Conceptual Analysis and Analytical Definitions in Frege

Gilead Bar-Elli

[Abstract: Conceptual analysis, it is argued, is not analysis of a concept but of its sense. Five Fregean philosophical principles are presented as constituting a framework for a theory of logical or conceptual analysis, which I call analytical explication. These principles, scattered and sometime latent in his writings are operative in Frege's critique of other views and in his constructive development of his own view. The proposed conception of analytical explication is also connected to Frege's notion of analytical definition, which is of paramount importance, if only because most of Frege's celebrated definitions are of that sort.]

Although Frege's interests were obviously focused in logic and the philosophy of mathematics (mainly in the foundations of arithmetic) his way of handling them is marked by broad and deep philosophical views, and by being involved in some general philosophical positions and doctrines, sometimes only hinted at and sometimes more fully developed. These, without in any way depriving Frege's technical achievements of their significance, may outlast the technical validity of his logistic project and are independent of it.

A central such doctrine, which in fact is the heart of what Frege thought he was doing, is his notion of **logical analysis** – its goals, its importance, and the constraints governing it. This is often construed as conceptual analysis, which is a very common notion, also in the Fregean literature, but it is often involved with some confusion, and some preliminary remarks seem here in order. For the mature Frege, just as proper names have reference (*Bedeutung*) and sense (*Sinn*), so do predicative expressions such as "... is a number", "... is a circle", "... is a man", "... is bigger than ---" etc. The references of these expressions are functions from objects to truth-values, and this for Frege is a very general category of entities including concepts and relations.

"Conceptual analysis" is not Frege's term. Moreover, its wide use notwithstanding, strictly, from a mature Fregean point of view, it is inappropriate: Concepts are references of (one-place) predicative expressions, they are functions from objects to truth-values, and as such have no structure with determinate components and cannot be analyzed; only their senses can.

One of the main reasons for that is that in general, the reference of a part (of a complex expression) is no part of the reference of the whole. Taking, for instance, "the capital of Sweden" as a complex name referring to Stockholm, Sweden is no part of the capital of Sweden, and, assuming it to be the most populated city of Scandinavia, Scandinavia is also no part of it, nor are the references of numerous other terms in expressions referring to it; Likewise, 5, 3 and the minus function are no parts of the number 2, which is the reference of "5-3"; similarly for concepts and functions in general, as can be seen by considering several definitions of a concept, e.g. a circle.[1] This is no small matter, for it is crucial for understanding how different expressions with different constituents can refer to the same thing (object, or function), which is vital for science, mathematics and thought in general.[2] An

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1 See Frege's "Notes for Ludwig Darmstaedter" (1919), PW 255, (FR 364-5), though there are some unfortunate formulations in earlier writings. In his *The Interpretation of Frege's Philosophy*, Duckworth, 1981, Dummett rightly regards the idea that the reference of a part (of a functional expression) is part of the reference of the whole "manifestly absurd" (265).

2 This central strand in Frege found an emphatic conclusion in Wittgenstein's doctrine in the *Tractatus* that "Objects are simple" (2.02), and that "Every statement about complexes can be resolved into a statement about their constituents..." (2.0201), in which the intention I surmise, is "Every statement that is apparently or allegedly about complexes..." for the upshot of the doctrine

important corollary is that although the references of the parts determine that of the whole, the reference of a whole and that of a part do not determine the references of the other parts. For example, the references of "a" and of "F" determine that of "a is F", while the references of "a" and of "a is F" do not determine the reference of "F".

This is of course entirely different with regard to sense (*Sinn*): In general, the sense of a part is a part of the sense of the whole. Senses have structure and determinate constituents (though there may be exceptions to this).[3] What we usually call conceptual analysis is therefore not an analysis of a concept but of its sense. Though, for simplicity, I speak here of concepts, their referring predicates and their senses, much of what I say here applies to other kinds of reference (objects, functions) and to their senses, in particular thoughts, which are senses of propositions and are often what is analyzed. Keeping this in mind, the intention in my using "conceptual analysis" here, in accord with its wide use, is I hope sufficiently clear. It also explains why much of what we shall say about analysis is focused on the notion of sense.

It seems that a minimal requirement of a logical (conceptual) analysis is that analysandum (A) and analysans (B) refer to the same concept. "The same" is

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is that there are no complexes. In relating it to Frege, one should remember that Wittgenstein rejected Frege's doctrine of sense. I shall not go into this here.

3 Facts, for Frege, are true thoughts - senses of true propositions. In the Appendix to Part I of *Philosophical Grammar* (tr. A. Kenny, Oxford 1974) Wittgenstein argues that facts are no complexes. In connection to the problem of complexity he writes: "To say that a red circle is composed of redness and circularity, or is a complex with these component parts, is a misuse of these words and is misleading (Frege was aware of this and told me)" (200).

problematic here because concepts are categorically different from objects, and identity (sameness, equality) strictly applies only to objects. Hence we need a similar equivalence relation applicable to concepts – call it simply "equivalence". Concepts (of first level) are equivalent according to Frege when they have the same extension – when they apply to the same objects. This is the criterion of their equivalence.[4]

Now, two main questions arise regarding analysis.

1. Does any change in the extension of A and B amount to such a difference in the concepts that the analysis would be incorrect? Can't we endorse an analysis, say of number, which results in some change in the extension of A, particularly where, like e.g. with the naturals, the integers, or the rationals, the extension of B includes that of A? Frege was notoriously quite rigid on this question and thought that any change in extension is a change in concepts. We shall not deal with this question here.

2. Assuming that in an acceptable analysis A and B refer to equivalent concepts, unless the case is trivial they obviously have different senses. What should be the relationship between these senses to render B as an appropriate analysis of A? This is obviously a central question – perhaps the central question – of the nature of analysis, for obviously not any expression referring to an equivalent concept would be regarded as a conceptual analysis of A. Further constrains are required. Some were traditionally recognized and are not particularly Fregean, as for instance the requirement of **non-circularity**: Analysans should not include or presuppose the analysandum, as for instance when we may say that a circle is a plane curve enclosing the maximum area of a given arc length; that it is the shape that has isoperimetric

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[4] This is stated already in FC and is implied by the famous axiom V of Gg. Frege elaborates on it in "Comments on Sense and Reference", PW 118-125 (in FR, 172-180).

equality etc. There are numerous such equivalence-theorems, in mathematics and outside it, that would not count as proper analyses. In this respect a certain hierarchy of dependence-relations is often assumed and the requirement is that analysans should be couched in terms that are more basic in the hierarchy than analysandum. I shall not discuss these traditional requirements here and rather focus on further, particularly Fregean principles. I shall present five such principles as forming together a partial basis for a theory of conceptual analysis and as constraining Frege's notion of analytical definition.

The Principle of implications enrichment – Frege was relatively explicit that the goal of logical analysis of (a sense of) a concept (or of proposition about it), which he compares to chemical analysis of a substance into its elements[5], is that it, and the definition based on it, enable us to prove or expose implications involving the concept, which we approve, but couldn't prove, or didn't even see, before; Hence also its great importance: it not only reveals new implication links, but also brings to the fore the concepts on which the analyzed relies.

This idea of logical analysis finds expression already in the Introduction to *Begriffsschrift* of 1879 and persists throughout Frege's writings. A clear late expression from his "Logic in Mathematics" of 1914 (LM) is:

> For it may not be possible to prove a truth containing a complex constituent so long as that constituent remains unanalyzed; but it may be possible, given an analysis, to prove it from truths in which the elements of the analysis occur (PW 209/226).

[5] Frege uses the analogy on various place, e.g. "Boole's Logical Calculus and My Concept Script" (PW, 36/40), "On concept and Object" (FR, 182/193), "Logic in Mathematics" (PW, 208/225), "Foundation of Geometry II", *Collected Papers* (CP, 302/303). Numbers to the right of a slash refer to the German edition.

I shall call this feature of logical analysis the principle of **implications enrichment**.

It should be noted that this is also important for Frege's notion of consistency (e.g. in his debate with Hilbert), which, for Frege, is a property of a set of thoughts, not of uninterpreted sentences. If these thoughts are not fully analyzed, the risk of inconsistency, he thought, is always there.[6]

This feature of his conception of logical analysis has been widely recognized.[7] It is in fact rooted in a general conception of meaning expounded already in *Begriffsschrift* of 1879 (B), that what he called there "the conceptual content" (*begriffliche Inhalt*) of a proposition is constituted by its implication relations: Two propositions have the same conceptual content if they have the same implication relations, i.e. if they imply (perhaps with other premises) and are implied by the same propositions. The implication relations are conceived here as constitutive of the content of a proposition. This may also explain the deep philosophical reason for Frege's calling his logical language *Begriffsschrift* – concept script.[8] For, a logical language is designed to conspicuously display the implication

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6 This has been presented in detail e.g. in P. Blanchette (2007). In FG Frege argues that axioms, as traditionally understood, are (expressions of) true thoughts, and as such their consistency is assured (FG, I, in CP 275/321). He further argues in FG II that Hilbert's axioms are really used as (2^{nd} order) definitions of concepts. Their consistency (and independence), is therefore determined by the consistency (and independence) of the characteristics (*Merkmale*) of these concepts, which are the result of their analysis.

7 Blanchette (2007); Shieh (2008).

8 I expanded on this in Bar-Elli (1986). Some of Frege's early remarks suggest this to be in line with the Leibnizean notion of *lingua characterica*, and he probably took the term *Begriffsschrift* from an article by Adolf Trendelenburg about Leibniz. However,

relations of a proposition as a function of its inner structure; and to achieve this, the concepts (strictly – concept-senses) of the proposition must be fully analyzed.

It will be claimed, however that this is insufficient, and Frege was much less explicit on further constraints governing such analyses and definitions. These will be our main concern. I shall present some further principles of Frege's philosophy, primarily of his conception of sense (*Sinn*), and suggest that all together they may be seen as forming a basis for a theory of conceptual analysis.[9] These principles are scattered and sometimes quite latent in his writings, but operative both in his critiques of other views and in his constructive development of his own.

The Principle of About – A central thesis of Frege's *Foundation of Arithmetic* of 1884 (FA) is that numerical propositions like 'There are three apples on the table' "**contain an assertion about a concept**" [*eine Aussage von einem begriffe enthalte*, FA §§ 46, 55]. This formulation, unlike many expositions of Frege's view, does not speak of a property of a concept, but is phrased in accord with the context principle in terms of **propositions**. Moreover, it is phrased in terms of the notion of **about**, which is a propositional notion – a relation between a proposition and things it is about.[10] This, as I argued elsewhere, is a

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in his late "Notes for Ludwig Darmstaedter" of 1919 (PW, 253/273) Frege emphasized the difference between his conception and Leibniz's, and even regretted choosing the term for its allusions to Leibniz's conception.

9 Mind the introductory remarks on this expression.

10 This is missed in some presentations of Frege's view. For instance in his "Frege's Theory of Number" (reprinted as ch. 6 in (1983) Parsons presents the thesis by saying: "Having a certain cardinal number is a property of a concept". Frege's formulation is not only more cautious in avoiding parson's somewhat unclear "having

fundamental notion in Frege, posited at the basis of a fundamental Fregean principle, namely that any meaningful proposition is about something(s).[11] Frege later proposed that the things – whether objects or functions (including concepts and relations) – a proposition is about are the references of its constituent terms,[12] which also implied that a proposition cannot be about something not explicitly referred to by a constituent of it.[13] I shall treat both claims as one thesis and call it "**Frege's principle of about**".

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a number", but more importantly in being explicitly phrased in terms of proposition and the notion of about.

11 Bar-Elli (1996), ch. 7. I suggested there, against Dummett and many others, to found the realistic basis of Frege's notion of reference on that of about, rather than on the naming relation – a suggestion that seems to me better both philosophically and exegetically. The centrality of the idea of about in Frege occurs already in the early "Dialogue with Puenjer" (PW, 66-7/59-60) and persists throughout his writings.

12 Cf. Dummett (1981), p. 196 ff. Though Dummett there seems to assume that reference is primary and we explicate the notion of about in terms of it, whereas I suggested in Bar-Elli (1996), ch. 7 the opposite direction.

13 This explicitly appears in SR (58/28). In FA it is explicit regarding objects: "**It is impossible to speak of an object without in some way designating or naming it**" (60), and implicit in the course of the general argument there regarding anything. It is re-affirmed in many of Frege's later writings. For example, in "Logic in Mathematics" of 1914 he writes: "[Myth and fiction apart] **a proper name must designate something and in a sentence containing a proper name we are making a statement about that which it designates, about its meaning** [*Bedeutung*]" (PW 225/243; see also e.g"On Schoenflies" in PW, 180/194-5).

It is stated by Frege as a basic premise for his construal of quantified propositions, like universal propositions of the form 'All As are Bs', and of numerical propositions like the above, as consisting in an assertion about concepts.[14] In the Introduction to Basic Laws of Arithmetic of 1893 (BL) p. 5/ix Frege presents this as his main discovery in FA. It also lies at the basis of his opposition to "implicit definitions" and of his polemics with Hilbert: One of Frege's main claims there is that such definitions do not determine meaning uniquely, and that "**One surely needs to know what one is talking about**" (*wovon man aussagt*, PW 213/230). This talk of the constituents of a proposition involves substantial views about its logical form, which determines these constituents and on which, Frege thought, vernacular language and traditional grammatical analyses may be profoundly misleading.

Frege's principle of about is often a quite effective constraint on explicating the sense of a proposition when we know what objects (or concepts) the proposition is about, like when we deal with regular empirical objects such as tables and apples, colors and persons.[15] In such cases knowing the objects – their nature and ways of identification, standard ways of justifying claims about them, etc. – may pose constraints on explicating the thought and the senses (*Sinne*) of the terms concerned, for these senses should express ways in which these things (supposedly known to us) are given. It is for this reason that, say, modeling propositions about tables and apples in arithmetic would not count as explicating their senses or as analytic definitions of them (or of the terms referring to them; more on this later). It is much less effective, however, and in fact quite problematic when we deal, e.g. with abstract objects like numbers. If we don't know what objects the numbers are, we may lack enough constraints on explicating their senses. And this, of

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[14] Bar-Elli (1996) in particular 7.D.

[15] This has been sometimes called "identity-dependent" proposition; see e.g. Blackburn (1984).

course, is the pivotal question of FA: "**When we make a statement of number, what is that of which we assert something**?" [*von wem durch eine Zahlangabe etwas ausgesagt werde,* FA § 45].

The principle of about is also the reason why Frege did not even consider founding the analyticity of geometry on the possibility of modeling it in arithmetic, and ultimately in logic. Frege was of course aware of such possibilities. One might think that they should enable him to turn the reduction of arithmetic to logic into a reduction of geometry to logic (or the logical definitions of arithmetical terms into logical definitions of the geometrical ones), and thus establishing the analyticity of geometry. But he never proposed it. The reason for this may be stated in another context in FA: "**But surely, everything geometrical must be given originally by intuition**" (*alles Geometriche muss doch wohl urspruenglich anschaulich sein*, FA 75). It persists in his later writings. In a letter to Hilbert of 27.12.99 (PMC 37) Frege wrote: "I call axioms propositions that are true but are not proved because **our knowledge of them flows from a source very different from the logical source**, a source that might be called **spatial intuition**". Spatial intuition is the source of knowledge of geometry (about which he and Hilbert were talking), and of how the geometrical concepts are given to us. An arithmetical or logical modeling of geometry would be pointless as their analysis, for it would not respect the ways geometrical objects and concepts are given to us by intuition. In other words, it would not respect what geometrical propositions are about. By the same reasoning the principle explains the pains Frege took in arguing for the logical nature of arithmetic and of numbers in justifying his analysis of them and disqualifying others. This amounts to a philosophical requirement of an analysis and of analytical definition to the effect that it respects the way (or the standard way) the defined is given to us – its sense. The principle of about is thus connected to Frege's core idea of sense, on which more will be said in the sequel.

I use "requirement" or "constraint" rather than "criterion" or "necessary condition", and speak rather vaguely of "respecting" the way the analyzed is given, partly because as noted above, in some cases we may not be clear on what it is that a proposition is about. Yet, even in these "hard" cases the principle is not void, for we may know enough to know that what the proposition is about is not what a proposed analysis is about (the case of geometry and numbers may serve as an example), and this may be a good reason to disqualify the analysis.

We said that the principle of about may constrain explicating the sense of propositions that deal with objects, like tables and apples, whose nature we know, but that it seems problematic with regards to other objects, like numbers, where we cannot presume such knowledge. Frege's way of overcoming this difficulty, if only partially, was to reverse the order of explanation, namely to explicate the senses of these objects – how they are given to us – by their being constituents of the senses of propositions about them, which are independently explicated, and when this is achieved satisfactorily we may get a grip on what these objects are, or can be. Thus, by explicating the sense of many numerical propositions of the form "There are n Fs" and of what they are really about, we can get a grip on the meaning of n. This, being the general course of argument of FA, is admittedly put here in a quite general and programmatic way, for it is not at the centre of our concerns, but I trust it that anyone familiar with FA can guess the direction in which a fuller account would proceed.

The Context Principle – Frege understood the notion of about realistically: A proposition cannot be about something (whether an object or a function) that does not exist.[16] Holding this he had to show that any referring term has a reference. Frege in fact

[16] This is a substantial thesis of BL and occurs in many of his writings after that.

But it is already clearly stated in his "17 Key Sentences on Logic" (PW 174), proves this for all referring terms in BL, but in FA he appeals to a broader view relying on a strong version of the **context principle**, according to which if a proposition containing these terms has a clear sense, which determines its truth-conditions, this **suffices for securing the terms references**. Generally, Frege's context principle says that only in the context of a proposition does a term have meaning (FA, Introduction). This can naturally be understood as a general thesis about meaning, namely, that the meaning of a term is its contribution to the meaning of propositions containing it. In FA Frege did not yet have the systematic sense/reference distinction,[17] but in its strong version the principle applies to both and states, as said above, that the meaning (sense) of a proposition secures reference to its terms:

> Only in a proposition do the words really have a meaning [...] It is enough if the proposition taken as a whole has a sense; it is this that confers on its parts also their content. [Es genuegt, wenn der Satz als Ganzes einen Sinn hat; dadurch enhalten auch seine Theile ihren Inhalt] (§ 60; cf. § 62).

This was emphasized by Dummett, who construed *Inhalt* here as reference.[18] Many scholars object and proclaim that this goes against Frege's view that a term may have sense but lack reference.[19]

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which is probably an early piece (see Dummett (1981b), pp. 66, 77). In article 10 he states: "The sentence 'Leo Sachse is a man' is the expression for a thought only if 'Leo Sachse' designates something (*etwas bezeichnet*)".

17 But see note 26 below.

18 According to the strong interpretation, the logical formula displaying the analyzed form of e.g. "there are 3 apples on the table" secures sense to the proposition, thus conferring reference to "3", which doesn't occur in the formula. But there are weaker interpretations in which only the constituents of the formula are conferred reference.

Following e.g. Evans, I believe quite the opposite – that it is this possibility of a referenceless sense that goes against Frege's principles. Although some of Frege's formulations (especially in SR) seem to admit this possibility, they do not, I believe, express his better and considered view. It is rather this possibility which is incoherent with Frege's basic principles, like his adherence to the bivalence principle of classical logic, for a sentence with a referenceless component is neither true nor false, in defiance of this principle. For a similar reason it doesn't cohere with his view that Thought is essentially true or false. It also hardly makes sense on his view that sense is a way a reference is given to us, for this hardly makes sense when there is no reference. For a similar reason the possibility concerned would hardly make sense on Frege's view that a meaningful proposition is about the references of its terms. All these strongly support the view that a referenceless sense is a problematic, indeed incoherent, idea. And indeed, most of Frege's pronouncements on the topic go in this direction.[20] Whenever a term lacks reference, this shows that it only seems to have a sense, but in fact doesn't. Strictly, **sense for Frege is supervenient on reference**. Yet, he conceived of certain **extensions** here in trying to account for our seeming understanding of expressions which only seem to have sense. These extensions relate, according to Frege, mainly to vernacular expressions in mythical or poetical contexts, which he regarded as devoid of sense in the strict sense. When sentences are used to express genuine thoughts (which are their senses), they cannot

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[19] For an early such objection see e.g. Parsons ibid, p. 156. See also Dummett (1981), pp. 160-171. Dummett emphasizes that Frege, though admitting it, saw it as a defect in a natural language, and that he thought that semantics cannot allow sentences lacking truth value, but less cautious ascriptions of the view to Frege persists still today.

[20] See e.g. "On Schoenflies" in PW, 180/194-5.

contain constituents that are devoid of reference, for then they would be neither true nor false. This is a much discussed exegetical issue into which I shall not get further here.

The Core idea of Sense – The sense of a proposition is built up by the senses of its constituents, and these senses express ways in which their references are given to us or presented to us – their *Art des Gegebenseins*.[21] This is Frege's main characterization of sense – its **core idea**. It should be distinguished from another one, made part of the Fregean lore especially by Dummett, construing sense as a "route to the reference" or a "way the reference of a term is determined".

This, I think, has a rather slim basis in Frege.[22] And it differs from Frege's main characterization of sense in its orientation and overtones: Frege's core idea – ways things in the world are given us – is epistemic, directed as it were form the world to us; the latter (Dummett's) is anchored in the philosophy of language, directed as it were from language to the world. The former hardly makes sense for terms lacking reference; the latter can tolerate such an idea more naturally. However, this is not to imply that Frege's notion is

[21] The way it is introduced in SR and used widely later. I shall not be strict here in distinguishing sense of a term from sense of its reference and trust it that the context makes clear which is meant. Frege's "core idea" makes ascribing sense to a reference quite natural, and he often speaks of a sense as attached to a reference (e.g. in SR 27, FR 153). Likewise in talking of definitions I shall use "the defined" for a term and for its sense.

[22] A quite common (not Dummett's) explication of this "way of determining" as a condition whose (sole) satisfier is the reference seems flatly wrong, as it suggests a predicative construal of the sense/reference relationship to which Frege explicitly opposed (His letter to Husserl of May 1891, PMC 61-3; SR, 64/34-35 in TPF (158 in FR); PW 194/211

detached from the philosophy of language. On the contrary – they are tightly linked. The links, however, are more roundabout and sophisticated than may be suggested by Dummett's formulation and often presented by others. Things in the world are given to us in ways that are constrained and displayed by the meanings of the linguistic expressions that refer to them. Frege's core idea of sense may therefore be briefly put thus: A sense of a term is a way its reference is given to us as this is expressed by the (linguistic) meaning of the term referring to it.

This appeal to language as constituting ways things in the world are given to us marks Frege's divergence from Kant, and one of his main revolutions in philosophy. But this appeal to language should not undermine the general epistemic orientation of Frege's notion of sense as expressing a way something in the world is given to us. Explicating the sense of a term should therefore respect this way its reference is given us.[23] It is for this reason, as noted above in discussing the principle of about, that Frege was not satisfied with the mere possibility of modeling arithmetic in logic, but devoted much effort to arguing that numbers, as they are used in numerical propositions and in arithmetic, are logical objects and are given to us as such.

The Justificatory Significance of Sense – This notion of sense, encapsulating ways in which things in the world are given to us, has **justificatory significance**, particularly in forming an epistemic, non-deductive justification of basic truths (axioms) which cannot be derived from other more elementary truths. I have defended elsewhere its ascription to Frege and argued that it is crucial for understanding his notions of analyticity and apriority,

[23] In BL and later Frege often talks of sense as a constituent of a thought (which is the sense of a proposition). This, though having the merit of being in accord with the context principle, can be interpreted in various ways and can hardly settle the issue between the two characterizations discussed in the text.

particularly in applying them to basic truths, for these notions, he insists, concern only justification (Bar-Elli, 2010).[24] If accounting for the analyticity of arithmetical propositions and for the apriority of geometrical ones is considered vital for their analysis (as Frege, I believe thought), the justificatory role of sense becomes predominant. I shall not repeat it here, but add another aspect of it that has not been given, I believe, its due attention. In his extensive attack on formalistic approaches in *Grundgesetze der Arithmetik* vol. II Frege contrasts what he calls "meaningful arithmetic" with "formal arithmetic". In attacking the latter, Frege's main arguments (and worries) concern the possibility of accounting for the **applicability** and the **justification** of a system of rules and axioms − arithmetic in that case. These he argues must be couched in terms of senses and references and are therefore not available to formalists.

He argues at length that a formalistic approach cannot account for the applicability of arithmetic:

> Arithmetic with no thought as its content will also be without possibility of application (*Anwendung*) [...] Why can arithmetical equations be applied? Only because they express thoughts [...] It is applicability alone which

24 To the references given there for Frege's thinking of non-derivational justification (of axioms) one could add the above letter to Hilbert of 27.12.1999 (PMC 37) in which Frege writes: "I call axioms propositions that are true but are not proved because **our knowledge of them flows from a source very different from the logical source**, a source that might be called spatial intuition". Spatial intuition is mentioned here because they were talking about geometry, but the point I believe is general, and it shows that knowledge of axioms have a "source" from which it "flows"; appeal to this source and the way knowledge of the axioms flow from it is a form of non-derivational justification of the axioms and knowledge of them.

> elevates arithmetic from a game to the rank of a science (Gg II §91; TPF 187).

This applicability he emphasizes "cannot be an accident", and calls for grounding and account, which the formalist cannot provide (ibid. §89). In § 92 there he argues that since arithmetic and numbers have applications in various domains, its applicability is an internal arithmetic issue, and "**to this end it is necessary above all that the arithmetician attach a sense to his formulas**".

However, it is not only the universal applicability of arithmetic which necessitates appeal to sense and reference; it is also the need to **justify** its base – rules and axioms: "**It is necessary that formulas express a sense and that the rules be grounded** (*Begruendung finden*) **in the reference of the signs**". (ibid. §92 p. 188; cf. § 94). In §101 he continues to argue that in meaningful arithmetic "We start from what is known about integers, **from connexions grounded in their nature** (*in deren Wesen begruendet sind*)". It is evident therefore that for Frege rules (and axioms) of arithmetic (and any genuine theory) are grounded and justified by the senses of its constituent-signs (which, he claims, are unavailable to the formalist).

We have briefly surveyed some philosophical doctrines, which, even if Frege, as some would claim, did not express as clearly as one could wish, are fundamental in his philosophy. Chief among them are: 1) the principle of implications enrichment by logical analysis; 2) the principle of about, including its corollary that understanding a proposition involves knowing what it is about; 3) the strong context principle, including its corollary that explicating the sense of a proposition secures reference to its constituents; 4) the core idea of sense and its corollary that explicating the sense of a term should answer to the ways its reference is given to us; and 5) the justificatory role of sense – the view that the sense of a term – a way its reference is given to us – may have a justificatory role, particularly in justifying basic truths containing it, which cannot be derived from other, more elementary ones.

Clearly, each of these principles deserves further clarification and elaboration, which partially I have tried to carry out elsewhere. The point I wish to suggest here is that this cluster of ideas forms a basis for a theory of logical analysis in the sense that they are principles such an analysis should respect. As remarked above, I use this rather vague expression rather than talk of them as criteria or necessary conditions mainly because on some cases their applicability may be deemed unclear: It may be unclear, with respect to the principle of implication enrichment, whether e.g. some implications are ones we approve; It may be unclear, with respect to the principle of about, what some propositions are about; It may be unclear what is the way something is given to us as expressed by a term referring to it, etc. However, here as elsewhere, the fact that a principle is not always applicable does not mean that it is always inapplicable. We have given above many Fregean examples of their applicability, and will give some more in the sequel in discussing them as philosophical constraints on analytical definitions, to which I now turn.

Definitions are of course crucial for Frege's project, and he was well aware of that to the point he could write that arithmetic is derivable from definitions by means of logical laws (rather than from logic by means of definitions; see e.g. his letter to Liebmann, PMC 92-94). He repeatedly criticized definitions given by other mathematicians and logicians, and posed some stringent conditions for proper definitions (FA §§94-103; Frege expands on it in *Grundgesetze*, vol II §§55-164, tr. in part in FR 258-290 and in TPF). To mention some: a definition should be **conservative** – logically, the defined is eliminable and no thought should be provable from it that is not provable without it, although psychologically it may be indispensible, and proofs and thinking without it would be intolerably cumbersome;[25] It should be **complete** – Frege objected to partial or piecemeal definitions determining meaning only in some contexts, leaving open the meaning of the term in others;[26] This includes the demand that a definition of a concept should be "sharp" – it should determine for any object whether or not the concept applies to it; It should determine the meaning of the defined **uniquely**; It should be given in terms of ***Merkmale*** – concepts of the same level as the defined, keeping strictly the distinction between first and second level predicates.

The ultimate reason for these requirements is to prevent or minimize risk of inconsistencies. Actual non-appearance of contradiction cannot suffice here: "**Rigoure of a proof remains an illusion, however complete the chain of inferences may be, if the definitions are only justified retrospectively by the non-appearance of any contradiction**" (FA, Introduction IX; cf. also FA §94, and BL vol. II, §55).

But Frege was not only well aware of the importance of formal requirements of definitions, but also of their philosophically problematic nature and of philosophical requirements they should satisfy. Already in the Introduction to FA he writes:

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[25] In explaining that in thinking and talking we are often unaware of the sense of our words, Frege writes: "So if from a logical point of view definitions are at bottom quite inessential, they are nevertheless of great importance for thinking as this actually takes place in human beings" ("Logic in Mathematics", PW, 209/226).

[26] These requirements are well known and I shall not expand on them. For this and the change in Frege's view on piecemeal or "contextual definitions" from FA (in which he thought that such definitions always depend on some justification for the existence and uniqueness assumptions), to BL (in which he thought that such justifications are in general impractical) and for the bearing of the issue on the status of Hume's principle in his system, see Shieh (2008), § 3.

> To those who feel inclined to criticize my definitions as unnatural, I would suggest that the point here is not whether they are natural but whether they **go to the root of the matter** [*den Kern der Sache treffend*] and are logically beyond criticism (FA, p. xi).

"Natural" probably means here that anyone would recognize the definition as giving the sense of what is defined, or even as offering a synonym. As if anticipating later objections, Frege is well aware here that his definitions are not statements of synonymy or of an even weaker meaning relation that would make them "natural".[27] He is also claiming that satisfying the formal-logical requirements, being "logically beyond criticism", is perhaps necessary but not sufficient. Definitions should also "go to the root of the matter". What does that mean if synonymy and naturalness are too strong,

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27 The claim was raised by Husserl shortly after the publication of FA. Parsons cites Follesdal as claiming that in his *Aufsaeze and Rezensionen*, 11-12 Husserl developed a distinction parallel to Frege's sense/reference one independently of and prior to Frege (see Follesdal's article cited by Parsons in *From Kant to Husserl*, Harvard, 2010, n. 22 p.145). Let me remark in this connection that although Frege's systematic *Sinn/Bedeutung* distinction appears in his papers of 1891-1892, the basic idea of the distinction appears already in § 8 of *Begriffsschrift* of 1879 (confined there to identity statements), and in § 62 of FA, both of which we know Husserl to have read shortly after their publication. So it is still possible that Husserl's distinction was inspired or influenced by his reading these Fregean sources before 1890.

modeling and even sameness of extension[28] – too weak, and being logically beyond criticism – insufficient?

Frege often distinguishes between definitions within a constructed system and explanations of the senses of terms, which are external and preparatory to the system, and serve, especially for primitive terms, in smoothing understanding and communication. He calls these explanations *Erlauterungen* (translated in FG II, "elucidations"). But something akin is often encountered also within a system, when we deal with definitions of familiar terms with a long standing use. And this is the case with most definitions in Frege's own systems.

In his posthumous "Logic in Mathematics", Frege distinguished "constructive" (*aufbauende*) definitions from "analytical" (*zerlegende*) definitions[29]. The former are in fact stipulations for shorts of complicated expressions. They are, he says, like "**receptacle for the sense… we conceal a very complex sense as in a receptacle – we also need definitions so that we can cram this sense into the receptacle and also take it out again**" (PW 209/226; cf. FG in CP 274/320). The latter, analytical definitions, are definitions of terms in use, whose meanings are at least partially known and need in some way to be maintained and clarified by the definition. These are of course the focus of our (and his) interests, for they are the results of what he calls "logical analysis". For them the distinction between the *propaedeutic* analysis and the definition of a term in use within the system is somewhat blurred, for the former is evidently the rationale for the latter.

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28 In his review of Husserl Frege notoriously seems to adopt an extensionalist view. This has been often criticized as plainly insufficient. See e.g. Dummett (1981), 148-54; Shieh (2008).

29 The English "Analytical" here should not be confused with his notion of analyticity or analytical judgment (see Bar-Elli, (2010)).

Frege then distinguishes two main sorts of analytical definitions. In the first, the agreement between the complex sense of the definition and the sense of the "**long established simple sign […] can only be recognized by an immediate insight**" [*unmittelbaren Einleuchten*]. In such a case, he says, the definition should be better termed an axiom. Frege seems to assume here that if the senses of A and B are clearly recognized one cannot be in doubt on whether or not they agree (ibid. 211/228). But what of the other cases where such a doubt can arise?

Frege's answer divides into two parts, one of which is in fact what has come to be known, following Carnap, as "**explication**".[30] This amounts to a suggestion to replace a term in use, whose meaning may be obscure, vague, only dimly or partially grasped, by a new term, precise and well defined within a theory. For such replacement to count as explication or analytic definition there should be a significant overlap in the use conditions of the two terms, which will include the paradigmatic and clear-cut cases of the former. This may be considered an extensional condition, and the whole idea, as Carnap emphasized, a **pragmatic suggestion** for replacing, for specific purposes, a term in use by a new precisely defined one. This has no pretension of being correct or of the latter providing logical analysis of the former, but only of being pragmatically expedient. Frege uses *Erlaeuterungen* here, which was translated in PW as "illustrative examples". Whether or not the translation is a happy one, this seems to be only one kind of *Erlaeuterung*.

30 This is distinct, though related to Frege's use of *Erlaeuterung* (translated as "explication" in FG II, 300/301, which is closer to Carnap's "explanation of the explicandum". Carnap attended Frege's course "Logic in Mathematics" in 1914 (see his "Intellectual Autobiography" in Schilpp, ed. (1963). The article in PW consists of the notes for this course, so there can be little doubt that Carnap took the idea of explication in ch. 2 of his *Foundations of Probablity* from Frege.

Another kind, sometimes translated as "elucidation", is e.g. "**sentences containing an empty sign, the other constituents of which are known**", which, as Frege says, may "**provide a clue to what is to be understood by the sign**", PW, 214/232).

However, in the second part of his answer Frege goes beyond this and writes:

> If this [whether the senses agree] is open to question although we can clearly recognize the sense of the complex expression from the way it is put together, then the reason must lie in the fact that we do not have a clear grasp of the sense of the simple sign, but that its outlines are confused as if we saw it through a mist. **The effect of the logical analysis of which we spoke will then be precisely this – to articulate the sense clearly** [*Das Ergebnis jener logischen Zerlegung wird dann eben sein, dass der Sinn deutlich herausgearbeitet worden sein*]. Work of this kind is very useful (ibid. 211/228).

This passage is of great importance, both in itself and in understanding Frege's attitude towards his own definitions in FA (as well as in B and BL which are almost all of that kind). First, it says that we may question whether two senses agree even when in fact they do, which implies that our conviction that A and B differ in sense may be mistaken – it may arise even when in fact their senses agree.[31] This is rooted in Frege's view, clearly expressed in this passage, that a sense can be only partially or imperfectly grasped, and this is crucial for understanding his notion of logical analysis. For, a successful analysis articulates clearly a sense that was unclearly grasped (or that stands to it in particular

31 This is a feature of the objectivity of sense that is often ignored. It is an important reservation to a widely accepted doctrine of the transparency of sense, namely that if two senses agree one who understands them cannot fail to realize that. This can at most be said of one who understands them fully. But this is a very problematic notion.

relations).[32] Secondly, Frege's ambitions seem here obviously much higher than Carnap's. For, he implies, what Carnap tried to avoid, that an analytical definition is the result of logical analysis of the old term and may amount to a **conceptual analysis** of it (i.e. of its sense), where the question of the correctness of this analysis is pertinent. To distinguish it from Carnap's explication, I shall therefore call it "**analytical explication**".

But unfortunately Frege stops there and does not answer his initial question, except for the self-evident case, and does not say how we can judge the correctness of an analytical definition, and to what such a judgment amounts. We can thus say that Frege here answers one aspect of the "paradox of analysis"[33] and explains that analysis can be important and informative in giving a clear and clearly recognized expression for an unclear and

32 The point was elaborated by Tyler Burge, e.g. in his 1990, though Burge argued that a successful analysis need not preserve the sense of the analysandum. This was recently challenge e.g. by Nelson (2008), who argued that at least since 1914 Frege's notion of analysis did require such sense-preserving. What can change, according to Nelson is not sense but what he calls "conception". I shall not go here into the cogency of the suggestion and its point, for as Nelson himself admits it is non-Fregean and in fact goes against the sense/reference distinction.

33 See his Review of Husserl's *Philosophie der Arithmetik* (FR 224), in which, following Husserl, he gives a clear formulation of what has come to be known as the paradox of analysis. Notoriously, as Dummett pointed out already in ch. 2 of his (1991a) and many had further explained (e.g. Shieh (2008) he doesn't answer it there, and the view he presented there clashes with both his view in FA and his later views, e.g. in LM.

unclearly recognized concept[34]. He seems to presume here that such an analysis is meaningful and expresses an objective thought. But he doesn't state what this thought is and what the constraints on its correctness are.

Some form of the above "extensional condition" is obviously necessary, so that analysans should meet at least paradigmatic cases of the applications of the analysandum. But Frege, as suggested above, aimed higher. As the term "analytical definition" suggests and as Frege makes explicit in the above passage, it amounts to a logical (conceptual) analysis, and as we saw before it should "go to the root of the matter". Frege poses the question of how we can know, or what it means that such an analysis and the definition resulting from it are correct. But he doesn't really answer it except for the case that it is immediately evident. If the problem was not extremely difficult and one of philosophy's perennial ones, this would be most surprising and almost inexcusable. For, as stated above, almost all of Frege's important definitions are of that kind and beset this problem. Without at least some hints at the answer we might feel at a loss in understanding the heart of his enterprise – what he thought he was doing in his logical analyses.

The problem of the correctness of analysis (and the related "paradox of analysis") in Frege were recognized already by Dummett (e.g. 1991 (see pp. 30-31, 143; see also his 1991a ch. 2), where he states that Frege did not have a satisfactory answer to it. Dummett suggests that an analytic definition ought "to come as close as possible to capturing the existing sense" (i.e. of the analyzed; 1991, 152). But, Dummett's added explanations notwithstanding, "as close as possible" is too vague and "capturing the existing sense" is unclear – in fact explicating it is our main problem.

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[34] Mind the introductory remarks on the problematic nature of analyzing a concept, for strictly, analysis is of sense rather than of reference (concept, function or object).

Two more recent papers dealing in detail with the subject are Blanchette (2007) and Shieh (2008).

Blanchette rightly emphasizes that the Frege-Hilbert controversy depends on realizing that for Frege implication and consistency concern thoughts and contents (fully analyzed) while for Hilbert they concern formal sentences. She explains in detail that for Frege the consistency of a theory is no guaranty of the consistency of its models (where a model of a theory shares its logical form while re-interpreting the non-logical terms), whereas for Hilbert the passage is unproblematic The reason is that for Frege consistency and implication concern thoughts, and therefore depend on their full analysis and meaning. Thus the consistency of a theory T^g is no guaranty of that of T^r, for this depends on the full meaning of the (non-logical) terms of T^r (see end of FG I, in CP 375/284).

She talks of analysis of propositions, and says: "Frege takes it that the two propositions [the analyzed and the analyzing] are sufficiently similar that they share logical grounds and entailments", and later she says more cautiously: "thoughts expressed by *analysandum*-sentence and *analysans*-sentence are sufficiently similar that the logical grounds and implications of one are a sure guide to those of the other. This is the assumption that underlies all of Frege's appeals to conceptual analysis in the course of his foundational investigations." It is not entirely clear what "a sure guide" is, and I shall not expand here on the problems applying the notion of analysis to propositions rather than their constituents. But my main worry is that Blanchette seems to focus on what we called the principle of implication enrichment, and what is lacking in her account is appreciation of the other principles we discussed and their role in forming philosophical constraints on conceptual analysis and analytical definitions. It is far from clear that sharing grounds and implications capture all of them.

Shieh (2008) traces the tension between Frege's views on definition in FA and LM.[35] There are some minor points I find faulty in his presentation. One is that Shieh (along with many scholars) talks (e.g. in § 4) of "the structure of a concept". If "concept" is used in Frege's mature sense, this, as noticed above, is objectionable − a concept in this sense, does not have a structure (cannot be complex), only its sense can. Another is that throughout his discussion Shieh appeals to the "sense-identity" requirement, i.e. that in an analytic definition the senses of the definiens and definiendum should be identical. Though the point is disputable, I think that senses for Frege are not objects,[36] and therefore talking of identity here is problematic anyhow, and particularly with regard to analytic definitions. Shieh identifies a thought with the truth-conditions of a sentence, which Frege never did − for Frege a thought expressed by a sentence determines its truth-conditions but is a far richer notion, expressing a particular way in which these truth-conditions are grasped.

But even waiving these worries, I think that Shieh's sharp dichotomy between senses being either identical or completely disjoint ignores interim possibilities, like Dummett's "analytic equivalence" (1991a ch. 2) and the possibility presented above − the explicatory relation between senses: Senses that do not fully agree can still stand in this explicatory relation, satisfying our above conditions, and thus be not fully disjoint.

Following the above sharp dichotomy, in discussing the problem of analytic definitions in LM Shieh concludes that in the new system, based on strict definitions "they or their senses **have nothing to do** with arithmetic as we have used it previously" (end of §5, my emphasis). He also says that according to Frege's late view (e.g. in LM) "the proofs of *Grundgesetze* are of mathematical statements **unrelated in sense** to existing

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35 This was described and discussed in detail already in Dummett's "Frege and the Paradox of Analysis", ch. 2 in his (1981a).

36 I have elaborated on this in Bar-Elli (2014).

arithmetical statements". This seems implausible and foreign to the main tenor of Frege's view.

We have seen before that other scholars (e.g. Nelson 2008) also assume that Frege's notion of analysis requires that the senses of analysans and analysandum be the same. I cannot get here into a detailed discussion of the issue, but will only remark that Frege doesn't explicitly say, even in LM, that a successful analysis always results in their having the same sense. His formulations (e.g. his talking in the above quote of whether senses "agree") allow for a more flexible view of the relationships between analysans and analysandum, in which their senses correspond, agree etc., but are not strictly the same. When an analysis terminates in a definition, and this is incorporated into a system, then one can say that the senses are the same, simply because the one (definiens) fixes the other (definiendum). But this is not the case with which we are concerned here. By the main tenor of our discussion it seems that in a significant analysis, sense according to Frege, is not strictly preserved. In any case, whether we say, as I think we should, that analysis may result in some change of sense, or with Nelson that the senses must be the same, while what changes is one's "conceptions",[37] we still face the problem of the correctness of the analysis, which these constrains are supposed to govern. A successful analysis, I surmise, results in a clear articulation of a sense that determines the same concept as the original one, and stands to the original sense in the explicatory relation governed by the principles we spelled out above.

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37 Relegating the change to something like Nelson's finer grained "conceptions", even if spelled out clearly, would amount to inserting a wedge between sense grasped, and understanding a linguistic expression, which is constituted by these finer-grained notions. This may be quite a high price to pay.

Shieh's positive suggestion amounts to an epistemic twist of the original problem. He briefly mentions Burge's distinction between expressing a sense and grasping the sense expressed, and claims that the former is independent of the latter. He relates this to the difference between a full and clear grasp of a sense and a partial and unclear one, which he construes in terms of the ability to derive the rules of use (of a term) from "a grasp of the rules governing the use of it" (next to the last paragraph). One could perhaps think that the apparent circularity here is due to a slip at the end of the above quote, which should perhaps be "...governing the use of its defining terms". But this charitable reading is doubtful, for Shieh in fact repeats the circularity by construing the grasp of a sense so that "we would have to derive our [rule-governed] uses of those expressions from our knowledge of their senses". This in itself could be granted, but in the subsequent sentence he implies that knowing the sense of an expression is grasping "the rules governing its use" (§ 6). This again seems to render the view hopelessly circular. But even though the above difference between a partial and full grasp is indeed Fregean, it is doubtful whether its construal in terms of "rules of use" and their derivation is. A detailed argument for this will take us too far afield, and Shieh's remarks on that are too brief to allow a detailed examination.

In general, what I believe makes these explications of Frege's notion of analysis insufficient is a lack of full cognition of the philosophical constraints on explicating sense in Frege's theory. Evidently, I don't pretend to have a full answer to the question of analysis. But I would propose that our above principles suggest philosophical constraints and conditions for achieving this: Analytical definition, analytical explication and logical (conceptual) analysis in general should expose and enable to prove implication relations we intuitively wish to hold concerning the analysandum (Implication enrichment principle); they should retain and clarify what propositions including the terms concerned are understood to be about (principle of about); they should articulate the sense of these propositions in a way that secures their constituents reference in accordance with the strong context principle; they should explicate this sense in a way that is responsive to the way its reference is given us (the core idea of sense); and they should articulate the senses concerned – ways their references are given to us – in a way that forms a basis for justifying basic truths and axioms about these references.

Philosophical constrains on definitions has been a major concern in philosophy at least since Plato and Aristotle. It was a major topic in the debate between realism and nominalism and between extensionalists and essentialists throughout the ages. Frege had many reservations about classical forms of definition and did not participate in these debates in these terms. But his views about analytical definitions, his polemics against many other approaches and his own constructive way in FA and other writings, suggest a new path in this celebrated history, and our five principles are the basis of it.

These principles are the basis of much of Frege's criticism of other approaches. The principle of implication enrichment is often appealed to in his criticizing various analyses and definitions based on them, which do not show up in proofs and do not reveal any implication relation, and therefore are either senseless or of no use (see for instance, his long critiques of Weierstrass in "Logic in Mathematics" 221/239 ff.). The principle of about is basic, for instance, in his criticism and rejection of formalistic approaches. This and the context principle are basis of many of his polemics against empirical and psychological approaches. The justificatory role of sense is, I believe, important in understanding his departure from Kant on the notions of analytic and a priori. But, perhaps even more importantly, these principles together with the core idea of sense are basic in the constructive development of his own argument in FA.

Now, as stated above, although Frege doesn't say so explicitly, most of his own definitions e.g. in FA, are what we called analytical explications. And the above principles are implicit in the course of argument of FA. The extensional condition is met by the proof that the definitions concerned offer a logical model of the basic truths of arithmetic (the so called "Peano axioms"). The principle of about is basic in Frege's view that quantified statements are about concepts, and in the rational of his doctrine of logical objects, for, logical truths, like any proposition, must be about some things. It is also appealed to in regarding the universal generality of arithmetic as a reason for regarding it as part of logic (and this, as we have seen, was the basis of Frege's account for the universal applicability of arithmetic). Accordingly, the definitions also explicate and clarify what the arithmetical propositions are "really" about – namely, logical objects and functions defined over them. They explicate the senses of the arithmetical propositions in terms of logical notions whose senses are beyond doubt, thus securing the arithmetical terms references, which render the propositions containing them objectively true or false, in accordance with the strong context principle. These definitions also reveal how the arithmetical objects are given to us – namely, as logical objects given by basic features of our ability to think. They thus form a basis for epistemic, non-deductive justification of basic truths couched in their terms, which is required for regarding them as analytic. A parallel argument works e.g. for the apriority of geometry, but we shall not follow it here.

We may thus say in conclusion that where B is a successful logical (conceptual) analysis of A their senses determine the same reference (concept), while B clearly articulates a sense that stands to that of A in the particular explicatory relation constrained by our five principles. These principles, as noted above, may fall short of being necessary conditions applicable to any proposed analysis, but they are always pertinent in assessing it, and they may disqualify some analyses and motivate and support others.

As Frege makes clear on several occasions his main concerns in FA, and since the *Nachwort* for *Grundgesetze*, his concerns with extensions and the problem his basic law V encounters, were ultimately rooted in these philosophical aspects of his explicatory definitions.[38] For, alternative courses of bypassing these problems (e.g. in axiomatic set theory) would seem to him of limited philosophical significance if they did not form a basis for analytical explications of the nature of arithmetical and logical objects (i.e. extensions) and of how they are given to us. Frege states something like this in the concluding sentences of his appendix (*Nachwort*) to the second volume of Gg (1902). After discussing the problem raised by his axiom V and some possibilities of replacing it he says:

> The prime problem of arithmetic may be taken to be the problem: How do we apprehend logical objects, in particular numbers? what justifies us in recognizing numbers as objects?

Frege then expresses his current belief that the question has not been finally solved, but that his efforts were on the right track. The ultimate reason for this is the philosophical principles of an analytical explication that shaped this track, some of which were proposed above. And although I have talked of these principles separately, I hope it is clear that there

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[38] Frege had qualms about extensions prior to learning of Russell's Paradox and admitted his axiom V to be not as evident as other logical axioms. But after 1903 he gradually became more and more skeptical about the appeal to extensions, and the reduction of arithmetic to pure logic. This is a fairly well known story I shall not repeat here. See a review of this e.g. in Parson's ch. 5 of *From Kant to Husserl*, Harvard 2010.

are deep inter-connections between them, to the point we can talk of them together as constituting a skeleton of a theory of logical (philosophical) analysis.[39]

Gilead Bar-Elli, Jerusalem, June 2015

[39] Most of the above Fregean Principles were raised by him in the course of his logistic project that we would now perhaps call reducing arithmetic to logic. Though there are many kinds of reduction in many areas, I believe (without being able to argue for it here) that for many of them, particularly in "open" domains like e.g. the mental, in order to carry the philosophical significance many people attach to them, they should satisfy the conditions of analytical explication sketched above.

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