$

Therefore, when all the other players exert effort α', a small increase in effort above α' by an unranked player increases this player's payoff by α', which is exactly the increase in the total social welfare provided that the informed player reports truthfully.

the ranked players act as budget breakers.[27] At the first stage, the unranked players act as residual claimants and therefore maximize overall utility, which implies an efficient report.

This example implements efficiency contingent on the informed player being unranked. To see that the efficient allocation cannot be achieved if the informed player is from set $\mathcal{A}$, note that in this example the utility of all ranked agents is the same. The utility of an informed, ranked player (and hence all ranked agents) is increasing in the report α' whenever it exceeds the true α. Intuitively, what happens is that the uninformed agents "pay all the costs." In order to provide the unranked player(s) with incentives to exert efforts, the unranked players' share of the output should be substantially more volatile with increasing output. If the informed player exaggerates the report α', all players increase their efforts. However, given the report, the output is below the expected output and the unranked agents (to be provided with efficient incentives) suffer more than the ranked agents. This destroys the reporting incentives of a ranked informed player at the first stage.$\triangleleft$

5 Concluding remarks

The present paper analyzes the question of what constitutes the coordinative essence of leadership in team structures when (some of) the team members are privately informed about some aspect of the profitability of a joint project. Such proprietary information arises naturally if, for example, some team member occupies a role in a predefined organizational structure by virtue of which she acquires and disseminates information. Our model extensions allow both for private information that is dispersed among otherwise symmetric team members and for ex-ante asymmetries among team members. We show in both cases that, to some extent, our main efficiency result can be retained.

An immediate implication of our symmetric analysis is that efficiency can be achieved in a project selection problem. Consider a situation in which some team members propose projects and are privately informed about their productivity. That is, each member is a potential leader of one of the projects. However, the team is able to execute only one of the projects due to limited resources. Consider a mechanism in which the agents report the productivity parameters at the first stage. Then the project that maximizes joint welfare (based on the reports) is chosen. At the second stage, the mechanism described in Proposition 1 is used to implement efficient efforts. In this model each privately informed player reports his signal truthfully and the best project is selected.

Connecting to the literature on leading by example represented by Hermalin (1998), if the leader is allowed to choose her effort before the other players and this effort is observable by her team partners, the leader's effort serves as a signal of her private information on the team

[27] For the ranked players, nothing changes from our base model analysis.

productivity parameter. As a result, the leader is unable to choose effort which is inconsistent with her (implicit) report, thereby reducing the dimensionality of the problem. In this case, there exists a simpler ranking-based sharing rule which implements the efficient outcome and depends only on observed output. A detailed analysis of this case is provided in Appendix A.

An example which emphasizes the coordination aspect and features the main elements of our model environment can be found in the "trench whistle," which officers in most armies used to communicate isolated timing information to their teams during the First World War. Its high-pitched sound was used to coordinate large scale attacks. At the sound of the officers' whistles, the soldiers would go "over the top" of the trenches and attack the enemy. Note that signaling the attack at the wrong time could result in over- or underexertion of team efforts relative to the efficient level that would benefit or harm the standing of the whistling leader.

A final example of carefully designed incentive structures in partnerships is the nineteenth-century American whaling industry, beautifully described by Hilt (2006). The author reports how managing partners provided appropriate incentives to the whaler's captains and crews on their entirely unobservable multi-year expeditions. During the 1830s, *part of the industry* changed its structure from the previously unincorporated partnerships to corporative ownership. "This represented a significant departure from the traditional reliance on concentrated ownership to resolve incentive conflicts in the industry, and it failed: none of the whaling corporations survived beyond the 1840s, and few experienced much financial success, at a time the American whaling industry as a whole continued to expand." (Hilt, 2006, p. 198)

Appendix A: Leading by example

In this Appendix we return to the symmetric setting of Section 2, but we change the structure of the interaction by allowing the leader to choose her effort before the other players. This effort is assumed to be observable by her team members, with everything else the same as in the base model. We show that in such a case, there exists a simpler ranking-based sharing rule that implements the efficient outcome. This sharing rule will depend only on observed output.

To show this result we consider the following sequential game: at the first stage, the leader chooses effort e_1. Then, at the second stage, all other players $j \neq 1$ observe e_1 and choose their own efforts $e_j(e_1(\cdot))$. Following this stage, a noisy ranking of all players' efforts is realized. The winner receives a fraction s^1 of the team's output and each of the losers receives a share $(1 - s^1)/(n - 1)$.

In this environment, the sharing rule may be conditioned on the observed output y^* alone because the leader's effort e_1 is observed by all other players before they choose their own efforts. Thus, the leader's effort serves as a signal of the team's productivity parameter α. Moreover, and this is crucial, this time structure limits the strategic possibilities of the leader.

While in the original game—in which everyone chooses efforts simultaneously—the leader was able to deviate in both her report α' and the chosen effort (and so multi-dimensional deviations had to be taken into account), one of these channels is shut here. In the present structure, a misreport is more costly to the leader, as she cannot report α' and subsequently choose an effort that is inconsistent with this report. Therefore, a report of productivity α is not needed to implement the efficient allocation as it can be deduced from the leader's effort choice.

Proposition 4. *Assume the sequential game described above. The sharing rule consisting of*

$$s^1(y^*) = \frac{1}{n} + \frac{n-1}{n}\frac{c'(e^*(\breve{\alpha}(y^*))) - \frac{y_2\left(\breve{\alpha}(y^*);e^*(\breve{\alpha}(y^*)),\boldsymbol{e}^*_{-i}(\breve{\alpha}(y^*))\right)}{n}}{y^* f_1^1(e^*(\breve{\alpha}(y^*)),\boldsymbol{e}^*_{-i}(\breve{\alpha}(y^*)))} \tag{42}$$

and $s^j(y^*) = \frac{1-s^1(y^*)}{n-1}$ *for all* $j \neq 1$ *in which* $\breve{\alpha}(y^*)$ *is the solution to* $y^* = y(\alpha; e_1(\alpha), \boldsymbol{e}_{-1}(\alpha))$ *implements efficient efforts.*

Proof. We show that this sharing rule induces *i*) the leader to choose $e_1 = e^*(\alpha)$, and *ii*) all the other agents to follow the leader and choose e_1 as well. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents follow the described strategy. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$y\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right)\left[f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)s^1(y^*) + \left(1 - f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right)\frac{1-s^1(y^*)}{n-1}\right] - c(e). \tag{43}$$

The derivative of the last expression with respect to e is given by

$$\begin{aligned} &y_2\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right)\left[f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)s^1(y^*) + \left(1 - f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right)\frac{1-s^1(y^*)}{n-1}\right] \\ &- c'(e) + y\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right)\left[f_1^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\left(\frac{ns^1(y^*)}{n-1} - \frac{1}{n-1}\right)\right. \\ &\left. + s^{1'}(y^*)\left(f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right) - \frac{\left(1 - f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right)}{n-1}\right)y_2\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right]. \end{aligned} \tag{44}$$

For $e = e_1$ to be an equilibrium, it must be the case that the last derivative at point $e = e_1$ is 0. Since $f^1\left(e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha)\right) = 1/n$, we get

$$\begin{aligned} &c'(e^*(\alpha)) - \frac{y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha))}{n} \\ &= y(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha))\left[f_1^1\left(e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha)\right)\left(\frac{ns^1(y^*)}{n-1} - \frac{1}{n-1}\right)\right]. \end{aligned} \tag{45}$$

Inserting

$$s^1(y^*) = \frac{1}{n} + \frac{n-1}{n}\frac{c'(e^*(\breve{\alpha}(y^*))) - \frac{y_2\left(\breve{\alpha}(y^*);e^*(\breve{\alpha}(y^*)),\boldsymbol{e}^*_{-j}(\breve{\alpha}(y^*))\right)}{n}}{y^* f_1^1(e^*(\breve{\alpha}(y^*)),\boldsymbol{e}^*_{-j}(\breve{\alpha}(y^*)))} \tag{46}$$

and noticing that in equilibrium $\breve{\alpha}(y^*) = \alpha$ gives the required condition.

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α is given by

$$y(\alpha; e_1, \ldots, e_1)\frac{1}{n} - c(e_1). \tag{47}$$

This is maximized at

$$c'(e_1) = \frac{1}{n}\sum_{j=2}^{n+1} y_j(\alpha; e_1, \ldots, e_1). \tag{48}$$

By symmetry, $y_j(\alpha; e_1, \ldots, e_1) = y_k(\alpha; e_1, \ldots, e_1)$ for $j, k \in \{2, \ldots, n+1\}$. Thus, the efficient effort level, given state of the world α, maximizes player 1's payoff. □

Example 7. We continue our example with $y = \alpha \sum_i e_i$, $c(e_i) = e_i^2/2$, and $f^1(e_i, \boldsymbol{e}_{-i}) = e_i^r / \sum e_j^r$ by replacing the simultaneous game with the sequential structure described above. In this sequential game, a contest with shares

$$s^1 = \frac{(n-1)+r}{nr}, \qquad s^j = \frac{r-1}{nr} \text{ for } j \neq 1 \tag{49}$$

implements efficiency. To see this, note that on observing the effort choice of player 1, e_1, players $j \neq 1$ believe that the productivity parameter of the team is $\alpha = e_1$. Given sharing rule (49), it is a best reply of the uninformed players to follow their leader by choosing exactly $e_j = e_1$. At the first stage, anticipating that the uninformed players are going to follow suit by choosing $e_j = e_1$, player 1's best strategy is to choose effort $e_1 = \alpha$, thus communicating the true state of world and implementing efficiency. Note that sharing rule (49) is independent of output. That this is the case is due only to the fact that our example uses a linear production function; output independence is not obtained in general.◁

In a setup with linear production and quadratic effort costs, Hermalin (1998, p. 1192) finds that leading by example is superior to a range of other mechanisms. Nevertheless, leading by example fails to achieve full efficiency because the usual moral hazard problem persists. The reason for this failure is the fixed sharing rule that Hermalin (1998) uses throughout the paper. We show that a well-designed contest, by orchestrating competition among the players, removes the free-riding incentives while ensuring truthful information revelation.

In the discussion of Section 3 we explain that fixed shares can generally not provide incentives for the efficient provision of efforts. This remains true with upfront exertion of observable efforts by a privately informed leader. The fact that efficiency is easier to implement in the sequential structure than in the simultaneous structure echoes the finding in Zhou & Chen (2015) that in network structures, players' contributions increase when they move sequentially rather than simultaneously.

We have thus far assumed that only final output is contractible. If, however, the leader's upfront effort is also contractible, then the next Proposition states that there exists a sharing rule that provides the correct incentives to all agents and conditions only on the leader's observed effort.

Proposition 5. *Assume the sequential game described above with contractible leader's effort. In this game, there exists a sharing rule that depends solely upon the leader's effort that implements efficiency.*

Proof. We show that the following winner's share

$$s^1(e_1) = \frac{1}{n} + \frac{n-1}{n} \frac{c'(e_1) - \frac{y_2(\alpha^{**}(e_1); e_1, \boldsymbol{e}_1)}{n}}{y(\alpha^{**}(e_1); \boldsymbol{e}_1) f_1^1(e_1, \boldsymbol{e}_1)} \tag{50}$$

where $\alpha^{**}(e_1)$ is the solution to $e^*(\alpha^{**}) = e_1$, together with $s^j(y^*) = \frac{1-s^1(y^*)}{n-1}$ for all $j \neq 1$, induces *i)* the leader to choose $e_1 = e^*(\alpha)$, and *ii)* all the other agents to follow the leader and choose e_1 as well. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents choose $e_1 = e^*(\alpha)$. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$y\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right) \left[f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right) s^1 + \left(1 - f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right) \frac{1-s^1}{n-1} \right] - c(e). \tag{51}$$

Since $s^1(e_1)$ is independent of the realized output, agent j cannot affect it. The derivative of the last expression with respect to e is given by

$$\begin{aligned} &y_2\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right) \left[f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right) s^1 + \left(1 - f^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right)\right) \frac{1-s^1}{n-1} \right] \\ &\quad + y\left(\alpha; e, \boldsymbol{e}^*_{-j}(\alpha)\right) \left[f_1^1\left(e, \boldsymbol{e}^*_{-j}(\alpha)\right) \left(\frac{ns^1}{n-1} - \frac{1}{n-1} \right) \right] - c'(e). \end{aligned} \tag{52}$$

For $e = e_1$ to be an equilibrium, it must be the case that the derivative is 0 at this point. Since $f^1(e^*(\alpha), \boldsymbol{e_{-j}}^*(\alpha)) = 1/n$ and $c'(e^*(\alpha)) = y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha))$, we obtain

$$c'(e^*(\alpha)) - \frac{y_2\left(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-j}(\alpha)\right)}{n} = y(\alpha; \boldsymbol{e}^*(\alpha)) \left[f_1^1(e^*(\alpha), \boldsymbol{e_{-j}}^*(\alpha)) \left(\frac{ns^1}{n-1} - \frac{1}{n-1} \right) \right] \tag{53}$$

which implies

$$s^1(e_1) = \frac{1}{n} + \frac{n-1}{n} \frac{c'(e_1) - \frac{y_2(\alpha; e_1, \boldsymbol{e}_1)}{n}}{y(\alpha^{**}; \boldsymbol{e}_1) f_1^1(e_1, \boldsymbol{e}_1)}. \tag{54}$$

Since in equilibrium $\alpha^{**}(e_1) = \alpha$, player j indeed chooses e_1 under sharing rule (50).

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α is given by

$$y(\alpha; e_1, \ldots, e_1) \frac{1}{n} - c(e_1). \tag{55}$$

This is maximized at

$$c'(e_1) = \frac{1}{n} \sum_{j=2}^{n+1} y_j(\alpha; e_1, \ldots, e_1). \tag{56}$$

By symmetry, $y_j(\alpha; e_1, \ldots, e_1) = y_k(\alpha; e_1, \ldots, e_1)$ for $j, k \in \{2, \ldots, n+1\}$. Thus, the efficient effort level, given the state of the world α, maximizes player 1's payoff. □

Appendix B: Proofs

Proof of Proposition 1. We prove the Proposition in two steps. In step 1 we show that, given the leader's report α', it is a mutually best response for the leader and the other players to choose effort $e^*(\alpha')$ under sharing rule (8). In step 2 we show that, anticipating the equilibrium at the second stage, the leader's optimal strategy is to report truthfully.

Step 1. For a given report α', given that every other player chooses $e^*(\alpha')$, the team-leader's first-order condition with respect to effort choice is

$$\begin{aligned}\frac{\partial u_1(e_1, \boldsymbol{e}^*_{-1}(\alpha'), \alpha)}{\partial e_1} &= y_2(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha') \right) \\ &+ y(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n} f_1^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha') \right) \\ &+ y(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) y_2(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) s_1^\ell(y^*, \alpha') \right) \\ &- c'(e_1),\end{aligned} \tag{57}$$

which equals

$$\begin{aligned}&y_2(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha') \right) - c'(e_1) \\ &+ y(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n-1} f_1^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha') \right) \right) \\ &+ y(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) y_2(\alpha; e_1, \boldsymbol{e}^*_{-1}(\alpha')) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) s_1^\ell(y^*, \alpha') \right)\end{aligned} \tag{58}$$

where $f_1^\ell(e_1, \boldsymbol{e}_{-1}) = \frac{\partial}{\partial e_1} f^\ell(e_1, \boldsymbol{e}_{-1})$. The equality holds because of equation (2), budget-balancedness, and $\sum_\ell f_1^\ell(e_1, \boldsymbol{e}^*_{-1}(\alpha')) = \sum_\ell s_1^\ell(y^*, \alpha') = 0$. In the case of equal effort levels of all agents, $e_1 = e^*(\alpha')$, $f^\ell(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')) = 1/n$ for all $\ell = 1, \ldots, n$. Then, setting the first-order condition with respect to effort choice to zero gives

$$\begin{aligned} 0 \;&= \frac{\partial u_1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'), \alpha)}{\partial e_1} = \frac{y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))}{n} \\ &+ y^* \left(\sum_{\ell=1}^{n-1} f_1^\ell(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha')\right) \right) - c'(e^*(\alpha')) \end{aligned} \tag{59}$$

with output in equilibrium $y^* = y(\alpha, ne^*(\alpha'))$. Therefore, we get that

$$\sum_{\ell=1}^{n-1} f_1^\ell(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha')\right) = \frac{c'(e^*(\alpha')) - y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))/n}{y^*}. \tag{60}$$

Under the simple prize structure with $s^2(y^*, \alpha') = \cdots = s^n(y^*, \alpha')$, this gives

$$f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')) \left(s^1(y^*, \alpha') - s^n(y^*, \alpha')\right) = \frac{c'(e^*(\alpha')) - y_2(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))/n}{y^*}. \tag{61}$$

This equation is satisfied under sharing rule (8), proving that it is indeed a best reply of the leader to choose $e^*(\alpha')$ given every other player choosing $e^*(\alpha')$. A similar argument implies that all the uninformed agents also prefer to exert the report-contingent efficient effort level $e^*(\alpha')$ for any report of the team leader α'.

Step 2. We now have to show that, at the first stage, player 1 announces the true signal. At the reporting stage, player 1 who exerts efforts $e^*(\alpha')$ reports α' so as to maximize expected utility (6), i.e.,

$$\max_{\alpha'} u_1(e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha'), \alpha) \; = \max_{\alpha'} \left(\frac{y\left(\alpha; e^*(\alpha'), \boldsymbol{e}^*_{-1}(\alpha')\right)}{n} - c(e^*(\alpha')) \right). \tag{62}$$

The first-order condition is

$$\frac{1}{n} \sum_{j=2}^{n+1} y_j(\alpha; e^*(\alpha'), \ldots, e^*(\alpha')) e^{*\prime}(\alpha') - c'(e^*(\alpha')) e^{*\prime}(\alpha') = 0. \tag{63}$$

Recall that symmetry implies $y_j(\alpha; e^*(\alpha'), \ldots, e^*(\alpha')) = y_l(\alpha; e^*(\alpha'), \ldots, e^*(\alpha'))$ for $j, l \in \{2, \ldots, n+1\}$. Therefore, (63) can be rewritten as

$$y_j(\alpha; e^*(\alpha'), \ldots, e^*(\alpha')) = c'(e^*(\alpha')) \tag{64}$$

which exactly coincides with the first-best solution (4). Thus, player 1's expected utility (62) is maximized at the report $\alpha' = \alpha$, implying that player 1 reports truthfully. All team members will find it optimal to accept the contract because each player j will get $1/n$ of the generated efficient social surplus, which must be positive because $y_2(\alpha; 0, \boldsymbol{e}^*_{-j}(\alpha')) > 0$ and $c'(0) = 0$. Therefore, the players' ex-post efficient efforts $e^*(\alpha)$ are implementable. $\square$

Proof of Corollary 1. Recall that the winner's share is

$$s^1(y^*, \alpha') = \frac{1}{n} + \frac{n-1}{ny^*}\left[\frac{c'(e^*(\alpha'))}{f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))} - \frac{y_2(\alpha(y^*, \alpha'); e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))}{f_1^1(e^*(\alpha'), \boldsymbol{e}^*_{-i}(\alpha'))n}\right]. \tag{65}$$

Given the truth-telling behavior of the team leader, we can rewrite the equilibrium winner's share as follows:

$$\begin{aligned}
s^1(y^*, \alpha) &= \frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*} - \frac{y_2(\alpha(y^*, \alpha); e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*n}\right] \\
&= \frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*} - \frac{y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*n}\right] \\
&= \frac{1}{n} + \frac{n-1}{n}\left[\frac{y_2(\alpha; ne^*(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*} - \frac{y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{f_1^1(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))y^*n}\right] \\
&= \frac{1}{n} + \left(\frac{n-1}{n}\right)^2 \frac{1}{f_1^1\left(e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha)\right)} \frac{y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))}{y(\alpha; \boldsymbol{e}^*(\alpha))},
\end{aligned} \tag{66}$$

where the second line follows since $\alpha = \alpha(y^*, \alpha)$ and the third line follows since in the efficient allocation we have $c'(e^*(\alpha)) = y_2(\alpha; e^*(\alpha), \boldsymbol{e}^*_{-i}(\alpha))$. $\square$

Proof of Proposition 2. As in the previous proof, denote by $s^1(y^*, \boldsymbol{\alpha}')$ the share of the winner of the contest if the realized output is y^* and the reported signals are $\boldsymbol{\alpha}' = (\alpha'_1, \ldots, \alpha'_n)$. We will show that the following sharing rule implements both truthful reports at the first stage and efficient efforts at the second stage:

$$\begin{aligned}
s^1(y^*, \boldsymbol{\alpha}') = \frac{1}{n} &+ \frac{n-1}{ny^* f_1^1\left(e^*(\boldsymbol{\alpha}'), \boldsymbol{e}^*_{-i}(\boldsymbol{\alpha}')\right)} \\
&\left[c'(e^*(\boldsymbol{\alpha}')) - \frac{y_{n+1}\left(\alpha(y^*, \boldsymbol{\alpha}'), \boldsymbol{\alpha}'_{-1}; \boldsymbol{e}^*(\boldsymbol{\alpha}')\right)}{n}\right]
\end{aligned} \tag{67}$$

where $s^j(y^*, \boldsymbol{\alpha}') = (1 - s^1(y^*, \boldsymbol{\alpha}'))/(n-1)$, for $j \in \{2, \ldots, n\}$, and $\alpha(y^*, \boldsymbol{\alpha}')$ is the solution to $y^* = y\left(\alpha, \boldsymbol{\alpha}'_{-1}; \boldsymbol{e}^*(\boldsymbol{\alpha}')\right)$.

Due to symmetry it suffices for us to analyze only the incentives of agent 1. The expected utility of agent 1 at the second stage after reports of $\boldsymbol{\alpha}'$ if the state of the world is $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$, all the others exert efforts of $e^*(\boldsymbol{\alpha}')$, while player 1 exerts efforts of e is given by

$$y^*\left[f^1\left(e, \boldsymbol{e}^*_{-1}(\boldsymbol{\alpha}')\right) s^1(y^*, \boldsymbol{\alpha}') + \left(1 - f^1\left(e, \boldsymbol{e}^*_{-1}(\boldsymbol{\alpha}')\right)\right) \frac{1 - s^1(y^*, \boldsymbol{\alpha}')}{n-1}\right] - c(e). \tag{68}$$

The derivative with respect to e is

$$
\begin{aligned}
& y_{n+1}\left(\boldsymbol{\alpha}'; e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)\left[f^{1}\left(e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right) s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)\right. \\
& \left.+\left(1-f^{1}\left(e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)\right) \frac{1-s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)}{n-1}\right] \\
& +y^{*}\left\{f_{1}^{1}\left(e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)\left[s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)-\frac{1-s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)}{n-1}\right]\right. \\
& \left.+s_{1}^{1}\left(y^{*}, \boldsymbol{\alpha}'\right) y_{n+1}\left(\boldsymbol{\alpha}'; e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)\left[f^{1}\left(e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)-\frac{1-f^{1}\left(e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)}{n-1}\right]\right\}-c'(e) .
\end{aligned} \tag{69}
$$

We show that this derivative is equal to zero for $e=e^{*}\left(\boldsymbol{\alpha}'\right)$. Note that for $e=e^{*}\left(\boldsymbol{\alpha}'\right)$ we can rewrite the last expression as

$$
y_{n+1}\left(\boldsymbol{\alpha}'; e, \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right) \frac{1}{n}-c'\left(e^{*}\left(\boldsymbol{\alpha}'\right)\right)+y^{*} f_{1}^{1}\left(e^{*}\left(\boldsymbol{\alpha}'\right), \boldsymbol{e}_{-1}^{*}\left(\boldsymbol{\alpha}'\right)\right)\left[\frac{n}{n-1} s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)-\frac{1}{n-1}\right] . \tag{70}
$$

Assuming that all players but player 1 report their signals truthfully and that $\alpha_{j}'=\alpha_{j}$ for all $j \neq 1$, we can rewrite the last expression as follows:

$$
\begin{aligned}
& y_{n+1}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}; e, \boldsymbol{e}_{-1}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right) \frac{1}{n}-c'\left(e^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right) \\
& +y^{*} f_{1}^{1}\left(e^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right), \boldsymbol{e}_{-1}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)\left[\frac{n}{n-1} s1\left(y^{*}, \alpha_{1}', \boldsymbol{\alpha}_{-1}\right)-\frac{1}{n-1}\right] .
\end{aligned} \tag{71}
$$

This expression equals zero with the given sharing rule $s^{1}\left(y^{*}, \boldsymbol{\alpha}'\right)$. Thus it is a best reply of player 1 to choose $e^{*}(\boldsymbol{\alpha}')$.

At the first stage, agent 1 reports α_{1}' to maximize his expected utility, given that his opponents report their types truthfully. Since our sharing rule implements efficient efforts as ex-post equilibrium, it induces truth telling even if agent 1 knows the signals of all the other players.

$$
\max_{\alpha_{1}'}\left(\frac{y\left(\alpha_{1}, \boldsymbol{\alpha}_{-1}; \boldsymbol{e}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)}{n}-c\left(e^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)\right) . \tag{72}
$$

The first-order condition with respect to α_{1}' is

$$
\frac{1}{n} \sum_{j=1}^{n} y_{n+j}\left(\alpha_{1}, \boldsymbol{\alpha}_{-1}; \boldsymbol{e}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right) e_{1}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)-c'\left(e^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right) e_{1}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)=0 . \tag{73}
$$

Symmetry implies that $y_{n+j}\left(\alpha_{1}, \boldsymbol{\alpha}_{-1}; \boldsymbol{e}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)=y_{n+l}\left(\alpha_{1}, \boldsymbol{\alpha}_{-1}; \boldsymbol{e}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)$ for $j, l \in\{1, \ldots, n\}$. Therefore, we can rewrite the last equality as

$$
y_{n+j}\left(\alpha_{1}, \boldsymbol{\alpha}_{-1}; \boldsymbol{e}^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right)=c'\left(e^{*}\left(\alpha_{1}', \boldsymbol{\alpha}_{-1}\right)\right), \tag{74}
$$

which holds at $\alpha_{1}'=\alpha_{1}$ and coincides with the first-best. □

Proof of Proposition 3. Consider first the case in which either set A or B has at least two members, i.e., either $n_1 \geq 2$ and $n_2 \geq 1$ or $n_1 \geq 2$ and $n_2 \geq 1$. We first derive (15) from the effort stage for given report α' by imposing the restriction that it is a mutually best reply of each player to choose $e^*(\alpha')$. The derivation shows that for implementing report-contingent efficiency at the second stage, there is a degree of freedom in the sharing rule captured by $C(\alpha')$ that is independent of the realized output. We then proceed to show that at the first reporting stage, $C(\alpha')$ can be chosen to provide exactly the right incentive for the leader to report truthfully.

Effort stage. When the state of the world is α, the reported state is α', and all other agents exert efficient efforts given the report, then the expected utility of informed player 1 if she exerts effort e_1 is

$$\begin{aligned} u_1(e_1, \boldsymbol{e}^*_{-1}(\alpha'), \alpha) = & \frac{e_1}{e_1 + (n_1 - 1)\, e^*_A(\alpha') + n_2 e^*_B(\alpha')} s^A(y^*, \alpha')\, y^* \\ & + \frac{n_2 e^*_B(\alpha')}{e_1 + (n_1 - 1)\, e^*_A(\alpha') + n_2 e^*_B(\alpha')} \frac{1 - s^B(y^*, \alpha')}{n_1} y^* - \frac{\gamma e_1^2}{2} \end{aligned} \tag{75}$$

where $y^* = \alpha(e_1 + (n_1 - 1)e^*_A(\alpha') + n_2 e^*_B(\alpha'))$. Setting the derivative with respect to e_1 to zero and substituting $e_1 = e^*_A(\alpha') = \frac{\alpha'}{\gamma}$ and $e^*_B(\alpha') = \alpha'$ gives

$$\alpha\left(\alpha\alpha'\left(n_1 s^A_1(y^*, \alpha') - \gamma n_2 s^B_1(y^*, \alpha')\right) + \gamma n_1 s^A(y^*, \alpha')\right) - \alpha'\gamma n_1 = 0. \tag{76}$$

Solving for $s^A(y^*, \alpha')$ gives

$$s^A(y^*, \alpha') = \frac{\alpha'\left(\alpha^2\left(\gamma n_2 s^B_1(y^*, \alpha') - n_1 s^A_1(y^*, \alpha')\right) + \gamma n_1\right)}{\alpha\gamma n_1}. \tag{77}$$

Analyzing the expected utility maximization problem of an agent from group B in the same way yields $s^B(y^*, \alpha')$:

$$s^B(y^*, \alpha') = \frac{\alpha'\left(\alpha^2\left(n_1 s^A_1(y^*, \alpha') - \gamma n_2 s^B_1(y^*, \alpha')\right) + \gamma n_2\right)}{\alpha\gamma n_2}. \tag{78}$$

Putting together expressions (77) and (78) and substituting $\alpha = \frac{\gamma y^*}{\alpha'(n_1 + \gamma n_2)}$ leads to

$$\begin{aligned} s^A(y^*, \alpha') &= \frac{\gamma y^{*2}\left(\gamma n_2 s^B_1(y^*, \alpha') - n_1 s^A_1(y^*, \alpha')\right) + \alpha'^2 n_1 (n_1 + \gamma n_2)^2}{\gamma n_1 y^* (n_1 + \gamma n_2)}, \\ s^B(y^*, \alpha') &= \frac{\gamma y^{*2}\left(n_1 s^A_1(y^*, \alpha') - \gamma n_2 s^B_1(y^*, \alpha')\right) + \alpha'^2 n_2 (n_1 + \gamma n_2)^2}{\gamma n_2 y^* (n_1 + \gamma n_2)}. \end{aligned} \tag{79}$$

From this we get

$$n_1 s^A(y^*, \alpha') + n_2 s^B(y^*, \alpha') = \frac{\alpha'^2 (n_1 + n_2)(n_1 + \gamma n_2)}{\gamma y^*} \tag{80}$$

with derivative

$$n_1 s_1^A(y^*,\alpha') + n_2 s_1^B(y^*,\alpha') = -\frac{\alpha'^2(n_1+n_2)(n_1+\gamma n_2)}{\gamma y^{*2}}. \tag{81}$$

Using this in combination with the first equation in (79) gives the differential equations

$$\alpha'^2\left(n_1^2+\gamma n_1(n_2-1)-\gamma n_2\right)(n_1+\gamma n_2) = \gamma n_1 y^*\left((n_1+\gamma n_2)s^A(y^*,\alpha') + (\gamma+1)y^* s_1^A(y^*,\alpha')\right). \tag{82}$$

which are solved by

$$s^A(y^*,\alpha') = \frac{\alpha'^2(n_1+\gamma n_2)\left(n_1^2+\gamma n_1(n_2-1)-\gamma n_2\right)}{\gamma n_1 y^*(n_1+\gamma(n_2-1)-1)} + C(\alpha')((\gamma+1)y^*)^{-\frac{n_1+\gamma n_2}{\gamma+1}}, \tag{83}$$

where $C(\alpha')$ is some constant for the given α'. Using (80), we get

$$s^B(y^*,\alpha') = \frac{\alpha'^2(n_1+\gamma n_2)(n_1(n_2-1)+n_2(\gamma n_2-1))}{\gamma y^*(n_1+\gamma(n_2-1)-1)n_2} - \frac{n_1 C(\alpha')((\gamma+1)y^*)^{-\frac{n_1+\gamma n_2}{\gamma+1}}}{n_2}. \tag{84}$$

Report stage. At the reporting stage, the expected utility of player 1 if the state of the world is α and he reports α' is given by

$$e_A^*(\alpha')\,\alpha s^A(y^*,\alpha') + \tfrac{n_2}{n_1} e_B^*(\alpha')\,\alpha\left(1-s^B(y^*,\alpha')\right) - \tfrac{\gamma\left(e_A^*(\alpha')\right)^2}{2}. \tag{85}$$

Plugging $e_A^*(\alpha') = \frac{\alpha'}{\gamma}$, $e_B^*(\alpha') = \alpha'$, $s^A(y^*,\alpha')$, and $s^B(y^*,\alpha')$ into (83)–(84) and substituting $y^* = \frac{\alpha'\alpha}{\gamma}(n_1+\gamma n_2)$ yields

$$\begin{aligned} &\frac{1}{\gamma}C(\alpha')\alpha\alpha'(\gamma+1)\left(\frac{\alpha\alpha'(\gamma+1)(n_1+\gamma n_2)}{\gamma}\right)^{-\frac{n_1+\gamma n_2}{\gamma+1}} \\ &\quad+\frac{\alpha'\left(\alpha' n_1^2+n_1(\alpha'+\gamma(\alpha'+2\alpha n_2-\alpha' n_2))-2\gamma n_2(\alpha+\gamma(\alpha-\alpha n_2+\alpha' n_2))\right)}{2\gamma n_1(n_1+\gamma(n_2-1)-1)}. \end{aligned} \tag{86}$$

The derivative of this expression with respect to α' is

$$\begin{aligned} &\alpha n_1\left(\frac{\alpha\alpha'(\gamma+1)(n_1+\gamma n_2)}{\gamma}\right)^{-\frac{n_1+\gamma n_2}{\gamma+1}}(\alpha'(\gamma+1)C'(\alpha')+C(\alpha')(\gamma-n_1-\gamma n_2+1)) \\ &\quad+\frac{\alpha' n_1^2+n_1(\alpha'+\gamma(\alpha'+n_2(\alpha-\alpha')))-\gamma n_2(\alpha+\gamma(\alpha-\alpha n_2+2\alpha' n_2))}{n_1+\gamma(n_2-1)-1}. \end{aligned} \tag{87}$$

This must be equal to zero for $\alpha'=\alpha$; that is, $C(\alpha')$ must satisfy

$$\begin{aligned} &n_1(n_1+\gamma(n_2-1)-1)\left(\alpha'(\gamma+1)C'(\alpha')+C(\alpha')(\gamma-n_1-\gamma n_2+1)\right) \\ +\;&(\gamma+n_1+\gamma n_2+1)(n_1-\gamma n_2)\left(\tfrac{(\alpha')^2(\gamma+1)(n_1+\gamma n_2)}{\gamma}\right)^{\frac{n_1+\gamma n_2}{\gamma+1}} = 0, \end{aligned} \tag{88}$$

which gives

$$C(\alpha') = \alpha'^{\frac{n_1-n_2}{\gamma+1}+n_2-1}C2 + \frac{(\gamma n_2 - n_1)\left(\frac{\alpha'^2(\gamma+1)(n_1+\gamma n_2)}{\gamma}\right)^{\frac{n_1+\gamma n_2}{\gamma+1}}}{n_1(n_1+\gamma(n_2-1)-1)} \tag{89}$$

for constant $C2$. Setting $C2$ to zero yields the claimed sharing rule.

Now consider the case in which both sets A and B have only a single member, i.e., $n_1 = n_2 = 1$. A derivation following exactly the same steps as above yields the shares

$$\begin{aligned} s^A(y^*, \alpha') &= \frac{\alpha'^2(1-\gamma)\log(y^*)}{\gamma y^*} + \frac{C(\alpha')}{y^*}, \\ s^B(y^*, \alpha') &= \frac{\alpha'^2(2(\gamma+1)+(\gamma-1)\log(y^*))}{\gamma y^*} - \frac{C(\alpha')}{y^*} \end{aligned} \tag{90}$$

for constant

$$C(\alpha) = \frac{\alpha'^2(\gamma+1) + \alpha'^2(\gamma-1)\log\left(\alpha'^2\frac{(\gamma+1)}{\gamma}\right)}{\gamma} + C2 \tag{91}$$

for arbitrary constant $C2$. □

Derivation of Example 2. We show that the following sharing rule implements efficient efforts

$$\begin{aligned} s1\left(y^*, \alpha_1', \alpha_2'\right) &= \frac{\left(\alpha_1' + \beta\alpha_2'\right)^2(1-\gamma)\log(y^*)}{\gamma y^*} + \frac{C(\alpha_1', \alpha_2')}{y^*} \\ s2\left(y^*, \alpha_1', \alpha_2'\right) &= \frac{\left(\alpha_1' + \beta\alpha_2'\right)^2(2(\gamma+1)+(\gamma-1)\log(y^*))}{\gamma y^*} - \frac{C(\alpha_1', \alpha_2')}{y^*}, \end{aligned} \tag{92}$$

where

$$C(\alpha_1', \alpha_2') = \frac{(\alpha_1' + \alpha_2'\beta)^2\left((\gamma-1)\log\left(\frac{(\gamma+1)(\alpha_1'+\alpha_2'\beta)^2}{\gamma}\right) + \gamma + 1\right)}{\gamma}, \tag{93}$$

where $sj\left(y^*, \alpha_1', \alpha_2'\right)$ and $1 - sj\left(y^*, \alpha_1', \alpha_2'\right)$ are the winning and losing shares, respectively, if player $j \in \{1, 2\}$ wins.

At the effort stage, the expected utility of player 1 if he exerts effort e_1, the true state of the world is (α_1, α_2), the reports are (α_1', α_2'), and player 2 exerts effort $e_2^*\left(\alpha_1', \alpha_2'\right)$ is

$$\frac{e_1}{e_1 + e_2^*\left(\alpha_1', \alpha_2'\right)} s1\left(y^*, \alpha_1', \alpha_2'\right) y^* + \frac{e_2^*\left(\alpha_1', \alpha_2'\right)}{e_1 + e_2^*\left(\alpha_1', \alpha_2'\right)}\left(1 - s2\left(y^*, \alpha_1', \alpha_2'\right)\right) y^* - c_1(e_1). \tag{94}$$

Plugging in $e_2^*\left(\alpha_1', \alpha_2'\right) = \alpha_1' + \beta\alpha_2'$ and $y^* = (\alpha_1 + \beta\alpha_2)(e_1 + e_2^*(\alpha_1', \alpha_2'))$ yields

$$e_1 s1\left(y^*, \alpha_1', \alpha_2'\right)(\alpha_1 + \beta\alpha_2) + \left(\alpha_1' + \beta\alpha_2'\right)\left(1 - s2\left(y^*, \alpha_1', \alpha_2'\right)\right)(\alpha_1 + \beta\alpha_2) - \gamma\frac{e_1^2}{2}. \tag{95}$$

Taking the derivative with respect to e_1 and equalizing it to zero for $e_1 = \frac{\alpha_1' + \beta\alpha_2'}{\gamma}$ gives

$$\begin{aligned} & s1\left(y^*, \alpha_1', \alpha_2'\right)(\alpha_1 + \beta\alpha_2) + \frac{\alpha_1' + \beta\alpha_2'}{\gamma} s1_1\left(y^*, \alpha_1', \alpha_2'\right)(\alpha_1 + \beta\alpha_2)^2 - \\ & - \left(\alpha_1' + \beta\alpha_2'\right) s2_1\left(y^*, \alpha_1', \alpha_2'\right)(\alpha_1 + \beta\alpha_2)^2 - \left(\alpha_1' + \beta\alpha_2'\right) = 0. \end{aligned} \tag{96}$$

The expected utility of player 2 if he exerts effort e_2, the true state of the world is (α_1, α_2), the reports are (α_1', α_2'), and player 1 exerts effort $e_1^*(\alpha_1', \alpha_2')$ is

$$\frac{e_2}{e_2 + e_1^*(\alpha_1', \alpha_2')} s2(y^*, \alpha_1', \alpha_2') y^* + \frac{e_1^*(\alpha_1', \alpha_2')}{e_2 + e_1^*(\alpha_1', \alpha_2')} (1 - s1(y^*, \alpha_1', \alpha_2')) y^* - c_2(e_2). \quad (97)$$

Taking the derivative with respective to e_2 and equalizing it to zero for $e_2 = \alpha_1' + \beta\alpha_2'$ gives

$$\begin{aligned} & s2(y^*, \alpha_1', \alpha_2')(\alpha_1 + \beta\alpha_2) + (\alpha_1' + \beta\alpha_2') s2_1(y^*, \alpha_1', \alpha_2')(\alpha_1 + \beta\alpha_2)^2 - \\ & -\frac{\alpha_1' + \beta\alpha_2'}{\gamma} s1_1(y^*, \alpha_1', \alpha_2')(\alpha_1 + \beta\alpha_2)^2 - (\alpha_1' + \beta\alpha_2') = 0. \end{aligned} \quad (98)$$

The shares (92) are the solutions to the two differential equations (96) and (98). Thus, given the claimed sharing rule, the players' mutual best replies are to choose effort $e_i^*(\alpha_1', \alpha_2')$.

We now proceed to the first stage. The expected utility of player 1 if the state of the world is (α_1, α_2), player 2 reports truthfully, player 1 reports α_1', and both players exert effort $e_i^*(\alpha_1', \alpha_2)$ is given by

$$e_1^*(\alpha_1', \alpha_2) s1(y^*, \alpha_1', \alpha_2)(\alpha_1 + \beta\alpha_2) + e_2^*(\alpha_1', \alpha_2)(1 - s2(y^*, \alpha_1', \alpha_2))(\alpha_1 + \beta\alpha_2) - c_1(e_1^*(\alpha_1', \alpha_2)). \quad (99)$$

Plugging in $s1(y^*, \alpha_1', \alpha_2)$, $s2(y^*, \alpha_1', \alpha_2)$, and $e_1^*(\alpha_1', \alpha_2) = \frac{\alpha_1' + \beta\alpha_2}{\gamma}$, and $e_2^*(\alpha_1', \alpha_2) = \alpha_1' + \beta\alpha_2$, and rearranging yields

$$\begin{aligned} & \frac{1}{2\gamma}\Bigg(2\gamma C(\alpha_1', \alpha_2') \\ & - (\alpha_1' + \alpha_2'\beta)\Bigg(2(\gamma - 1)(\alpha_1' + \alpha_2'\beta)\log\left(\frac{(\gamma + 1)(\alpha_1 + \alpha_2\beta)(\alpha_1' + \alpha_2'\beta)}{\gamma}\right) \\ & - 2\gamma(\alpha_1 + \alpha_2\beta - 2\alpha_2'\beta) + 4\alpha_1'\gamma + \alpha_1' + \alpha_2'\beta\Bigg)\Bigg). \end{aligned} \quad (100)$$

Taking the derivative with respect to α_1' and equalizing it to zero at $\alpha_1 = \alpha_1' = \alpha_1$ gives

$$C_1(\alpha_1', \alpha_2') = \frac{2(\alpha_1' + \alpha_2'\beta)\left((\gamma - 1)\log\left(\frac{(\gamma+1)(\alpha_1' + \alpha_2'\beta)^2}{\gamma}\right) + 2\gamma\right)}{\gamma}. \quad (101)$$

Solving player 2's problem in the same way gives

$$C_2(\alpha_1', \alpha_2') = \frac{2\beta(\alpha_1' + \alpha_2'\beta)\left((\gamma - 1)\log\left(\frac{(\gamma+1)(\alpha_1' + \alpha_2'\beta)^2}{\gamma}\right) + 2\gamma\right)}{\gamma}. \quad (102)$$

The differential equations (101) and (102) are solved by

$$\begin{aligned} C(\alpha_1', \alpha_2') &= \frac{(\alpha_1' + \alpha_2'\beta)^2\left((\gamma - 1)\log\left(\frac{(\gamma+1)(\alpha_1' + \alpha_2'\beta)^2}{\gamma}\right) + \gamma + 1\right)}{\gamma} + C2(\alpha_2'), \\ C(\alpha_1', \alpha_2') &= \frac{(\alpha_1' + \alpha_2'\beta)^2\left((\gamma - 1)\log\left(\frac{(\gamma+1)(\alpha_1' + \alpha_2'\beta)^2}{\gamma}\right) + \gamma + 1\right)}{\gamma} + C1(\alpha_1') \end{aligned} \quad (103)$$

for constants $C1(\alpha_1')$ and $C2(\alpha_2')$. Setting $C1(\alpha_1') = C2(\alpha_2') = 0$ delivers the claimed sharing rules. □

Derivation of Example 3. Consider the second stage first. We want to check the possibility of inducing efficient effort even if the player deviated at the first stage. Consider player 1. His utility from effort e after a report of α_1' if the second player reports truthfully $\alpha_2' = \alpha_2$ and exerts the efficient effort $e_2^* = \alpha_2$ is given by

$$\left[\frac{e}{e+\alpha_2} s1\left(\alpha_1', \alpha_2, y\right) + \frac{\alpha_2}{e+\alpha_2}\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)\right] y - \frac{e^2}{2}. \tag{104}$$

Taking the derivative with respect to e and equalizing the expression to zero for $e = \alpha_1'$ gives

$$\begin{aligned}&\left[\frac{\alpha_2 s1\left(\alpha_1', \alpha_2, y\right) - \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\left(\alpha_1' + \alpha_2\right)^2} + \alpha_1 \frac{\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right)}{\alpha_1' + \alpha_2}\right] y \\ &+ \alpha_1 \frac{\alpha_1' s1\left(\alpha_1', \alpha_2, y\right) + \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\alpha_1' + \alpha_2} - \alpha_1' = 0.\end{aligned} \tag{105}$$

In order to provide the players with efficient incentives, the last identity must hold for any $\alpha_1, \alpha_1', \alpha_2$. Derive both sides of the last identity with respect to α_1. We get

$$\begin{aligned}&\alpha_1' \left[\frac{\alpha_2 s1\left(\alpha_1', \alpha_2, y\right) - \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\left(\alpha_1' + \alpha_2\right)^2}\right. \\ &\left. + \alpha_1 \frac{\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right)}{\alpha_1' + \alpha_2}\right] + y \frac{\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right)}{\alpha_1' + \alpha_2} \\ &+ \frac{\alpha_1' s1\left(\alpha_1', \alpha_2, y\right) + \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\alpha_1' + \alpha_2} = 0.\end{aligned} \tag{106}$$

Taking the derivative with respect to α_1 gives

$$\alpha_1' \frac{\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right)}{\alpha_1' + \alpha_2} + \alpha_1' \frac{\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right)}{\alpha_1' + \alpha_2} = 0 \tag{107}$$

implying that

$$\alpha_1' s1_3\left(\alpha_1', \alpha_2, y\right) - \alpha_2 s2_3\left(\alpha_1', \alpha_2, y\right) = 0. \tag{108}$$

Plugging this back into (106) yields

$$\alpha_1' \frac{\alpha_2 s1\left(\alpha_1', \alpha_2, y\right) - \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\left(\alpha_1' + \alpha_2\right)^2} + \frac{\alpha_1' s1\left(\alpha_1', \alpha_2, y\right) + \alpha_2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right)}{\alpha_1' + \alpha_2} = 0. \tag{109}$$

Multiplying by $\left(\alpha_1' + \alpha_2\right)^2$ and rearranging gives

$$\alpha_1' \alpha_2 s1\left(\alpha_1', \alpha_2, y\right) + \left(\alpha_1'\right)^2 s1\left(\alpha_1', \alpha_2, y\right) + \alpha_2 \alpha_1' s1\left(\alpha_1', \alpha_2, y\right) + \left(\alpha_2\right)^2\left(1 - s2\left(\alpha_1', \alpha_2, y\right)\right) = 0. \tag{110}$$

Since it must hold for any α_1, let us take the derivative of the last equality with respect to α_1. We get

$$\alpha_1' \left[\alpha_1'\alpha_2 s1_3\left(\alpha_1',\alpha_2,y\right)+\left(\alpha_1'\right)^2 s1_3\left(\alpha_1',\alpha_2,y\right)+\alpha_2\alpha_1' s1_3\left(\alpha_1',\alpha_2,y\right)+\left(\alpha_2\right)^2 s2_3\left(\alpha_1',\alpha_2,y\right)\right]=0. \tag{111}$$

The only solution to the system

$$\begin{aligned}\alpha_1'\alpha_2 s1_3\left(\alpha_1',\alpha_2,y\right)+\left(\alpha_1'\right)^2 s1_3\left(\alpha_1',\alpha_2,y\right)+\alpha_2\alpha_1' s1_3\left(\alpha_1',\alpha_2,y\right)+\left(\alpha_2\right)^2 s2_3\left(\alpha_1',\alpha_2,y\right)=0\\ \alpha_1' s1_3\left(\alpha_1',\alpha_2,y\right)-\alpha_2 s2_3\left(\alpha_1',\alpha_2,y\right)=0\end{aligned} \tag{112}$$

is $s1_3\left(\alpha_1',\alpha_2,y\right)=s2_3\left(\alpha_1',\alpha_2,y\right)=0$. Similarly, if player 2 assumes that player 1 truthfully reports his type and exerts efficient effort at the second stage, we get

$$\begin{aligned}&\left[\frac{\alpha_1 s1\left(\alpha_1,\alpha_2',y\right)-\alpha_1\left(1-s2\left(\alpha_1,\alpha_2',y\right)\right)}{\left(\alpha_1+\alpha_2'\right)^2}+\alpha_2\frac{\alpha_2' s2_3\left(\alpha_1,\alpha_2',y\right)-\alpha_1 s1_3\left(\alpha_1,\alpha_2',y\right)}{\alpha_1+\alpha_2'}\right]y\\ &+\alpha_2\frac{\alpha_2' s2\left(\alpha_1,\alpha_2',y\right)+\alpha_1\left(1-s1\left(\alpha_1,\alpha_2',y\right)\right)}{\alpha_1+\alpha_2'}-\alpha_2'=0.\end{aligned} \tag{113}$$

Given that $s2_3\left(\alpha_1,\alpha_2',y\right)=s1_3\left(\alpha_1,\alpha_2',y\right)=0$ and taking the derivative with respect to α_2 gives

$$\alpha_2'\frac{\alpha_1 s1\left(\alpha_1,\alpha_2',y\right)-\alpha_1\left(1-s2\left(\alpha_1,\alpha_2',y\right)\right)}{\left(\alpha_1+\alpha_2'\right)^2}+\frac{\alpha_2' s2\left(\alpha_1,\alpha_2',y\right)+\alpha_1\left(1-s1\left(\alpha_1,\alpha_2',y\right)\right)}{\alpha_1+\alpha_2'}=0. \tag{114}$$

However, there is no solution to the system with $s1\left(\alpha_1,\alpha_2,y\right)\geq 0$,

$$\begin{aligned}\alpha_2\frac{\alpha_1\left[s1\left(\alpha_1,\alpha_2,y\right)+s2\left(\alpha_1,\alpha_2,y\right)-1\right]}{\left(\alpha_1+\alpha_2\right)^2}+\frac{\alpha_2 s2\left(\alpha_1,\alpha_2,y\right)+\alpha_1\left(1-s1\left(\alpha_1,\alpha_2,y\right)\right)}{\alpha_1+\alpha_2}&=0\\ \alpha_1\frac{\alpha_2\left[s1\left(\alpha_1,\alpha_2,y\right)+s2\left(\alpha_1,\alpha_2,y\right)-1\right]}{\left(\alpha_1+\alpha_2\right)^2}+\frac{\alpha_1 s1\left(\alpha_1,\alpha_2,y\right)+\alpha_2\left(1-s2\left(\alpha_1,\alpha_2,y\right)\right)}{\alpha_1+\alpha_2}&=0.\quad\square\end{aligned}$$

Derivation of the sharing rule for Example 6 (incomplete ranking). First consider player $i\in\mathcal{A}$. Given the reported state α' when the true state is α, player i exerts effort e_i, while the other $n-1$ players exert efficient report-contingent efforts $e^*(\alpha')=\alpha'$. In this case, player i's expected utility is

$$\alpha\left(e_i+(n-1)\alpha'\right)\left(\frac{e_i^r}{e_i^r+(n_1-1)(\alpha')^r}s1\left(y^*,\alpha'\right)+\frac{(n_1-1)(\alpha')^r}{e_i^r+(n_1-1)(\alpha')^r}s2\left(y^*,\alpha'\right)\right)-\frac{e_i^2}{2}. \tag{115}$$

The derivative with respect to e_i is

$$\begin{aligned}&\frac{\alpha e_i^r \mathrm{s1}(y^*,\alpha')}{e_i^r+(n_1-1)(\alpha')^r}+\frac{\alpha(n_1-1)(\alpha')^r\mathrm{s2}(y^*,\alpha')}{e_i^r+(n_1-1)(\alpha')^r}+\frac{\alpha^2((n-1)\alpha'+e_i)e_i^r\mathrm{s1}_1(y^*,\alpha')}{e_i^r+(n_1-1)(\alpha')^r}\\&+\frac{\alpha^2((n-1)\alpha'+e_i)(n_1-1)(\alpha')^r\mathrm{s2}_1(y^*,\alpha')}{e_i^r+(n_1-1)(\alpha')^r}\\&+\frac{\alpha r((n-1)\alpha'+e_i)e_i^{r-1}(n_1-1)\alpha'^r}{(e_i^r+(n_1-1)(\alpha')^r)^2}(\mathrm{s1}(y^*,\alpha')-\mathrm{s2}(y^*,\alpha'))-e_i.\end{aligned} \tag{116}$$

Setting this to zero at $e_i=\alpha'$ gives the first-order condition

$$\begin{aligned}&n_1\alpha\left(s1(y^*,\alpha')+(n_1-1)s2(y^*,\alpha')\right)+\alpha nr(n_1-1)(s1(y^*,\alpha')-s2(y^*,\alpha'))\\&+\alpha^2 nn_1\alpha'\left(s1_1(y^*,\alpha')+(n_1-1)s2_1(y^*,\alpha')\right)-n_1^2\alpha'=0.\end{aligned} \tag{117}$$

Replacing $\alpha=y^*/(n\alpha')$ and rearranging yields

$$\begin{aligned}&y^*n_1\left(s1(y^*,\alpha')+(n_1-1)s2(y^*,\alpha')\right)+y^*rn(n_1-1)(s1(y^*,\alpha')-s2(y^*,\alpha'))\\&+y^{*2}n_1(s1_1(y^*,\alpha')+(n_1-1)s2_1(y^*,\alpha'))-n_1^2\alpha'^2n=0.\end{aligned} \tag{118}$$

We now check the incentives of player $j\in\mathcal{B}$ ($\mathcal{B}$ contains the informed player). His expected utility if the state of the world is α and the other $n-1$ players exert effort $e^*(\alpha')=\alpha'$ while player j exerts effort e_j is

$$\alpha\left(e_j+(n-1)\alpha'\right)\frac{\left(1-s1\left(y^*,\alpha'\right)-(n_1-1)s2\left(y^*,\alpha'\right)\right)}{n_2}-\frac{e_j^2}{2}. \tag{119}$$

The derivative with respect to e_j is

$$\frac{1}{n_2}\left(y^*\left(-\alpha \mathrm{s1}_1(y^*,\alpha')-\alpha(n_1-1)\mathrm{s2}_1(y^*,\alpha')\right)+\alpha(-\mathrm{s1}(y^*,\alpha')-(n_1-1)\mathrm{s2}(y^*,\alpha')+1)\right)-e_j. \tag{120}$$

Setting this to zero at $e_j=\alpha'$ gives the first-order condition

$$\alpha(1-s1(y^*,\alpha')-(n_1-1)s2(y^*,\alpha'))-n\alpha^2\alpha'(s1_1(y^*,\alpha')+(n_1-1)s2_1(y^*,\alpha'))-n_2\alpha'=0. \tag{121}$$

Replacing $\alpha=y/(n\alpha')$ and rearranging, we get

$$y^*(1-s1(y^*,\alpha')-(n_1-1)s2(y^*,\alpha'))-y^{*2}(s1_1(y^*,\alpha')+(n_1-1)s2_1(y^*,\alpha'))-nn_2\alpha'^2=0. \tag{122}$$

Rearranging this gives

$$s1_1(y^*,\alpha')+(n_1-1)s2_1(y^*,\alpha')=\frac{y^*(1-s1(y^*,\alpha')-(n_1-1)s2(y^*,\alpha'))-nn_2\alpha'^2}{y^{*2}}. \tag{123}$$

Plugging (123) into (117) and rearranging, we get

$$\begin{aligned}s1(y^*,\alpha')-s2(y^*,\alpha')&=\frac{nn_1n_2\alpha'^2+n_1^2n\alpha'^2-n_1y^*}{y^*rn(n_1-1)}\\&=\frac{n^2n_1\alpha'^2-n_1y^*}{y^*rn(n_1-1)}.\end{aligned} \tag{124}$$

Taking the derivative with respect to y^* gives

$$s1_1(y^*,\alpha') - s2_1(y^*,\alpha') = -\frac{n_1 n \alpha'^2}{y^{*2} r(n_1-1)}. \tag{125}$$

From (124) and (125), we get

$$\begin{aligned} s1_1(y^*,\alpha') &= s2_1(y^*,\alpha') - \frac{n_1 n \alpha'^2}{y^{*2} r(n_1-1)} \\ s1(y^*,\alpha') &= s2(y^*,\alpha') + \frac{n^2 n_1 \alpha'^2 - n_1 y^*}{y^* r n(n_1-1)}. \end{aligned} \tag{126}$$

Plugging these back into (123), we get the differential equation

$$n_1 s2_1(y^*,\alpha') - \frac{n_1 n \alpha'^2}{y^{*2} r(n_1-1)} = \frac{y^* - n n_2 \alpha'^2}{y^{*2}} - \frac{1}{y^*}\left(n_1 s2(y^*,\alpha') + \frac{n^2 n_1 \alpha'^2 - n_1 y^*}{y^* r n(n_1-1)}\right). \tag{127}$$

Rearranging gives

$$y^* s2_1(y^*,\alpha') + s2(y^*,\alpha') = \frac{y^* - n n_2 \alpha'^2}{y^* n_1} + \frac{1}{rn(n_1-1)}, \tag{128}$$

which is solved by

$$s2(y^*,\alpha') = \frac{1}{n_1} + \frac{1}{nr(n_1-1)} - \frac{n n_2 \alpha'^2 \log(y^*)}{n_1 y^*} + \frac{C(\alpha')}{y^*} \tag{129}$$

for constant $C(\alpha')$. The other share is obtained by inserting $s_2(y^*,\alpha')$ into (126), which gives

$$s1(y^*,\alpha') = \frac{1}{n_1} - \frac{1}{rn} + \frac{n n_1 \alpha'^2}{y^* r(n_1-1)} - \frac{n n_2 \alpha'^2 \log(y^*)}{n_1 y^*} + \frac{C(\alpha')}{y^*}. \tag{130}$$

We now check the reporting incentives for the unranked and privately informed player from set $\mathcal{B}$. For $e^*(\alpha') = \alpha'$ and $y^* = \alpha n \alpha'$, (119) transforms into the first-stage maximization problem

$$y^*\left(\frac{1 - s1\left(y^*,\alpha'\right) - (n_1-1)s2\left(y^*,\alpha'\right)}{n_2}\right) - \frac{(\alpha')^2}{2}. \tag{131}$$

Note that from the shares (39)

$$s1(y^*,\alpha') + (n_1-1)s2(y^*,\alpha') = 1 + \frac{n n_1 \alpha'^2}{y^* r(n_1-1)} - \frac{n n_2 \alpha'^2 \log(y^*)}{y^*} + \frac{n_1 C(\alpha')}{y^*} \tag{132}$$

after plugging in the expression for y^* we get

$$1 + \frac{n_1 \alpha'}{\alpha r(n_1-1)} - \frac{n_2 \alpha' \log(n\alpha'\alpha)}{\alpha} + \frac{n_1 C(\alpha')}{n\alpha'\alpha}. \tag{133}$$

The derivative of (131) with respect to α' gives the differential equation

$$\frac{\alpha(-2nn_1 + (n-1)(n_1-1)n_2 r + 2n(n_1-1)n_2 r \log(n\alpha^2))}{(n_1-1)n_2 r} - \frac{n_1 C'(\alpha')}{n_2} \tag{134}$$

which can be solved to yield the constant (40). Inserting this constant gives

$$2\alpha'(n_1 + n_2)\log\left(\frac{\alpha}{\alpha'}\right). \tag{135}$$

Setting this derivative to zero yields $\alpha' = \alpha$. Hence the unranked player has the appropriate incentives to report truthfully under the claimed sharing rule (39) with constant (40). □

Appendix C: Equilibrium existence

We now examine in which cases the efficient efforts implemented through sharing rule (8) constitute an equilibrium. Since equilibrium existence depends on the detailed specification of the curvature of the ranking technology, the production function, and costs, we switch to a particular class in which we demonstrate that the exertion of efficient efforts constitutes a global utility maximum under our proposed sharing rule (8).

This is not the only case in which equilibria exist in our model. To illustrate this, we add an example of a commonly used model setup in which our proposed equilibrium exists. This example falls outside the class of specifications investigated in the following Proposition.

Proposition 6. *We restrict attention to the class of problems consisting of output $y(\alpha; e_1 + (n-1)e^*(\alpha')) = \alpha\bar{w}(e_1 + (n-1)e^*(\alpha'))$, symmetric cost $c(e) = (e^x)/x$, for $x > 1$, $\bar{w} > 0$, and generalized Tullock contest success technology with precision parameter $r > 0$. Moreover, we restrict permissible α to the compact range $[a, na]$ for $a > 0$. A sufficient condition for efficient effort provision by every player and truthful type reporting by player 1 to be an equilibrium in this class is that $x = r$.*

Proof of Proposition 6. We start with the second-stage effort choice problem given any report α'. Consider the objective

$$u_1\left(e_1, \boldsymbol{e}^*_{-1}(\alpha'), \alpha\right) = y(\alpha, e_1)\left(\frac{(1-f^1(e_1))(1-s^1(e_1))}{n-1} + f^1(e_1)s^1(e_1)\right) - c(e_1) \tag{136}$$

where

$$s^1(e_1) = \frac{(n-1)\left(\frac{c'^*(\alpha')}{f^1_1(e^*(\alpha'))} - \frac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf^1_1(e^*(\alpha'))}\right)}{ny(\alpha, e_1)} + \frac{1}{n}. \tag{137}$$

We use the shorthand notation $y(\alpha, \hat{e}) = y(\alpha; \hat{e} + (n-1)e^*(\alpha'))$, $f^1(\hat{e}) = f^1(\hat{e}, \boldsymbol{e_{-i}}^*(\alpha'))$, with

$\hat{e} \in \{e_1, e^*(\alpha')\}$, and similarly for all other expressions. Then, $\dfrac{\partial u_1}{\partial e_1} =$

$$
\begin{aligned}
&y(\alpha, e_1)\Bigg[\frac{(1-f^1(e_1))\left(\frac{(n-1)y_2(\alpha,e_1)\mu}{ny(\alpha,e_1)^2}+\frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha,e_1)}\right)}{n-1} \\
&+ f^1(e_1)\Bigg(-\frac{(n-1)y_2(\alpha,e_1)\mu}{ny(\alpha,e_1)^2}-\frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha,e_1)}\Bigg) \\
&+ f_1^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha,e_1)}+\frac{1}{n}\right)-\frac{f_1^1(e_1)\left(-\frac{(n-1)\mu}{ny(\alpha,e_1)}-\frac{1}{n}+1\right)}{n-1}\Bigg] \\
&+ y_2(\alpha,e_1)\left\{f^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha,e_1)}+\frac{1}{n}\right)+\frac{(1-f^1(e_1))\left(-\frac{(n-1)\mu}{ny(\alpha,e_1)}-\frac{1}{n}+1\right)}{n-1}\right\} \\
&- c'(e_1)
\end{aligned}
\tag{138}
$$

where $\mu = \dfrac{c'^*(\alpha')}{f_1^1(e^*(\alpha'))} - \dfrac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf_1^1(e^*(\alpha'))}$. (138) simplifies to

$$
\begin{aligned}
\frac{\partial u_1}{\partial e_1} = &\frac{y_2(\alpha,e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1),e^*(\alpha'))}{n}\right) \\
&- \tilde{\alpha}'(e_1)\frac{y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}.
\end{aligned}
\tag{139}
$$

Inserting

$$
\tilde{\alpha}'(e_1) = \frac{y_2(\alpha,e_1)}{y_1(\tilde{\alpha}(e_1),e^*(\alpha'))}
\tag{140}
$$

we obtain

$$
\begin{aligned}
\frac{\partial u_1}{\partial e_1} = &\frac{y_2(\alpha,e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1),e^*(\alpha'))}{n}\right) \\
&- \frac{y_2(\alpha,e_1)}{y_1(\tilde{\alpha}(e_1),e^*(\alpha'))}\frac{y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}.
\end{aligned}
\tag{141}
$$

For the linear case $y(\alpha,\hat{e}) = \alpha w(\hat{e}+(n-1)e^*(\alpha')) = \alpha w(\hat{e})$ using again the shortened notation for the function $w(\cdot)$ in the last step we get

$$
\begin{aligned}
\frac{\partial u_1}{\partial e_1} = &\frac{\alpha w'(e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{\alpha w(e_1)w'^*(\alpha'))}{nw(e^*(\alpha'))}\right) \\
&- \frac{\alpha w'(e_1)}{w(e^*(\alpha')}\frac{w'^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}
\end{aligned}
\tag{142}
$$

where we substitute the linear adjustment

$$
\tilde{\alpha}'(e_1) = \frac{\alpha w'(e_1)}{w(e^*(\alpha'))}.
\tag{143}
$$

Using linear $w(\hat{e}) = \bar{w}(\hat{e}+(n-1)e^*(\alpha'))$ and monomial cost $c(\hat{e}) = \hat{e}^x/x$, the first-order condition equals

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha\bar{w}}{n} - e_1^{x-1} + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha\bar{w}(e_1 + e^*(\alpha')(n-1))}{n^2 e^*(\alpha')}\right) \\ &\quad - \frac{\alpha}{ne^*(\alpha')}\frac{\bar{w}}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1) - 1\right\}.\end{aligned} \tag{144}$$

We use $ke^*(\alpha')$ in order to allow for any possible effort deviation. Then substituting $e_1 = ke^*(\alpha')$, the report-contingent efficient $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$, and Tullock technology into the ratio of success function slopes gives

$$\begin{aligned}\frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))} &= \left(\frac{(n-1)re_1^{r-1}e^*(\alpha')^r}{(e_1^r + (n-1)e^*(\alpha')^r)^2}\right) / \left(\frac{(n-1)r}{e^*(\alpha')n^2}\right) \\ &= \frac{e^*(\alpha')n^2(e_1 e^*(\alpha'))^r}{e_1\left(e_1^r + (n-1)e^*(\alpha')^r\right)^2} \\ &= \frac{n^2\left(k(\alpha'\bar{w})^{\frac{2}{x-1}}\right)^r}{k\left(\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r + (n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r\right)^2} \\ &= \frac{n^2 k^{r-1}}{(k^r + n - 1)^2}.\end{aligned} \tag{145}$$

Making the same substitutions step by step for the remainder of (144) , for $e_1 = ke^*(\alpha')$, yields

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha\bar{w}}{n} - (e^*(\alpha')k)^{x-1} + \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha\bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha')n^2}\right) \\ &\quad - \frac{\alpha\bar{w}(nf^1(e^*(\alpha')k) - 1)}{e^*(\alpha')n^3 f_1^1(e^*(\alpha'))},\end{aligned} \tag{146}$$

inserting Tullock technology yields

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha\bar{w}}{n} - (e^*(\alpha')k)^{x-1} + \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha\bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha')n^2}\right) \\ &\quad - \frac{\alpha\bar{w}\left(\frac{n}{(n-1)e^*(\alpha')^r(e^*(\alpha')k)^{-r}+1} - 1\right)}{(n-1)nr},\end{aligned} \tag{147}$$

and finally inserting $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$ yields

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha\bar{w}}{n} - \left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^{x-1} \\ &+ \frac{n^2k^{r-1}}{(k^r+n-1)^2}\left(\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^{x-1} - \frac{\alpha\bar{w}(\alpha'\bar{w})^{-\frac{1}{x-1}}\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}+(n-1)(\alpha'\bar{w})^{\frac{1}{x-1}}\right)}{n^2}\right) \\ &- \frac{\alpha\bar{w}\left(\frac{n}{(n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^{-r}+1} - 1\right)}{(n-1)nr}\end{aligned} \tag{148}$$

which simplifies to

$$\frac{\partial u_1}{\partial e_1} = \bar{w}\left(\frac{\alpha}{n} - \alpha' k^{x-1} - \frac{\alpha\,(k^r-1)}{nr\,(k^r+n-1)} + \frac{k^{r-1}\,(\alpha' n^2 - \alpha(k+n-1))}{(k^r+n-1)^2}\right). \tag{149}$$

As a special case, we substitute $x = r$ and get

$$\frac{\partial u_1}{\partial e_1} = \frac{n}{\alpha}k^{r-1}\left(\frac{\alpha' n^2 - \alpha(k+n-1)}{(k^r+n-1)^2} - \alpha'\right) + 1 - \frac{k^r-1}{r\,(k^r+n-1)}. \tag{150}$$

We need to find a condition that ensures that this expression is positive for $k < 1$ and negative for $k > 1$.

1. $k < 1$: We need to ensure that the right-hand side of

$$\frac{n}{\alpha}k^{r-1}\left(\frac{\alpha' n^2 - \alpha(k+n-1)}{(k^r+n-1)^2} - \alpha'\right) + 1 > \frac{k^r-1}{r\,(k^r+n-1)} \tag{151}$$

is negative whenever $k < 1$. In order for the left-hand side to be positive, we need

$$\frac{\alpha k^{1-r}}{n} + \frac{\alpha' n^2 - \alpha(k+n-1)}{(k^r+n-1)^2} > \alpha' \tag{152}$$

which is implied by

$$\alpha'\underbrace{\left(\frac{n^2}{(k^r+n-1)^2} - 1\right)}_{=A} + \alpha\underbrace{\left(\frac{k^{1-r}}{n} - \frac{1}{k^r+n-1}\right)}_{=B} > 0. \tag{153}$$

$A > 0$ for all $k < 1$ and $r > 0$ and $B > 0$ if $k < 1$ and

$$r > \frac{\log\left(\frac{k(n-1)}{n-k}\right)}{\log(k)} \geq \frac{n}{n-1} \tag{154}$$

where the final term on the right-hand side is the limit of the increasing log-ratio as $k \to 1$.

We showed that for $x = r$, it is the case that

$$\frac{\alpha}{n} - \alpha' k^{x-1} - \frac{\alpha\left(k^r - 1\right)}{nr\left(k^r + n - 1\right)} + \frac{k^{r-1}\left(\alpha' n^2 - \alpha(k + n - 1)\right)}{\left(k^r + n - 1\right)^2} > 0. \tag{155}$$

However, since the derivative of the left-hand side of the last inequality with respect to x is $-\alpha' k^{x-1} \log k$, which is positive for any $k < 1$, the last inequality holds for any $x \geq r$.

2. $k > 1$: We start from (149) and want to show that

$$\frac{k^{r-1}\left(\alpha' n^2 - \alpha(k + n - 1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^{x-1} < \frac{\alpha\left(k^r - 1\right)}{nr\left(k^r + n - 1\right)} - \frac{\alpha}{n} \tag{156}$$

is implied by

$$\frac{n}{\alpha k}\left(\frac{k^r\left(\alpha' n^2 - \alpha(k + n - 1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^x\right) + 1 < \frac{1}{\frac{nr}{k^r - 1} + r} \tag{157}$$

whose right-hand side is positive. Thus, we need to show that

$$\frac{n}{\alpha k}\left(\frac{k^r\left(\alpha' n^2 - \alpha(k + n - 1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^x\right) + 1 < 0 \tag{158}$$

or, equivalently, that

$$\frac{\alpha k^{1-r}}{\alpha' n} + \frac{\alpha' n^2 - \alpha(k + n - 1)}{\alpha'\left(k^r + n - 1\right)^2} < k^{x-r} \tag{159}$$

which is implied by

$$\frac{n^2}{\left(k^r + n - 1\right)^2} - \frac{\alpha(k + n - 1)}{\alpha'\left(k^r + n - 1\right)^2} < \frac{k^{-r}\left(\alpha' n k^x - \alpha k\right)}{\alpha' n} \tag{160}$$

which gives

$$\frac{\alpha k - \alpha' n k^x}{n k^r} < \frac{\alpha(k + n - 1) - \alpha' n^2}{\left(k^r + n - 1\right)^2}. \tag{161}$$

We restrict possible $\alpha \in [a, b = sa]$, with $s \leq n$, and—since (161) is linear in α on both sides—we obtain two subcases:

(a) Highest misreport $\alpha = a$, $\alpha' = b$: resulting in

$$\frac{ak^{-r}\left(k - snk^{x}\right)}{n} < \frac{a\left(k - sn^{2} + n - 1\right)}{\left(k^{r} + n - 1\right)^{2}} \tag{162}$$

which holds for x sufficiently higher than r. For instance, for $x = r$, we obtain

$$\frac{k}{k^{r}} - 1 < sn - 1 + \frac{n(k - 1 + n - sn^{2})}{(k^{r} - 1 + n)^{2}} \tag{163}$$

where the left-hand side is negative and the right-hand side is positive for $k > 1$ because

$$\begin{aligned} \frac{sn\left(k^{r} + n - 1\right)^{2} + n\left(k - 1 + n - sn^{2}\right) - \left(k^{r} - 1 + n\right)^{2}}{\left(k^{r} - 1 + n\right)^{2}} &> 0 \iff \\ ns\left(k^{r} + n - 1\right)^{2} - \left(k^{r} + n - 1\right)^{2} + n\left(k + n^{2}(-s) + n - 1\right) &> 0 \end{aligned} \tag{164}$$

which equals

$$(ns - 1)\left(k^{r} + n - 1\right)^{2} > n^{3}s - n(k + n - 1). \tag{165}$$

Recall that the left-hand side equals the right-hand side at $k = 1$ by construction. The left-hand side derivative is $2rk^{r-1}(ns - 1)\left(k^{r} + n - 1\right) > 0$ and the right-hand side derivative is $-n < 0$. Hence, for $k > 1$, (163) holds.

(b) Lowest misreport $\alpha = b$, $\alpha' = a$: resulting in

$$\frac{ak^{-r}\left(sk - nk^{x}\right)}{n} < \frac{a(k - 1 + n)s - an^{2}}{\left(k^{r} + n - 1\right)^{2}} \tag{166}$$

which also holds for x sufficiently higher than r. For instance, for $x = r$, we obtain

$$\frac{k}{k^{r}} - 1 < \frac{n}{s} - \frac{n\left(n^{2} - s(k + n - 1)\right)}{s\left(k^{r} + n - 1\right)^{2}} - 1 \tag{167}$$

where the left-hand side is negative for $k > 1$ and the right-hand side is positive if

$$\frac{n}{s} - 1 > \frac{n\left(n^{2} - s(k + n - 1)\right)}{s\left(k^{r} + n - 1\right)^{2}}. \tag{168}$$

We can rewrite the last inequality as follows

$$\begin{aligned} \frac{n - s}{s} &> \frac{n\left(n^{2} - s(k + n - 1)\right)}{s\left(k^{r} + n - 1\right)^{2}} \iff \\ (n - s)\left(k^{r} + n - 1\right)^{2} &> n^{3} - sn(k + n - 1). \end{aligned} \tag{169}$$

We have equality for $k = 1$. The derivative of the right-hand side of the last inequality is $-sn$, which is negative for $s > 0$, while the derivative of the left-hand side is $(n - s)\left(k^{r} + n - 1\right)2rk^{r-1}$, which is positive for $n > s$. Hence, for $k > 1$, (167) holds.

Given player 1's choice of $e_1 = e^*(\alpha')$ at the second stage,[28] we now move on to the reporting stage at which player 1 chooses α' in order to maximize utility

$$\begin{aligned} &\max_{\alpha'} \; y(\alpha, ne(\alpha'))\left(\sum_{\ell=1}^{n} f^{\ell}(\boldsymbol{e}^*(\alpha'))s^{\ell}(y^*, \alpha')\right) - c(e(\alpha')) \\ &= \max_{\alpha'} \;\; u_1(\boldsymbol{e}^*(\alpha'), \alpha) \\ &= y(\alpha, ne(\alpha'))\frac{1}{n} - c(e(\alpha')) \end{aligned} \tag{170}$$

because $f^{\ell}(\boldsymbol{e}^*(\alpha')) = 1/n$ for every ℓ and $\sum_{\ell} s^{\ell}(y^*, \alpha') = 1$. This yields the first-order condition

$$y_2(\alpha; ne(\alpha')) = c'(e(\alpha')), \tag{171}$$

which equals the social planner's efficiency condition. Therefore, if the solution to the planner's problem is unique, then player 1 shares the same objective and will choose to truthfully report $\alpha' = \alpha$. □

The following example shows that it is easy to find instances in violation of the sufficient conditions of Proposition 6 while still exhibiting the equilibrium identified in Proposition 1.

Example 8. Consider the following two-player example outside of the class for which we show existence in Proposition 6:[29] (i) square-root team production $y(\alpha, e_1, e_2) = \alpha\bar{w}\sqrt{e_1 + e_2}$ and (ii) "exponential difference" contest success function defined for two players as

$$f^1(e_1, e_2) \; = \frac{1}{1 + \exp\left(r(e_2 - e_1)\right)}, \text{ for } r > 0. \tag{172}$$

All other specifications are as in Proposition 6. Assume that player 2 behaves according to our equilibrium prescription, i.e., $e_2 = e^*(\alpha') = 8^{\frac{1}{1-2x}}(\alpha'\bar{w})^{\frac{2}{2x-1}}$ (from the solution to the planner's problem). In this example's setup, we obtain player 1's objective as $u_1(e_1, e^*_{-1}(\alpha'), \alpha) =$

$$\frac{\alpha\bar{w}(e_1 + e^*(\alpha'))\left(s^1(y^*, \alpha')\left(\exp(r(e_1 - e^*(\alpha'))) - 1\right) + 1\right)}{\exp(r(e_1 - e^*(\alpha'))) + 1} \tag{173}$$

where the equivalent of the sharing rule (8) is

$$s^1(y^*, \alpha') = \frac{1}{2}\left(1 - \frac{\alpha^*\bar{w}e_1 + \alpha^*\bar{w}e^*(\alpha') - 4e^*(\alpha')^x}{\alpha^*\bar{w}e_1e^*(\alpha')r + \alpha^*\bar{w}e^*(\alpha')^2 r}\right), \tag{174}$$

where $\alpha^* = y^*/(\bar{w}\sqrt{e_1 + e^*(\alpha')})$. Consider parameter values $x = 2$, $r = 2.5$, $\alpha \in [1, 50]$. A plot of player 1's objective against $e^*(\alpha')$ by player two in Figure 3 shows no profitable deviations. Hence, this example illustrates that there exist situations exhibiting the equilibrium behavior derived in Proposition 1 and lying outside of the class defined by the sufficient conditions presented in Proposition 6.◁

[28] Since the effort choice problem of the uninformed players is identical to that of the leader, this argument directly implies that $e_j = e^*(\alpha')$ for every $j > 1$ as well.

[29] Another example of an easy generalization is to substitute $\exp e$ for the monomial cost e^x/x used in the proof of Proposition 6.

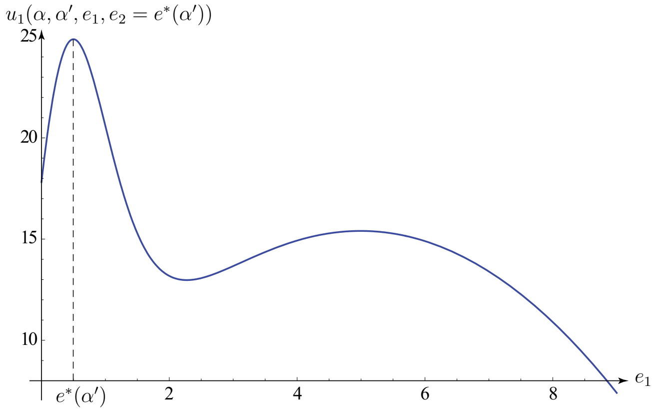

Figure 3: Possible deviations from $e^*(\alpha')$ for $\alpha \in [1, 50]$; the objective possess no other maxima.

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