

## *Identity in Frege's Begriffsschrift: Where Both Thau-Caplan and Heck Are Wrong*

GILEAD BAR-ELLI  
The Hebrew University of Jerusalem  
Jerusalem 91905  
ISRAEL

Frege's views on identity continue to provoke scholars, and rightly so. In particular his view in *Begriffsschrift* (Bs) of 1879, and its relation to his view in 'Über Sinn und Bedeutung' (SB) of 1892 deserve careful attention. The issues involved have a wider significance than Frege's specific views on identity in different periods, though these are important enough. They concern also the move from what I call below 'thin' semantics, which is exhausted in signs being assigned content, to a 'thick' semantics, in which 'modes of determining their content' (Bs), or *Sinne* (SB) are also concerned.

In 'What's Puzzling Gottlob Frege?' (2001) Michael Thau and Ben Caplan (T&C) argue that in SB Frege did not reject his earlier view of identity in Bs, and that the arguments raised there endorse those of Bs. In this, I believe they are right.<sup>1</sup> However, T&C adopt and enhance a widespread view, according to which the Bs theory of identity is 'metalinguistic.' By this they presumably mean that identity is a relation between signs and that an identity statement is about signs. This, I

---

1 I have argued so myself in the section on identity in the Introduction to my Hebrew translation of Bs (Jerusalem: Shalem 2003). I didn't know then of T&C's paper, and had argued for this view in lectures many years before that.

believe, is wrong, or at least misleading. I shall argue that the objectual vs. meta-linguistic opposition is anachronistic and may be misleading with regard to Bs, in a way that can distort its view. If, nevertheless, we insist on enforcing it on the text, the Bs view should rather be seen as objectual, not meta-linguistic in the above sense: identity, in Bs, is **identity of content**; and the contents of names are objects. But in order thus to talk of objects, Frege realized, we must regard identity statements as concerned **also** with the ways of determining these contents. So, at least, I shall argue.

T&C also argue that the meta-linguistic theory they find in Bs is never rejected or replaced by Frege in his later writings. I think this is wrong, but shall not discuss the matter here. I confine my remarks here only to some aspects of Bs and SB.

In his 'Frege on Identity and Identity Statements' (2003), Richard Heck accepts T&C's meta-linguistic construal of identity in Bs, on which, as stated above, I believe he is wrong, but rejects their view with regard to Frege's later writings. Heck insists on distinguishing the question of what identity is, from the question of what identity statements express. In Bs, according to him, Frege thought that identity is a relation between expressions; later (from the early 90s), he rejected it and thought that identity is a relation between objects. Heck seems to begin the late phase (as is customary) with 'Funktion und Begriff' (FB), from which he cites extensively. This should include SB, which was published after FB, but Heck doesn't discuss SB in detail.<sup>2</sup> As to what identity statements express, Heck is less clear. He claims that in *Basic Laws of Arithmetic* (of 1893, BL), Frege thought that '0=1' for instance 'denote the True if and only if zero is one. The thought it expresses is thus that zero is one, not that "0" denotes the same object as "1"' (95). This is of course right, but it may seem to underrate the main point, viz. that the thought expressed here is not just the identity of the objects, but their identity **as conceived under the particular corresponding senses** (of 'zero' and 'one'), which are constituents of the thought.

---

<sup>2</sup> SB is quite surprisingly silent on the matter and does not explicitly answer its opening question. A sort of an objectual construal of identity is however naturally suggested, and wouldn't be off mark.

## I The Main Point of the Identity Section (8) in Bs and Its Relation to SB

Both T&C and Heck (as almost everybody) seem to ignore a terminological point that reflects Frege's main innovation in the identity section (8) of Bs. Frege distinguishes there between names (Namen) and signs (Zeichen). The distinction is not explicitly stated, but it is used almost consistently in section 8. In the second sentence of the section, for instance, Frege writes:

'Während sonst die **Zeichen** lediglich Vertreter ihres Inhaltes sind ... sobald sie durch das Zeichen der Inhaltsgleichheit verbunden werden; denn es wird dadurch der Umstand bezeichnet, dass zwei **Namen** denselben Inhalt haben.'

('Whereas in other contexts **signs** are merely representatives of their content ... they suddenly display their own selves when they are combined by means of the sign for identity of content; for it expresses the circumstance that two **names** have the same content.' (Emphases added))

Throughout the section, except for the last sentence, Frege speaks of identity consistently in terms of names, i.e. signs endowed with modes of determining their contents,<sup>3</sup> whereas in the rest of the book he talks, where identities are not concerned, simply of signs. A sign, in Bs, just denotes its content; this exhausts its meaning. A name, in contrast, includes a mode of determination (Bestimmungsweise) of its content. This notion of mode of determination of content is akin to the later notion of sense (*Sinn*), and forms the main innovation of section 8 of Bs. It would be going too far, however, to identify it with the later notion of sense and to claim that Frege had the notion of sense already in Bs. For first, in Bs Frege appeals to mode of determination as an exception to his general semantics, an exception that is confined to identity statements, whereas his later notion of sense applies generally to any expression in any context. Second, the later notion of sense cannot be detached from the comprehensive and rich theory of sense and reference, which Frege had not yet have in Bs. In any case, it seems evident, as we shall see, that by bringing in names and modes of determining their contents into his account of identity in Bs Frege did not mean to deny that names (in identity statements) also denote their contents, and that such statements

---

<sup>3</sup> Heck's ignoring of the distinction between names and signs reflects in a mistranslation in which he says that 'Names "stand at times for their content, at times for themselves"' (Heck, 89). Frege writes Zeichen where Heck puts 'names.'

are about these contents. He wanted to say that **in addition**, they also include (and express) the ways in which the contents are determined.

The semantics of signs in Bs is a ‘thin,’ one-dimensional semantics — signs stand for their contents, and this exhausts their meaning. The correlation between a sign and its content is an arbitrary convention or stipulation. Signs are not inherently or internally connected to their contents. Talking about a sign is, therefore, meta-linguistic in a simple and straightforward way. The semantics of names, in contrast, is ‘thick’: a name does not only denote its content, but includes and expresses a way its content is determined. This, Frege emphasizes, is an objective feature that pertains to the ‘essence of things’ (*Wesen der Sache*), to use his terms. The connection here is therefore not arbitrary or conventional; it is essential and objective. This distinction is the heart of one of Frege’s main arguments there, namely, that identity statements are **not about signs** conceived as forms: ‘Die verschiedenen Namen für denselbe Inhalt nicht immer blos eine gleichgiltige Formsache sind’ (‘Different names for the same content are not always merely an irrelevant question of form’; Bs 15). A name, including as it does a mode of determining its content, is essentially connected to its content. Talk of names is therefore also talk of their contents and talk of names (in identity statements) is not simply meta-linguistic. The distinction between the linguistic and the meta-linguistic may thus be misleading here (it is definitely anachronistic with regard to Bs).

Having said that, in order to gear the following remarks to the articles of T&C and Heck, I shall distinguish in what follows between ‘the simple meta-linguistic view’ — that identity statements are only about linguistic **signs** — and ‘the complex meta-linguistic view’ — that identity statements are about **names** and their contents, where a name includes the mode of determination of its content. In using a sign in a (regular) proposition, according to Bs, the sign just goes proxy for its content (say, object), and the connection between them is arbitrary and conventional. In using a sign, we talk about its content, but since a sign is not essentially related to its content, and the relationship between them is conventional and arbitrary, talking of a sign does not pertain to its content, and is thus simply meta-linguistic. In using a name in a proposition of identity, on the other hand, it stands for the content **and expresses the way it is determined** (Frege says it ‘includes’ it); the connection between name and its content is therefore not arbitrary, but relates to the ‘essence of the thing.’ Talk of the name is thus complexly meta-linguistic. The main point of section 8 of Bs is the move from the simple to the complex view. Frege is up to show that the simple view is ‘an empty illusion’ (he speaks of the *Nichtigkeit dieses Scheines*). Frege introduces his explanation for this by saying:

'Dies erweckt zunächst den Anschein, als ob es sich hier um etwas handle, was dem *Ausdrücke* allein, nicht *dem Denken* angehöre.... Um die Nichtigkeit dieses Scheines klar zu legen wähle ich folgendes Beispiel aus der Geometrie (14).

('At first we have the impression that what we are dealing with pertains merely to the expression and not to the thought.... To show that this is an empty illusion, I take the following example from geometry').

This is a clear rejection of the simple meta-linguistic view. Frege then gives a geometrical example of an identity, and explains that in understanding it we must consider not only the signs and their contents but also the ways these contents are determined. A major point here is that this view does not preclude that the relation of identity is between contents (objects) and that identity statements are about contents. This characterizes what we have called the complex view. Frege's view there cannot be understood without the idea (perhaps obscure and, some would say, confused) that in talking about modes of determination of a content (included in names) we may *ipso facto* also talk about the content; and a relation between modes of determination of contents may thus be also a relation between the contents.<sup>4</sup>

This conception doesn't seem to fit squarely within the current linguistic/meta-linguistic distinction. But this should not detain us, for there are a lot of anachronisms that get into the way both T&C and Heck present the simple meta-linguistic view. This perhaps is inevitable, but it is important to be aware of it, and take the required precautions. The very idea of a meta-language is foreign to Frege of 1879. Heck also talks about identity in Bs as a function, and asks whether the arguments of this function are names or objects. But this has an anachronistic ring about it as well. For Frege of Bs, functions and relations, as well as arguments, are linguistic expressions, not the contents of them:

If in an expression, whose content need not be capable of becoming a judgment, a simple or a compound sign has one or more occurrences and if we regard that sign as replaceable in all or some of these occurrences by something else (but everywhere by the same thing), then we call the part that remains invariant in the expression a function, and the replaceable part the argument of the function. (Bs, section 9)

(... so nennen wir den hierbei unveränderlich erscheinenden Theil des Ausdrucks Function, den ersetzbaren ihr Argument).

---

<sup>4</sup> This, I believe, is true also of his later notions of sense and reference. I argued for that in Bar-Elli (1996), both as a general thesis, and in particular with respect to his view of oblique contexts and *ungerade Bedeutung* (see ch. 9).

It seems apparent (though the point has been debated) that here both function and argument are linguistic expressions.<sup>5</sup> Hence, for Frege of *Bs* it was most natural to speak of identity as a relation between names (including the mode of determination of their contents).

Frege in *Bs* did not yet have the later distinction between sense and reference, and he talked in terms of the one notion of content (*Inhalt*). Content was presented as a sort of abstraction from judgment, where one abstracts from the claim to truth in a judgment (*Bs*, section 2). Yet, Frege distinguished there between contents that can become a judgment (*beurtheilbare*), like the content of a propositional sign, and contents that cannot, like the content of a singular name or a functional expression. Since Frege later changed his view, and replaced talk of content by talk of either reference (*Bedeutung*) or sense (*Sinn*), it is difficult, and perhaps impossible to be strict here, but in most contexts in *Bs* his use of 'content' is much closer to his later reference than to his later sense. It seems clear, for instance, that in *Bs* contents determine the truth and falsity of judgments, as well as the logical potential of inferential relations, which are entirely 'extensional' in *Bs*. It is also clear that in the geometrical example Frege gives at 8 the point itself is the content of the name, and it is distinguished from the way of determining it. Hence it seems clear that at least in identity statements, the contents concerned are akin to Frege's later references — they are the objects themselves. Be it as it may, my point here against the simple meta-linguistic view, and for the claim that identity statements are also about the contents of their names, holds good even if we construe the notion of content somewhat differently.

---

5 It is doubtful whether Frege has been entirely consistent in this. Some scholars assume that function and argument are linguistic expressions in *Bs* (see for instance Carl (1996), 65), while others claim that function and argument are contents in *Bs*. Baker and Hacker, for instance, assume, without much ado, that function and argument in *Bs* are contents (1984, 133-85); in their recent (2003) they criticize the 'linguistic interpretation,' which they present as the standard one (held, among others, by Dummett, Geach and Kenny), and propose an alternative reading, based, *inter alia*, on a sharp distinction between 'expressions in *Bs*' and 'expressions in a natural language' (284-8). But even they present their reading quite cautiously as a possible one, whose difficulties are outweighed by its alleged merits. I believe that Frege, though he may have been somewhat careless about use and mention in *Bs*, was driven to his linguistic conception of function for weighty reasons, but I cannot expand on them here (see note 12 below). Like Demopoulos I believe that 'function' of the first chapter of *Bs* should be distinguished from 'procedure' (*Verfahren*) of the third, though Frege use the same letters for both (cf. Demopoulos (1995), n. 26 on 85). But these niceties are impertinent to the question discussed in the text, and I shall not get into them here.

The identity sign can occur between expressions for both kinds of content — those that can become judgments and those that cannot. This raises a problem: In his later view Frege regarded a sentence as a complex name referring to a truth value, regarded as an object.<sup>6</sup> It was then straightforward to regard propositional equivalence as identity. In *Bs*, however, Frege did not have the notion of a truth-value as an object denoted by a sentence, as in his later view. And yet he had one notion of identity applicable to all contents (or expressions). In particular, he did not distinguish between identity and propositional equivalence (we shall ignore here the problems pertaining to identity of functions). Since his main point in section 8 is that in identity statements we must consider not only the content (and not only the sign for the content), but also the way the content is determined, this must apply to the propositional case as well. (Had Frege been consistent here, his view that the identity sign may occur between sentences, and that in general the identity sign occurs between names, should have led him to regard sentences as names already in *Bs*.) But whereas we may have a fairly clear idea of the distinction between an object and the way it is determined, we seem to lose any grip on the idea with respect to propositional contents: what could the distinction amount to there?

We can grasp the distinction between the man Socrates and the way of determining it as the content of an expression, which is included in the name 'the great teacher of Plato.' Can we grasp, analogously, a distinction between the content of, say, 'the earth is round' and a way of determining this content, which is 'included' in the sentence? We can grasp of course the distinction between a content and a way of determining its truth. But this is not the issue here. The issue is whether we can intelligibly distinguish between a propositional content and a way of determining it (not its truth). The difficulty in providing any satisfactory account for this was, I believe, one of the main reasons for Frege's later view, in which he abandoned the notion of content in favor of the bifurcation into sense and reference, and postulated truth-values as objects named by sentences, and thoughts as their senses. But I shall not elaborate on the point here.

For these and other reasons, it may seem pointless to be very strict about Frege's formulations in comparing his view of these matters in *Bs* with his later views. Heck and the 'conservative' interpreters, may justly claim that so much the better for their view that much of this changed in the 90s. But for understanding *Bs* we should better try seeing through

---

<sup>6</sup> I expanded on the reasons for recognizing truth-values as objects in Bar-Elli (2001).

this haze of inaccuracies and anachronistic terminology into the real issue. And the real issue, I suggest, is the recognition, in Bs section 8, that the ‘thin’ semantics of signs standing for their contents is too narrow a framework for the semantics of identity statements; that we must also consider modes of determination of contents, which are ‘included,’ or expressed by the names for them. The semantics of names (in identity statements) is thick in that they do not only have contents and stand for them, but also include (and express) modes of determining these contents. These modes are objective, they are communicable and belong to the ‘essence of things,’ and they are expressed by names, in addition to these names denoting their contents. Let me emphasize again that in Bs Frege had not yet generalized this view to all expressions in all contexts — this, with a comprehensive theory of sense and reference was to come later, beginning in the articles of the 1890s. In Bs, his general semantics, except for identity statements, remained ‘thin.’

## II The Identity Relation, and What Identity Statements Are About

In the opening of section 8 of Bs Frege says that a statement of identity is about names, not about their contents. This is an unhappy formulation. Frege evidently meant that it is about names, and not **merely** about their contents. This should be evident not only from the title of the section (*Die Inhaltsgleichheit* — The Identity of Content); it is repeated in the body of the section, where Frege speaks repeatedly of the sign of identity as a ‘sign for the identity of content.’ This would be most implausible if Frege thought that identity were not a relation between contents. The content, as mentioned above, can be one that can become a judgement (*Beurteilbare*) — by his later view, a thought — or one that cannot — in his later view, an object or function.

That identity statements are also about the contents of their names is evident also from the main tenor of the section, where Frege argues that a mode of determination of a content is not a matter of arbitrary stipulation concerning signs and their form, but has to do with the ‘essence of things’:

Die verschiedenen Namen für denselben Inhalt nicht immer blos eine gleichgiltige Formsache sind, sondern dass sie das **Wesen der Sache** selbst betreffen, wenn sie mit verschiedenen Bestimmungsweisen zusammenhängen. (15)

(Different names for the same content are not always merely an irrelevant question of form; rather, they concern the essence of the thing, if they are associated with different ways of determining the content.) (I changed here the Bauer-Mengelberg translation).



This means that there is nothing arbitrary or conventional in this connection, and since a name 'includes' (or expresses) such a mode of determining a content, its connection to the content is also not arbitrary or conventional. This, to repeat, is not a subsidiary casual remark, but the gist of Frege's argument in the section. Earlier in the section he says that the possible impression that identity is a relation between mere arbitrary signs is 'an empty illusion.' Hence, the simple meta-linguistic theory is not Frege's view in Bs.

Moreover, it can't be his view unless one is prepared to ascribe to him (in the work in which he initiated modern logic!) the most elementary fallacy of ambiguity. For if we take the simple meta-linguistic theory literally, from 'Fa' and 'a=b' one cannot infer 'Fb,' since the two occurrences of 'a' (and the two occurrences of 'b') do not have the same content (do not refer to the same thing) — the first refers to the content of 'a,' while the second — to 'a' itself.

Frege, of course, endorses the law of identity — it is a fundamental principle of his logic (52), and he ends section 8 itself with it. It is hardly credible that Frege fell victim to this fallacy, and didn't notice that on his view of identity the principle turns out invalid, and that this view makes every 'mixed' context, in which identity occurs with regular contexts, incomprehensible. Hence, for anyone who would rather not ascribe to Frege these trivial fallacies, the simple meta-linguistic theory cannot be Frege's view in Bs.<sup>7</sup>

And in fact, as stated above, a close reading of section 8 of Bs makes it clear that Frege's main concern is to show that identity statements are not only about the contents of their signs (as is the case in e.g. conditional and negative statements), and not only about the signs (as the simple meta-linguistic view holds), but about the contents conceived by the modes of determination included in their names. One needs not adhere too stringently to the unhappy formulation of the first sentence (cited above), but try to understand the main tenor of section 8 of Bs.

We thus see that there are strong reasons to believe that in Bs Frege thought that identity statements are concerned with the contents of their names. On the other hand we have Frege's seemingly conflicting claim that identity is a relation between names. The way to reconcile these two, I have suggested, is to realize that a name, being endowed with a way of determining its content, is essentially and non-arbitrarily connected to its content, for the way of determining it is objective and pertains to 'the essence of the thing.' Hence, we can say, as I said above, that when

---

<sup>7</sup> This is different from, though related to, Heck's claim that the meta-linguistic view of identity makes quantified identity statements muddled (87).

names (not merely signs) are concerned, talk of names may also be talk of their contents.

Heck (like T&C and pretty much everybody)<sup>8</sup> claims that the Bs' theory of identity is simply meta-linguistic, and emphasizes, in support, Frege's wording of his (52), in which Frege writes: 'the content of c is identical with the content of d' etc. (Heck, 95). But this is poor evidence. First, 'the content of c' is a complex name of a content, not of a sign. Second, Frege writes in this way throughout Bs, with regard to all contexts, including regular ones, like simple monadic formulas, or conditionals, negations, etc. But these contexts are explicitly about the contents of their signs, and no meta-linguistic theory with regard to them is at issue at all. Even in the above wording of (52), which Heck quotes, Frege renders a simple conditional like 'If f(c) then f(d)' in his regular way in terms of 'the case where f(c) is affirmed and f(d) is denied' etc. A careful reading of the explanation attached there suggests that 'f(c)' and 'f(d)' are mentioned here. On Heck's reading, if followed consistently, this would imply that simple monadic predications and conditionals should be read 'meta-linguistically.' But no one would suspect Frege of proposing a meta-linguistic construal of regular contexts like 'f(d),' or 'if f(d) then g(c)' etc., and he explicitly says at the beginning of section 8, that in these contexts signs simply stand for their contents, so that in using them we simply talk about their contents.<sup>9</sup>

We can use the following list of distinctions for summarizing some of the above points:

1. The identity relation is a relation between:
  - a. Objects.
  - b. Signs.
  - c. Names.
  - d. Contents (objects) conceived under a particular mode of determination contained in their names.

---

8 Let me mention one more example. W. Taschek (1992) presented Frege's 'identity puzzle' in a way with which I am, in general, in sympathy. But with regard to Bs he also subscribes to the mistaken simple meta-linguistic view (see 770-1). This, I believe, hardly squares with his (justified) opposition to information-based rendering of content, and with his view that this rendering implies a meta-linguistic solution to the puzzle.

9 In general, it is often difficult to decide whether affirmation and denial are meant to apply to contents or to propositions (Sätze). A careful reading of 5, for instance, suggests that they apply to propositions. This may be one of the cases in which reading Bs with a strict distinction between use and mention may be quite embarrassing.

2. An identity statement expresses the thought that:
  - a. Two signs refer to the same thing (have the same content).
  - b. Two names (including the mode of determination) refer to the same thing.
3. An identity statement expresses the thought that:
  - a. The content (referent) of a sign is the same as the content (referent) of another.
  - b. The content (referent) of a name (including the mode of determination) is the same as the content (referent) of another.
4. An identity statement is about:
  - a. The names it contains.
  - b. The contents (objects) referred to by these names.
  - c. The contents (objects) as conceived under the modes of determination included in their names.

For the reasons expounded above, I propose that 1.d., 2.b. (or 3.b.; I take them to be equivalent),<sup>10</sup> and 4.c. are the real views expounded in Bs. We have found four main reasons for that: (a) Frege repeatedly talks in section 8 of identity of content, which is also the title of the section, and which would be most implausible if he thought that identity was merely a relation between expressions. (b) A Name, in distinction from a mere sign, includes a mode of determining its content, so that the connection between them is not arbitrary, which may render talk of names to be *ipso facto* talk of their contents. (c) The simple meta-linguistic view does not square with Frege's use of the law of identity, which is basic in his logic, and with 'mixed contexts.' (d) Frege's basic argument in section 8 would not make much sense on the simple meta-linguistic model, since he explicitly claims that signs (unlike names) are connected to contents arbitrarily, and his explicit and main point was to overcome this arbitrariness.

I believe that each of these makes a good case for my point here; all of them together make it overwhelming: in Bs Frege did not hold the simple meta-linguistic theory; if we are to enforce the meta-linguistic perspective on his views, we might say that he held the complex one: identity, being a relation between names, is also a relation between their contents,

---

<sup>10</sup> They are not strictly equivalent, for in 2 names are directly mentioned, while in 3 they are indirectly mentioned in mentioning their content. I still take them here as equivalent, in line with the view, expounded in the text, that since a name includes a mode of determining its content, in speaking of a name, one indirectly speaks of its content.

conceived or determined in particular ways, and an identity statement is about these contents.

### III The Change in SB

True — this, strangely enough, was confined in Bs to identity statements. But since identity is a logical notion, and is part of the basic notation of Bs, the recognition must have had a general significance, the full realization of which came in the early nineties. With removing this confinement to identity statements, the gist of the Bs view of identity is maintained in SB. It is in fact generalized there as pertaining to the entire language — to expressions in any context, not only to identity statements, as in Bs. SB presents many other important aspects — indeed a whole theory — of sense and reference, which are beyond our concerns here. I confine my remarks here only to the particular distinction between content and mode of determining it, which was shown to be crucial for understanding Frege's theory of identity in Bs. In SB the distinction between signs and names is rejected — all meaningful expressions are treated as including or expressing modes of determination of their contents (i.e. on the model of 'names' in identity statements in Bs), and thus a uniformity of the semantics in all contexts is attained, though the semantics is uniformly a thick one. SB could thus be regarded as a generalization of the Bs semantics of identity to all contexts.<sup>11</sup> What is rejected in SB of the Bs account is the 'thin' conception of the meaning of expressions (signs) in regular contexts (in which signs just stand for their contents), not the 'thick' conception of names in identity ones (in which they also include the mode of determining their contents).

The result of all this was the rejection, in SB, of the Bs notion of content (*Inhalt*), in favor of the bifurcation into reference (or meaning, *Bedeutung*) and sense (*Sinn*). A major reason for this was Frege's realization that the general 'thin' semantics of signs and contents in regular contexts in Bs could not allow for the vital distinction, recognized already in Bs with regard to identity statements, between content and a way of determining it: If one says that the latter is part of the former (included in it), identity statements carrying cognitive value would not be true, for their contents would be different; if, on the other hand, one says that it is not part of the content, it has, within the confinements of 'thin' semantics, no place and no role to play in understanding these identity statements. Frege's

---

11 This perhaps explains Frege's strange formulation, in SB, where he repeatedly talks of 'names or signs,' as if to blur his own previous distinction between them.

argument shows that both horns of the dilemma are unacceptable and we have to abandon the restricted limits of the thin semantics, and thicken it by the notion of modes of determination or Sinn. This, together with the above difficulty of applying the distinction to sentences and their contents were, I propose, among the chief reasons that led to the change.<sup>12</sup>

#### IV The 'Identity Puzzle,' Cognitive Value and Conceptual Content

I emphasized above that with regard to identity statements, Frege proposed in Bs a 'thick' conception in which names (not mere signs) include the way of determining their content. Now, in order to enhance the above dilemma, let us look in more detail into the possibility that this 'mode of determination of a content' is part of the content. We have seen above that it is not arbitrary or conventional, but pertains to the essence of things. Yet, it is clear that in Bs Frege did not regard it as part of the content. Supposing that he did, would make almost everything he says about content, particularly in section 8, unintelligible: For example, in the geometrical example he gives there point A and point B are proved to be the same; the content of 'A' and of 'B' is the point itself, and they differ in the ways it is determined as their content. If these ways were part of the content, their contents themselves would be different.

One must remember, as Frege advises the reader, that not any aspect of the content, but only 'conceptual content' is concerned in a Begriffsschrift (Bs, Introduction, and section 3). Conceptual content is defined as what matters for inferential relations. Whatever is not pertinent to these relations is not part of the conceptual content, and hence, is not logic's concern. In Bs Frege apparently did not appreciate that mode of determination is thus relevant to inferential relations (he doesn't mention any that would be thus affected), and therefore did not regard it as part of the content. This may explain his bizarre and ad hoc attitude, in which he marshaled the notion of mode of determination in order to account for what seemed to him then a peculiarity of identity statements.

---

12 Another major reason, which I shall not discuss here, was, I propose, somewhat similar difficulties Frege faces with regard to the notion of function in Bs. Here again it was necessary to make some distinctions, which the narrow scope of the thin semantics of signs and contents in Bs did not allow. I expanded on this in the Introduction to my Hebrew translation of Bs *ibid*.

Similar remarks apply, as we shall see, to cognitive value. Suppose we grant that two sentences, including different names, which denote the same thing but contain different ways of determining it, differ in cognitive value. Does this reflect in their inferential power? There isn't any mentioning of this in Bs.

It seems therefore that only in SB did Frege discover that there are contexts in which the mode of determination (what he then called *Sinn*) must be regarded as belonging to the conceptual content — namely, oblique contexts of 'indirect reference,' like belief contexts. Here *Sinne* must be brought into a proper account of inferential relations.<sup>13</sup> There are uses of 'belief' in which from 'x believes that Fa' we can infer 'x believes that Fb' only where 'a' and 'b' have the same *Sinn* or mode of determining their contents.

The introduction of *Sinne* is usually tied up with the 'identity puzzle' Frege presented in both Bs and SB. Heck considers three versions of the puzzle (88-90):

- 1)  $a=b$  is informative although  $a=a$  is not (they differ in cognitive value).
- 2) Fa and Fb differ in cognitive value, although 'a' and 'b' refer to the same thing.
- 3) 'P' and 'Q' may differ in cognitive value, although they denote the same truth value.

I shall not go here into the differences between these versions, and the question of which is Frege's. A long and venerable tradition has presented the 'puzzle of identity' that brought Frege to his celebrated distinction between sense and reference in SB, in terms of 'cognitive value.' Heck rightly emphasizes that the issue is not the difference between a sentence that has and one that does not have cognitive value; the issue is rather that two sentences with co-referring terms may have different cognitive values (89-91).<sup>14</sup>

---

13 I find it quite surprising that Taschek says that 'Frege nowhere offers a sustained and systematic discussion' of the logical relevance of cognitive differences of the kind he discusses (772), for this is a major topic in SB (particularly 37-39.). The discussion there seems to me more directly pertinent to the logical appraisal of differences in cognitive potential than the indirect way by which Taschek argues in section 3 of his paper.

14 This has been argued in Bar-Elli (1981).

He also argues that according to Frege, 'the thoughts expressed by certain sentences differ **because** the senses of the names contained in them differ' (93). Evidently, there are contexts in which this 'because' is plain and straightforward: a thought consists of its constituent senses; hence a change in these senses causes a change in the thought. However, as a general claim about difference of sense (which is evidently Heck's intention here), this, I believe is a mistake. For Frege, the main criterion for difference of sense is difference of thoughts (containing the senses), and the criterion for difference of thoughts is epistemic: thoughts differ if a competent speaker can believe the one without believing (or when disbelieving) the other. Hence, if Heck's emphasized 'because,' in the above citation, is epistemic, if he maintains that we know that two thoughts differ because we know some of their constituents to differ in sense, he is, I think mistaken. It is, in general, rather the other way around: If two thoughts differ, and they are expressed by two sentences whose sole difference is that the one contains an expression where the other contains a different one (possibly co-referential), these two expressions differ in sense.

This seems to me important, among other things, for the following reason: for Frege, sense belongs to logic, and logic, as stated above, is concerned only with 'conceptual content' — namely only with those features of sentences that make a difference to their inferential power (what can be inferred from them, together with other propositions, and from what they can be inferred). Now, it could be argued (and has been argued) that the notion of sense and differences of cognitive value may be interesting and important for, say, epistemology and philosophy of mind, but do not belong to logic, and are not a basis for a logical difference.<sup>15</sup> This, of course, was not Frege's view. He, therefore, must have pointed out contexts in which sense and cognitive value matter for inferential relations. And Frege's paradigm examples for these are epistemic cases of propositional attitudes, like propositional belief contexts. This, which perhaps had been hinted at but not been fully appreciated in Bs, was enunciated in SB. And with this appreciation came the full recognition of *Sinne* as belonging to the conceptual content (to sin with another anachronism) and their full insertion into his logic.

*Received December 2004*

*Revised May 2005*

*Revised August 2005*

---

<sup>15</sup> Cf. the discussion of the 'neo-Russellian' view in Taschek (1992), 778ff.

## References

### Frege's Writings:

- Bs — *Begriffsschrift*, Halle, 1879. Reprinted (I. Angelelli, ed.) 1964, 1998. English translation (S. Bauer-Mengelberg) in: *From Frege to Gödel*, J. Van Heijenoort, ed. Cambridge: Harvard University Press 1967.
- FB — 'Funktion und Begriff' (1891), reprinted in *Funktion, Begriff, Bedeutung* (G. Patzig, Heraus.), Göttingen: Vandenhoeck & Ruprecht 1975. English translation, 'Function and Concept' (P. Geach) in *Translations From the Philosophical Writings of Frege*, Geach and Black, eds. Oxford: Basil Blackwell 1952: 21-41.
- SB — 'Sinn und Bedeutung,' (1892) reprinted in *Funktion, Begriff, Bedeutung*, English translation 'On Sense and Reference' (M. Black) in Geach and Black: 56-78.
- BL — *Basic Laws of Arithmetic* (M. Furth, trans.). Berkley: California University Press 1964.

### Other Writers:

- Baker, G. and P. Hacker. 1984. *Frege: Logical Excavations*. Oxford: Blackwell.
- Baker, G. and P. Hacker. 2003. 'Function in Begriffsschrift.' *Synthese* 135/3: 273-97.
- Bar-Elli, G. 1981. 'Frege and the Determination of Reference.' *Erkenntnis* 16: 137-60.
- Bar-Elli, G. 1996. *The Sense of Reference — Intentionality in Frege*. Berlin and New York: Walter de Gruyter.
- Bar-Elli, G. 2001. 'Sense and Objectivity in Frege's Logic,' in *Building on Frege: New Essays About Sense, Content, and Concepts*, A. Newen, et al., eds. Stanford: CSLI. 91-112.
- Carl, W. 1994. *Frege's Theory of Sense and Reference: Its Origin and Scope*. Cambridge: Cambridge University Press.
- Demopoulos, W. 1995. 'Frege and the Rigorization of Analysis,' in *Frege's Philosophy of Mathematics*, W. Demopoulos, ed. Cambridge: Harvard University Press.
- Heck, R. 2003. 'Frege on Identity and Identity Statements.' *Canadian Journal of Philosophy* 33: 83-102.
- Taschek, W. 1992. 'Frege's Puzzle, Sense, and Information Content,' *Mind* 101: 267-91.
- Thau, M. and Caplan, B. 2001. 'What's Puzzling Gottlob Frege?' *Canadian Journal of Philosophy* 31: 159-200.