Momentum, Information Uncertainty, and Leverage - an Explanation Based on Recursive Preferences

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Abstract

Momentum payoffs concentrate in high information uncertainty and high credit risk firms and are virtually nonexistent otherwise. This paper rationalizes such momentum concentrations in consumption based equilibrium asset pricing. In our paradigm, dividend growth is mean reverting, expected dividend growth is persistent, the representative agent is endowed with stochastic differential utility of Duffie and Epstein (1992), and dividend streams are used for both consumption and debt repayment per Abel (1999). Employing reasonable risk aversion levels we are able to produce the observational momentum effects. Momentum profitability is large in the interaction between high levered and risky cash flow firms. It rapidly deteriorates and ultimately disappears as leverage or cash flow risk diminishes.

1 Introduction

Momentum effects in stock returns are robust. Fama and French (1996) show that momentum is the only deviation from the CAPM unexplained by the Fama and French (1993) model. Schwert (2003) demonstrates that profit opportunities, such as the size and value effects as well as equity premium predictability, typically disappear, reverse, or attenuate following their discovery. Momentum is an exception. Specifically, Jegadeesh and Titman (2001, 2002) document momentum profitability in the period after its discovery in Jegadeesh and Titman (1993). Korajczyk and Sadka (2004) find that momentum survives trading costs, whereas Avramov, Chordia, and Goyal (2006) show that the profitability of the other past-return anomaly, namely reversal, disappears in the presence of trading costs. Fama and French (2007) argue that momentum is among the few robust anomalies. Momentum robustness has generated a plethora of behavioral and rational explanations.[1]

Empirical work also uncovers momentum interactions. In particular, Hong, Lim, and Stein (2000) show that momentum profitability is especially prominent among small cap stocks. Zhang (2006) finds that momentum concentrates in high information uncertainty stocks, i.e., stocks with high return volatility, high cash flow volatility, or high analysts' earnings forecast dispersion. Avramov, Chordia, Jostova, and Philipov (2007) document that momentum prevails only among high credit risk stocks and is nonexistent otherwise.

[1]Behavioral: Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hon, Lim, and Stein (2000). Rational: Berk, Green, and Naik (1999) and Johnson (2002). Moskowitz and Grinblatt (1999) argue that industry momentum explains momentum in individual stocks. Avramov and Chordia (2006) show that while momentum is unexplained by risk based asset pricing models, removing business cycle effects from model mispricing completely eliminates momentum effects. These last two studies indicate that momentum could be attributable to missing factors in rational asset pricing models.

This paper shows that the momentum concentration in high information uncertainty and high credit risk stocks is consistent with consumption based equilibrium asset pricing. In particular, we develop a representative agent paradigm extending the Lucas (1978) economy. Here, dividend growth is mean reverting, expected dividend growth is persistent, and the representative agent is endowed with the recursive utility form of Duffie and Epstein (1992), which is the continuous time analog of the Epstein and Zin (1989) and Weil (1989) preferences. Moreover, based on the novel formulation of Abel (1999), our equilibrium clearing condition requires that the stream of dividends serve for both consumption and debt repayment. The leverage role in asset pricing goes back at least to Merton (1974).

Essentially, we extend the single-firm paradigm of Johnson (2002) along two important dimensions. First, whereas Johnson does not explicitly formulate investor preferences, here the dynamics of the underlying economic fundamentals is tied down to stochastic differential utility of the representative agent. In our set-up "dividends" represent the overall cash-flows of a firm, whereas "consumption" stands for cash flows from operations. Moreover, Johnson studies momentum effects, whereas we attempt to rationalize the concentration of momentum profitability among stocks with some particular styles. The question of interest is how momentum payoffs vary in equilibrium with financial leverage, which proxies for credit risk, as well as information uncertainty measures such as firm size as well as return and cash-flow volatilities.[2] Also related to our work is Sagi and Seasholes (2006) who study momentum focusing on real options characteristic of firms. They claim that firms with relatively high market value must have hidden growth options which generate high return autocorrelation.

[2]The leverage proxy for credit risk is sound. For one, there is a strong correlation (the time series mean of the cross sectional Spearman Rank Correlation is 0.34) between S&P credit rating and leverage. Moreover, leverage is a crucial determinant in modeling default risk.

Our setting is novel in that momentum is driven by two major themes - long-run risk coming up from Duffie-Epstein preferences as well as Abel's formulation of leverage. Specifically, under Duffie-Epstein preferences, an economic agent would prefer an early resolution of uncertainty about the growth rate of cash flows. When the growth rate is highly persistent, the resolution of uncertainty is quite slow. In addition, if the equilibrium path is exposed to high information uncertainty, as reflected by high volatility of growth rate of cash-flow, the risk premium required is even larger. That is, in the presence of highly persistent growth rate and high information uncertainty, the notion of long run risk, or cumulative risk over the investment horizon, emerges. Long run risk invokes high premium required for equity holding, as also noted by Bansal and Yaron (2004). The effects of persistence and high volatility of cash-flow growth rate on the risk premium are further exacerbated by non-linear effects attributable to leverage which slows down cash-flow growth.

The combination of long run risk and leverage generates the empirically documented concentration of momentum in stocks with high leverage and high information uncertainty. The mechanism works as follows. In the presence of leverage, the firm's operating cash-flow growth slows down and mean reverts rather slowly. Consider now winner and loser stocks. The winner is exposed to recurring good shocks to cash-flows whereas the loser realizes continuously poor shocks. The winner firm keeps on earning high returns from good performance, and the loser continues to do poorly over the investment horizon. The effect of leverage is to *continue* these stocks in their same respective growth path that mean-reverts slowly, such that the winner (loser) continues experiencing good (bad) growth rates and earns high (low) return thus creating momentum effects. Herein lies one prominent distinction from Johnson (2002). Specifically, unlike Johnson, we endogenously determine the market prices of risk from cash-flows based on the underlying dynamics of economic fundamentals. Indeed, the paradigm developed here is fairly general from the perspective of both preferences and dynamics of economic fundamentals. The utility specification breaks the tight association between the elasticity of intertemporal substitution and the risk aversion measure, which are reciprocals of each other under power preferences. Most importantly, we depart from risk-aversion being the primary candidate for risk premium and focus on long-run risk attributable to late resolution of uncertainty about growth of operating cash flows.

Based on simulations, we are able to generate high enough equity premium using reasonable risk aversion measures, operating cash-flow dynamics, and leverage. In the cross section, we are able to rationalize momentum interactions with credit risk and information uncertainty. In particular, we generate strong momentum effects in the interaction between high leverage and risky cash flow firms. We show that the autocorrelation between past return and future expected return is positive and increases with leverage and uncertainty in growth rates. Momentum effects rapidly deteriorate and ultimately disappear as leverage or cash flow risk diminishes.

Quantitatively, when operating cash flows are highly volatile, the expected return spread between the highest (winner) and lowest (loser) past return portfolios is 11.49% for high leverage and only 0.27% for low leverage stocks. The overall spreads are small when either the volatility of expected growth in cash flows is small or the expected growth in cash flows is not highly persistent. Indeed, while leverage is crucial, risk and persistence of cash flow growth are also important determinants of momentum effects in the cross section of returns. If operating cash-flows are highly persistent then the uncertainty about future cash flows gets resolved rather slowly, thereby inducing long-run risk. Moreover, higher growth rate volatility introduces bigger perturbations on the slow evolving growth rate making it more uncertain for our agent. Finally, we find that momentum is only mildly related to investor's risk aversion.

To summarize, it has been documented that momentum interacts with firm-level information uncertainty measures and credit conditions. Our collective evidence shows that equilibrium momentum indeed concentrates in the interaction between risky cash flows and high credit risk firms. In the presence of recursive preferences, financial leverage, and persistent growth, one can match equity premia, riskfree rate, and especially the observational momentum profitability at reasonable risk-aversion levels.

The paper proceeds as follows. Section 2 describes the economic setup including investor preferences and the dynamics of the underlying economic fundamentals. Section 3 derives the interaction of momentum with credit risk and information uncertainty. Section 4 reports the simulation results coming up from the theoretical formulations. Section 5 concludes and provides suggestions for future work. Technical details are in the appendix.

2 The economic setup

2.1 Preferences and Dynamics

In formulating investor preferences, we depart from the regular power utility specification. Theoretically, power preferences put a heavy restriction on elasticity of inter-temporal substitution (EIS) and risk aversion – they are reciprocals of each other – even when risk aversion and EIS are distinct economic quantities. EIS is about a deterministic consumption path as it measures the willingness to exchange consumption today for consumption tomorrow conditioned on a current riskfree interest rate, whereas risk aversion is about preference over a random quantity. Empirically, the power utility restriction gives rise to the equity premium and riskfree rate puzzles as well as the failure of the consumption based asset pricing model to explain the cross section dispersion in average stock returns.

Instead, we employ stochastic differential utility (SDU) of Duffie and Epstein (1992). The SDU is the continuous time analog of the Epstein-Zin (1989) and Weil's (1990) recursive preferences, which break the tight association between risk aversion and EIS. The SDU is identified by a pair of functions $(f^*, A(J))$, called an aggregator, where $A(J)$ is local risk-aversion and f^* represents the relative preference between immediate consumption and the certainty equivalent of utility derived from future consumption. An ordinally equivalent representation of the SDU is given by the normalized aggregator $(f, 0)$, which is formulated as

$$f(C, J) = \frac{\beta(1-\gamma)J}{1-\frac{1}{\psi}}\left[C^{1-\frac{1}{\psi}}\left((1-\gamma)J\right)^{\frac{\frac{1}{\psi}-1}{1-\gamma}} - 1\right]. \tag{1}$$

In equation (1), C denotes the current consumption, J is the continuation utility (or the value function) attributable to future consumption streams, ψ is the EIS, β is the discount rate standing for the time preference, and γ is the relative risk-aversion parameter. Under this convention, the time t value function of an agent can be written as

$$J_t = E_t \int_t^\infty f(c_s, J_s) ds. \tag{2}$$

We derive below an explicit solution for J_t assuming that $\psi = 1$. This assumption is innocuous. In particular, Bansal and Yaron (2004) and Ai (2007) show that $\psi = 1$ is the point of equivalence between wealth and substitution effects, but preferences over risk are still determined by risk-aversion. That is, if $\psi < 1$ ($\psi > 1$), the income (substitution) effect dominates. In other words, high growth rate can lead an agent to consume more (income effect) or invest more (substitution effect). Which course of action is undertaken depends upon the ψ parameter. Indeed, EIS primarily deals with determining the riskfree rate, which is not at the core of our study. The normalized aggregator based on $\psi = 1$ is the limit of (1) taking the form

$$f(C, J) = \beta(1-\gamma)J \left[\log C - \frac{\log(1-\gamma)J}{1-\gamma} \right]. \tag{3}$$

The observed dividend growth and unobserved expected dividend growth are formulated as

$$\frac{dD_t}{D_t} = X_t dt + \sigma_D dW_1, \tag{4}$$

$$dX_t = \kappa(\bar{X} - X_t)dt + \sigma_x dW_2, \tag{5}$$

where X_t is the expected dividend growth, κ stands for the speed of mean reversion, $\bar{X}$ is the long run mean of X_t, σ_D is the volatility of dividend growth, σ_x is the volatility of expected dividend growth, and the correlation between the two Brownian motions is ρ.

To account for leverage, we build on the novel formulation of Abel (1999). Abel is able to generate, in a fairly simple setting, low variability of the riskfree rate along with a large equity premium, both of which are patterns documented in the US economy in the post-war period. In particular, we assume that the equilibrium consumption is a portion of dividend

$$C = D^{\lambda}, \tag{6}$$

while the remainder of the dividend stream is distributed as adjustment cost or debt payment in the economy. The no-leverage case $\lambda = 1$ depicts an economy with no adjustment cost wherein the agent consumes the full dividend streams. In the $\lambda < 1$ case, which is at the core of our analysis, the rest of dividends go towards debt payment. Then stocks in this economy are residual claims on the consumption stream net debt payments.

In the presence of leverage, the consumption growth dynamics takes the form

$$\frac{dC}{C} = \mu_C(X_t)dt + \sigma_C dW_1, \tag{7}$$

where

$$\mu_C(X_t) = \lambda\left[X_t + \frac{1}{2}(\lambda - 1)\sigma_D^2\right] \tag{8}$$

$$\sigma_C = \lambda\sigma_D, \tag{9}$$

suggesting that expected consumption growth is slower than expected dividend growth as long as $\lambda < 1$. This has the following interpretation in our set-up. If the overall cash-flows of a firm, which follow the dynamics formulated in (4), experience a good shock, then the corresponding shock to operating cash-flows is dampened due to the presence of leverage. Further, the volatility of operating cash-flows diminishes in the presence of leverage.

The utility process J satisfies the Bellman equation with respect to equilibrium consumption

$$\mathcal{DJ}(C, X, t) + f(C, J) = 0 \tag{10}$$

where $\mathcal{DJ}$ is the differential operator applied to J with respect to $\{C, X, t\}$ with the boundary condition $J(C, x, T) = 0$. In the analysis that follows, we are interested in the equilibrium as $T \to \infty$. Thus, we drop the explicit time dependence assuming that the agent is infinitely long-lived and has reached the equilibrium over time. We can formulate an exact solution to the value function which is stated in the following proposition.

Proposition 1. *An exact solution to the differential equation in* (10) *is given by*

$$J(C_t, X_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \exp\left(u_1 X_t + u_2\right) \tag{11}$$

where

$$u_1 = \frac{(1-\gamma)\lambda}{\kappa+\beta} \tag{12}$$

$$u_2 = \frac{(1-\gamma)\lambda}{\beta}\left[\frac{(\lambda-1-\lambda\gamma)\sigma_D^2}{2} + \frac{\kappa\bar{X}}{\kappa+\beta} + \frac{(1-\gamma)\lambda}{\kappa+\beta}\left[\frac{\sigma_x^2}{2(\kappa+\beta)} + \sigma_D\sigma_x\rho\right]\right]. \tag{13}$$

Proof: see the appendix.

Unlike in power preferences, SDU incorporates the agent's value function in the current utility. Thus, expected growth rate enters into the agent's utility through the value function J. Indeed, expected growth rate has important implications for future consumption streams. Higher expected growth rate indicates higher expected future consumption and hence higher future utility which is reflected through higher current value function. Thus, J_X is positive. On the other hand, J_{XX} is negative suggesting that J is concave in X or J_X increases in X in diminishing rates. We also run some simulations to learn the impact of the value function on the current utility. The current utility is increasing (decreasing) in growth rates under low (high) leverage and low (high) uncertainty of future growth rates.

Moreover, the autocorrelation of X is important for understanding the expected growth rate effect on future utility. If expected growth rate is highly persistent (low κ) then high X_t implies high future expected growth rate, which, in turn, implies that the agent expects to consume more in the future. Thus J_X increases with expected growth rate persistence. Finally, J_X decreases in β. In particular, if the agent discounts the future more (higher β) then the impact of expected growth rate on the value function (future utility) diminishes.

The next section derives the pricing kernel dynamics, the asset return dynamics, the correlation between observed realized returns and expected returns, and especially the link between leverage, information uncertainty, and stock return momentum.

3 Asset Pricing

Duffie and Epstein (1992) show that the pricing kernel for SDU is given by $\Lambda_t = \exp(\int_0^t f_J ds) f_C$, where f_J and f_C are the derivatives of $f(C, J)$ in (3) with respect to J and C. Hence we can find the explicit pricing kernel dynamics as stated in the proposition below.

Proposition 2. *The pricing kernel dynamics is given by*

$$\frac{d\Lambda}{\Lambda} = -r_t^f dt - \lambda\gamma\sigma_D dW_1 + u_1\sigma_x dW_2 \tag{14}$$

where

$$\begin{aligned} r_t^f &= \lambda X_t + u_1\lambda\gamma\sigma_D\sigma_x\rho + \beta(u_2+1) - u_1\kappa\bar{X} - \frac{1}{2}\lambda\gamma\sigma_D^2(\lambda\gamma+1) - \frac{1}{2}\sigma_x^2 u_1^2 \\ &= \mu_C(X_t) + \beta - \sigma_C^2\gamma + \frac{(1-\gamma)\lambda}{\kappa+\beta}\sigma_C\sigma_x\rho. \end{aligned} \tag{15}$$

Proof: see the appendix.

Observe from equation (14) that leverage is an important determinant of both the drift and diffusion of the pricing kernel dynamics. Moreover, our equilibrium riskfree rate formulated in (15) has two attractive features compared to its power utility counterpart. For comparison, the corresponding riskfree rate for power utility is $r_t^f = \gamma\mu_C(X_t) + \beta - \frac{\gamma(\gamma+1)\sigma_C^2}{2}$.

Under power utility, the elasticity of intertemporal substitution is the inverse of risk-aversion. Under such a restriction, the riskfree rate based on the power utility is highly sensitive to expected consumption growth. In particular, a one-percent increase in the growth rate is followed by γ-percent increase in the riskfree rate - a nuisance at the heart of the riskfree rate puzzle. Indeed, the riskfree rate determines the intertemporal substitution effect between deterministic consumption streams, whereas risk-aversion determines the agent's preference over risky bets. Thus, risk aversion should not exert such a considerable influence on the sensitivity between consumption growth and risk-free rate. In the SDU case, there is a much more realistic one-to-one relationship between expected consumption growth and riskfree rate which is essentially due to the fact that we have picked a special case of unit elasticity of intertemporal substitution.

Moreover, for realistic values of μ_C and σ_C, the power utility riskfree rate is increasing under believable values of risk-aversion - which again misconstrues the nature of the riskfree rate. In other words, higher risk-aversion is required to match the equity-premia, it also correspondingly increases the riskfree rate. In our case, as in Duffie and Epstein (1992), the riskfree rate is strictly decreasing in risk-aversion. Thus, if indeed higher risk-aversion is needed to match the equity premia, it does not pose any challenge to match low riskfree rate. It should also be noted that the riskfree rate does vary through time with X_t.

We next establish the equilibrium dividend price ratio and the return dynamics.

Proposition 3. *The equilibrium price-dividend ratio $\frac{P_t}{D_t}$, denoted by $G(X_t)$, is*

$$G(X_t) = \int_0^\infty \exp(P_1(\tau)X_t + P_2(\tau))d\tau \tag{16}$$

where P_1 and P_2 are the solutions of a system of ODEs given in the appendix.
The instantaneous expected excess return is $\mu_t^R = (\lambda\gamma\sigma_D^2 - u_1\sigma_D\sigma_x\rho) + \frac{G_X}{G}(\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2)$.

Furthermore, the excess return dynamics is given by

$$
\begin{aligned}
dR_t &= \mu_t^R dt + \sigma_D dW_1 + \frac{G_X}{G}\sigma_x dW_2 && (17)\\
d\mu_t^R &= (\cdot)dt + \left(\frac{G_X}{G}\right)_X (\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2)\sigma_x dW_2. && (18)
\end{aligned}
$$

It immediately follows that the instantaneous covariance between realized and expected return is given by

$$
E_t\left[\left(dR_t - \mu_t^R dt\right)\left(d\mu_t^R - (\cdot)dt\right)\right] = \left(\frac{G_X}{G}\right)_X \left(\sigma_D\sigma_x\rho + \frac{G_X}{G}\sigma_x^2\right)(\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2). \quad (19)
$$

The leading term of the covariance can be written as

$$
\frac{GG_{XX} - (G_X)^2}{G^2} = \frac{1}{G^2}\left[\int_0^\infty \exp(\cdot)d\tau \int_0^\infty \exp(\cdot)P_1^2(\tau)d\tau - \left(\int_0^\infty \exp(\cdot)P_1(\tau)d\tau\right)^2\right] \quad (20)
$$

which is positive based on the Cauchy-Schwartz inequality applied to functions $P_1(\tau)\sqrt{\exp(\cdot)}$ and $\sqrt{\exp(\cdot)}$, both of which are integrable in the domain. As we show below, the term $\frac{G_X}{G}$ is always positive for $\lambda < 1$. The autocorrelation is positive unless $\rho < 0$ *and* $|\sigma_D\rho| > \frac{G_X}{G}\sigma_x$.

We can now go further and formulate the SDEs of realized cumulative excess returns for an investment horizon of l periods. In particular,

$$
R_{t,t+l} = R_t + \int_t^{t+l}\mu_s^R ds + \int_t^{t+l}\sigma_D dW_1 + \int_t^{t+l}\frac{G_X}{G}\sigma_x dW_2 \quad (21)
$$

To examine stock return momentum, we analyze the correlation between past and future returns. Past return is based on the time interval $0 \to t$, whereas future return is based on

$t \to l$.

3.1 Interpretation and implications

Thus far, our framework has focused on the aggregate economy. From the firm level perspective, dividends represent the overall cash-flows of a firm whereas consumption stands for the cash-flows from operating activities. Indeed, before we further develop the firm-level discussion, it is useful to point out the macro-economic underpinning of our theory.

3.1.1 Cash-Flow Growth

Dividend growth and consumption growth given in (4) and (7), respectively, follow different processes due to leverage. In response to a good shock in aggregate dividends, consumption grows at a slower pace. That is well supported by the US data. Average aggregate real dividend growth is about 3.2% whereas average real consumption growth is 1.8%. Moreover, the volatility of aggregate real dividend growth is 13.4% whereas the volatility of real consumption growth is 1.4%. Our model implies that due to leverage both the expected value and volatility of consumption growth are smaller than the corresponding quantities of aggregate dividends. Note also that in a levered economy while expected dividend growth mean-reverts at the rate of κ, consumption mean-reverts slower at the rate of $\lambda\kappa$. It is this slow and persistent growth rate of consumption, which in the context of our work is the growth rate of cash-flows from operations, that exacerbates long-run risk in the presence of high information uncertainty.

3.1.2 Price-Dividend ratio

The business cycle effect on the P/D ratio is instantly observable as

$$G_X = \frac{1-\lambda}{\kappa} \int_0^{\infty} \exp(P_1(\tau)X_t + P_2(\tau)) \left(1 - e^{-\kappa\tau}\right) d\tau. \tag{22}$$

Thus the P/D ratio is increasing in the growth rate X_t as long as $\lambda < 1$. The effect is most pronounced for low λ (high leverage), it deteriorates as λ grows (lower leverage), and ultimately vanishes as λ approaches one (no leverage). It should be noted that in the power utility case G_X is positive only if $\lambda\gamma < 1$. Essentially, this restriction puts an undue burden on γ to be less than $\frac{1}{\lambda}$. Here, the condition that the P/D ratio increases in the business cycle variable does not depend on risk-aversion. This appealing feature is due to the use of the stochastic differential utility. Notice also that the $\lambda < 1$ case points to a wealth effect in the economy in the presence of increasing economic growth rate. That is, from a positive shock in the growth rate of consumption, the agent feels richer thereby increasing savings by demanding more assets. This increases price relative to dividend.

The equilibrium D/P ratio is shown in Figure 1 for plausible values of the parameters underlying the stock return dynamics. The effect of risk-aversion is straightforward. Higher risk-aversion increases expected return thus reducing the current stock price or increasing the D/P ratio. The effect of leverage on the D/P ratio, however, is not straightforward.

Figure 1 shows that leverage and the D/P ratio are nonlinearly related. The D/P ratio first increases as λ grows and then decreases as λ approaches one. As λ starts to decrease from 1 i.e. as leverage increases, equity (the claim to risky consumption) becomes more risky which reduces price. In the no leverage ($\lambda = 1$) case, price is high because there is no leverage. It is intuitive to think that the reverse would take place at high leverage level (small λ). However, when leverage is high, consumption growth slows down drastically so as to nullify the long-run risk attributable to fluctuations in dividend growth. Thus, the risky asset ultimately becomes a claim to a constant level of consumption stream that has no risk associated with it. This increases the price and decreases the dividend yield.

3.1.3 Expected return and leverage

The expected excess stock return is given by

$$\mu_t^R = (\lambda\gamma\sigma_D^2 - u_1\sigma_D\sigma_x\rho) + G_X(\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2)\frac{D}{P}. \tag{23}$$

Thus, G_X – the response of the P/D ratio to the economic growth rate – guides the expected return dynamics. Notice that for stocks with high λ (low leverage) the ability of the dividend yield to forecast future returns diminishes. In particular, for $\lambda = 1$, $G_X = 0$ suggesting that all time series effects on expected stock return vanish completely. Likewise, at $\lambda = 0$, $\mu_t^R = 0$ which implies that in a fully levered economy, there is no consumption risk and the only security available is a bond with a constant stream of payment. As such, the risk-premia vanishes and the bond yields a rate of return equal to the discount rate β. The time-series properties of expected return which is absolutely essential for momentum, is present for intermediate values of $0 < \lambda < 1$.

Interestingly, observe from Figure 2 that even in the absence of time series effects on expected stock return, our model can still deliver the relatively high equity premium observed in the US over the past century. In particular, the second term in the first parenthesis in (23), $u_1\sigma_D\sigma_x\rho = \frac{(1-\gamma)\lambda\sigma_D\sigma_x\rho}{\kappa+\beta}$, which is a contribution of the stochastic differential utility, provides the explanation. Small values of κ and β could greatly magnify this expected return component, thus producing expected return values that match the high equity premium at a relatively low risk-aversion. This is the contribution of long-run risk - a highly persistent growth rate with small κ delivers a high enough equity-premium to match the data.

Figure 2 also displays the non-linear dependence of expected return on leverage. Clearly, at $\lambda = 0$, $\mu_t^R = 0$ and at $\lambda = 1$, excess return is constant. The time-series dependence and its interaction with leverage is provided at intermediate levels of λ. As leverage starts to increase (λ starts to decrease), equity becomes riskier and equity premia increases. However, at extreme levels of leverage aggregate consumption growth vanishes and so does the agent's long-run risk component. In a fully levered economy, consumption becomes a constant stream of payment which reduces all risks in the economy and the risk premia on equity is zero.

4 Understanding momentum interactions

As noted earlier, for $\lambda = 1$ or $\lambda = 0$ the momentum effect is nonexistent. In the analysis below, we focus on the more realistic $0 < \lambda < 1$ domain where the full dynamics of the dividend growth can be interacted with leverage. Our goal is to compute the correlation between observed return and expected return as well as the overall momentum profitability for 42 distinct specifications of parameters underlying the return dynamics with each specification standing for one particular class of economy. For each economy we simulate 5,000 paths for past return, expected return, and future return. Each simulation path pertains to a single firm. Indeed, our model ultimately delivers prominent predictions on the payoffs to momentum strategies implemented on winner and loser firms for each distinct economy.

The parameter settings are described in Table 1. In the simulation exercises the seven settings exhibited in Table 1 are interacted with six leverage levels - thus overall we consider 42 distinct economies. For all the seven [A-G] settings, we assume that the discount rate for our infinitely lived agent is small ($\beta = 0.01$) and the long run mean growth rate ($\bar{X}$) at 5%. Expected dividend growth is highly autoregressive for cases A-F with $\kappa = 0.05$ and in case G it is far less persistent with $\kappa = 0.20$. The claim going forward is that high variance in expected dividend growth rate and high persistence generates the momentum effect that is more pronounced for highly levered firms. High variance could emerge due to both high volatility in expected growth rate innovation (high σ_x) and highly persistent expected dividend growth (low κ).

In Table 1, settings A and B display high variance of dividend growth (high σ_D and σ_x), C and D display low, and E exhibits medium. The distinction between A and B (C and D) is higher risk-aversion parameter for B (D). Both E and F exhibit the same volatility of dividend growth (σ_D) but F displays lower expected growth rate volatility (σ_x). Setting G takes higher κ corresponding to less persistent expected dividend growth rate, and at the same time holds σ_D and σ_x at a moderate level.

The next section describes the simulations made to assess the impact of leverage, volatility of dividend growth, volatility of expected dividend growth, risk aversion, and persistence of expected dividend growth on momentum effects.

4.1 Correlation between realized and future return

We first examine the correlation between observed and future returns. We follow a universe of stocks with returns evolving from time $0 \rightarrow t$. We then sort the stocks based on observed returns into ten portfolios - from lowest to highest. Then we track each of the portfolios for some period $t \rightarrow l$. The momentum phenomenon is consistent with positive autocorrelation.

The simulated autocorrelations are displayed in Figure 3. The top panel shows the correlation of two sets of economies - one with high leverage ($\lambda = .15$) and the other with low leverage ($\lambda = .9$). The other parameters for these economies pertain to case A of Table 1. Specifically, both economies have high information uncertainty and are only distinguished by the leverage level. Our hypothesis is that stocks with high leverage should generate high momentum payoffs. Indeed our simulations indicate that the portfolios corresponding to high levered economy ($\lambda = .15$) display a higher correlation of future returns with past returns, than the low levered economy ($\lambda = .9$). Interestingly, notice that the highest correlations of future and past returns are in the extreme loser and winner portfolios.

The bottom panel exhibits two cases with same leverage but different amount of information uncertainty. Leverage is high for both cases ($\lambda = .15$) whereas information uncertainty is higher for case A than for case C. It shows that firms with high information uncertainty do reflect higher autocorrelation with past return than firms with low information uncertainty. Again, we observe the highest autocorrelations in the extreme loser and winner portfolios.

The collective evidence emerging from Figure 3 suggests that the momentum effect could simply emerge due to strong autocorrelation in the extreme percentiles. In unreported tests we show that the other momentum sources (e.g., the variance of expected returns) play a relatively smaller role in generating momentum payoffs. The important role of autocorrelation has an intuitive interpretation through the growth rate dynamics. In particular, if a stock has experienced successive good growth rates, then chances are it is very far away from its equilibrium growth rate $\bar{X}$. As such, it mean reverts slowly (as is the case for A and C) back to its equilibrium level, which means it will continue to earn high return until it fully mean reverts. Similarly, if a stock has been realizing successive poor growth rates then its return will be low and due to slow mean reversion it will continue to earn a lower return. Stocks in the non extreme portfolios are closer to their equilibrium and hence mean-revert quickly which reduces their autocorrelation.

4.2 Instantaneous expected return spreads

Table 2 exhibits expected excess return over a one-year period in which past cumulative returns have been classified into ten deciles, with column 1 (10) pertaining to the lowest (highest) observed returns. There are several insights emerging about the role that leverage, dividend growth volatility, expected dividend growth volatility, risk aversion, and persistence play in generating momentum effects. Here, momentum profitability is defined as the expected return spread between the highest and lowest observed return portfolios.

Momentum and leverage. Momentum profitability monotonically increases with leverage regardless of the case considered, with settings A, B, and E displaying large and economically meaningful expected return spreads. Focusing on A, the expected return spread is 11.49% [26.51%-15.02%] per year for $\lambda = 0.15$ while it is only 0.27% for $\lambda = 0.9$. Moving to B, the expected return spread is 13.16% for $\lambda = 0.15$ and is 1.71% for $\lambda = 0.9$. The corresponding figures for E are 9.87% and 0.26%. On the other hand, the overall spreads are small for the C, D, and G settings. This indicates that while leverage is crucial there are some other important determinants of momentum effects.

Before we move on, it should be noted that the expected returns reported in Table 2 might seem quite large, especially the ones indicating strong momentum effects. However, notice that the time discount parameter β is small (0.01) at this stage. We reexamine some of the sub-cases using higher time-discount parameter. The evidence is reported in Table 3. Indeed, increasing β brings down expected return at every decile to more realistic levels. The interesting evidence, however, is that the expected return spread between the highest and lowest deciles is preserved. To illustrate, for case A and $\lambda = 0.25$ the spread is 7.56%, for B and $\lambda = 0.15$ is 9.96%, for B and $\lambda = 0.9$ is 0.57%, and for E and $\lambda = 0.2$ is 5.87%.

Momentum and information uncertainty. Settings A, B, and E display the highest expected return spreads across the ten deciles, as noted earlier. For the other settings the spreads are much lower and are often even negligible. To illustrate, the highest spread for C is 2.95% [15.01%-12.06%], 4.86% for D, 1.05% for F, and only 0.29% for G. Cases A, B, and E are all characterized by high (0.07) to moderate (0.05) expected dividend growth volatility, which, from the firm's perspective, amounts to high volatility cash flow growth.

To this point we are able to rationalize previously documented momentum interactions. In particular Zhang (2006) finds that momentum concentrates in high information uncertainty stocks and points to behavioral interpretations. Avramov, Chordia, Jostova, and Philipov (2007) document that momentum prevails only among high credit risk stocks. Whereas such momentum-credit risk interaction could point to rational interpretations, this is purely an empirical finding thus far that has not been formalized in an equilibrium model. The collective evidence here shows that equilibrium momentum effects should concentrate in the interaction of risky cash flows and highly levered firms. Interestingly, neither leverage alone nor cash flow volatility alone are sufficient to generate momentum effects. Furthermore, this also shows the effect of long-run risk in understanding the momentum effect. As growth rates evolve over time, σ_x can perturb the growth rate path thereby subjecting the growth path to more uncertainty. For agents who prefer who early resolution of such uncertainties, a higher information uncertainty induces higher long-run risk as can be seen from (14).

Focusing on information uncertainty measures, a valid point to make is that the volatility of expected dividend growth (σ_x) is the primary force of momentum effects, whereas the volatility of the unexpected dividend growth (σ_D) plays a marginal role. Note in particular that cases E and F are virtually identical with the only exception being $\sigma_x = 0.05$ in E versus $\sigma_x = 0.02$ in F. Nevertheless, the expected return spreads in E are considerably higher for all leverage levels.

Momentum and risk aversion. Setting G features the highest risk aversion but nevertheless yields the lowest expected return spreads between the ten portfolios, ranging between 0.02% and 0.29%. The immediate takeout is that the risk aversion measure is not a key parameter in generating momentum effects. Let us also compare A versus B as well as C versus D. For cases B and D, the higher risk-aversion simply increases expected return at every level of leverage and still preserves expected return spread across the deciles. Indeed, the momentum profitability in B (D) is slightly higher than than of A (C), suggesting that risk aversion has some effect, albeit relatively small, in explaining the return spread.

Momentum and expected growth rate persistence. Case G is different from the previous settings in that it features the lowest autocorrelation of expected growth rate, which reduces the total variance of expected growth rate. But, focussing on (14), it is clear that higher κ will reduce long-run risk premia because now the growth rate will mean revert quickly to its equilibrium thus reducing long-run risk. Thus, the effect of persistence is also measuring the contribution of long-run risk in understanding momentum. In case G, the shocks to the system are exactly the same as in case E ($\sigma_D = 0.06$ and $\sigma_x = 0.05$). Therefore, with higher risk-aversion in G (10 versus 5) we can only expect the momentum effect of case E to be exacerbated, just like case B compared to case A. However, with higher κ the expected return spread across the high and low performing portfolios is minuscule, even at high levels of leverage. Therefore, high κ, which decreases persistence, exhibits no expected return differential that characterizes the momentum effect in the data. Persistence is indeed crucial in generating momentum effects and it overwhelms risk aversion.

4.3 Holding period return spreads

What makes the momentum effect a conundrum is the holding period profit. The strategy of buying winners and selling short losers produces 8-12% ex post payoffs according to Jegadeesh and Titman (1993). We next simulate holding period returns based on one year formation period and conventional holding periods of 3-12 months. Table 4 reports momentum profitability which is the return spread between the top and bottom past return deciles.

Consistent with the evidence reported thus far it follows that the high volatility case A, which produces high ex-ante expected excess returns, also generates high ex post holding period returns. Focusing on the one year holding period, momentum profitability is 10.35% for $\lambda = 0.15$ and is only 0.37% for $\lambda = 0.9$. The low volatility case D fails to generate high momentum profitability. For the one year holding period the momentum payoff is 3.2% for high leverage and 0.13% at low leverage. Furthermore, the moderate volatility case E generates moderate levels of investment returns ranging between 0.26% and 8.98% for the one year holding period. The evidence in Case G with the lowest autocorrelation confirms the earlier observation that holding period returns based on low autocorrelation and high volatility are small and similar in magnitude to the high autocorrelation low volatility case D (recall $\sigma_x = 0.05$ in case G while $\sigma_x = 0.03$ in case D). In summary, we confirm that momentum effects concentrate in firms with high leverage as well as highly volatile and persistent expected cash flows growth.

4.4 Interpretation

Our simulation experiments based on the framework developed here apply to single firms. At the single firm level, the macro quantities dividend and consumption streams reflect, respectively, the entire cash flows and cash flows from operating activities. In our setting, an economic agent prefers early resolution of uncertainty about future operating cash-flow stream. A positive shock to overall cash-flows corresponds to a much smaller shock to operating cash-flow due to the presence of leverage. Moreover, operating cash-flow growth mean reverts slower than the overall cash-flow growth. In particular, if overall cash-flow mean reverts at speed κ, operating cash-flow mean reverts at speed $\lambda\kappa$. This makes growth rates in operating cash-flows highly persistent. Furthermore, high information uncertainty (high σ_x) can cause bigger perturbations in the growth rate path thus subjecting cash-flow growth to further uncertainty. A combination of information uncertainty and persistence induces long-run risk for agents with Duffie-Epstein preferences and hence the relatively high risk premium. The evidence in this paper shows that the same factors that drive risk-premia in Duffie-Epstein preferences also give rise to the momentum effect. We simulate stock return dynamics under different interactions between information uncertainty (σ_x), speed of mean reversion (κ), and leverage (λ) and conclusively show the profitability of momentum strategies among high levered high information uncertainty stocks.

5 Conclusion

Previous work shows that momentum effects in stock returns are robust. Indeed, the apparent stock return momentum anomaly has invoked a plethora of behavioral and rational explanations. Previous work also uncovers momentum interactions. In particular, momentum concentrates in stocks with high return volatility, high cash flow volatility, small market capitalization, high analysts' earnings forecast dispersion, as well as high credit risk.

This paper analyzes such momentum concentrations from a consumption based equilibrium perspective. We ultimately show that the concentration of momentum in high information uncertainty as well as high credit risk stocks is perfectly consistent with rational asset pricing. Our economic setup is fairly general from the perspectives of both preferences and dynamics. The stochastic differential utility of Duffie and Epstein (1992) employed here breaks the tight association between the elasticity of inter-temporal substitution and the risk aversion measure. Moreover, our cash-flow dynamics, based on the novel formulation of Abel (1999), allows one to examine leverage in an innovative way by introducing a non-linear relationship between overall and operating cash-flow streams.

We use simulations to find out that our paradigm indeed predicts strong equilibrium momentum effects for the interaction between high leverage and risky cash flow firms. Momentum profitability deteriorates and ultimately disappears as either leverage or cash flow risk diminishes. Specifically, it is the long-run component of the cash-flow risk - from near-unit root and volatile growth rates, that is delivering the momentum phenomenon when interacted with leverage. More specifically, the correlation between past returns and future expected returns is positive and increases with leverage and information uncertainty.

Moreover, the expected return spread between the highest and lowest past year cumulative return portfolios increases with leverage. For example, when operating cash flows are highly volatile, the expected return spread is 11.49% for high leverage and only 0.27% for low leverage firms. On the other hand, the overall spreads are small when either the volatility of expected cash flows growth is small, or the expected growth in cash flows is not highly persistent, i.e., when long-run risk is low. This indicates that while leverage is crucial, risk and persistence in cash flows growth are both important determinants of momentum effects. The collective evidence thus shows that equilibrium momentum profitability concentrates in the interaction between risky cash flows and high levered firms which is perfectly consistent with data.

There are several suggestions for future work. First, currently leverage is exogenous. We ask: given a particular leverage level - what is the overall momentum effect? It would be quite appealing to endogenize leverage and make it a firm-decision variable. Next, our focus here has been on firm level interactions. It has also been shown that momentum displays strong business cycle effects and is also related to the state of the market. Our setting can readily be extended to analyze possible rational business cycle effects as well as market states in momentum profitability. Finally, from an empirical perspective, one could analyze the joint effect of leverage, expected dividend growth risk, and dividend growth persistence on the cross section of average returns in general and market anomalies in particular.

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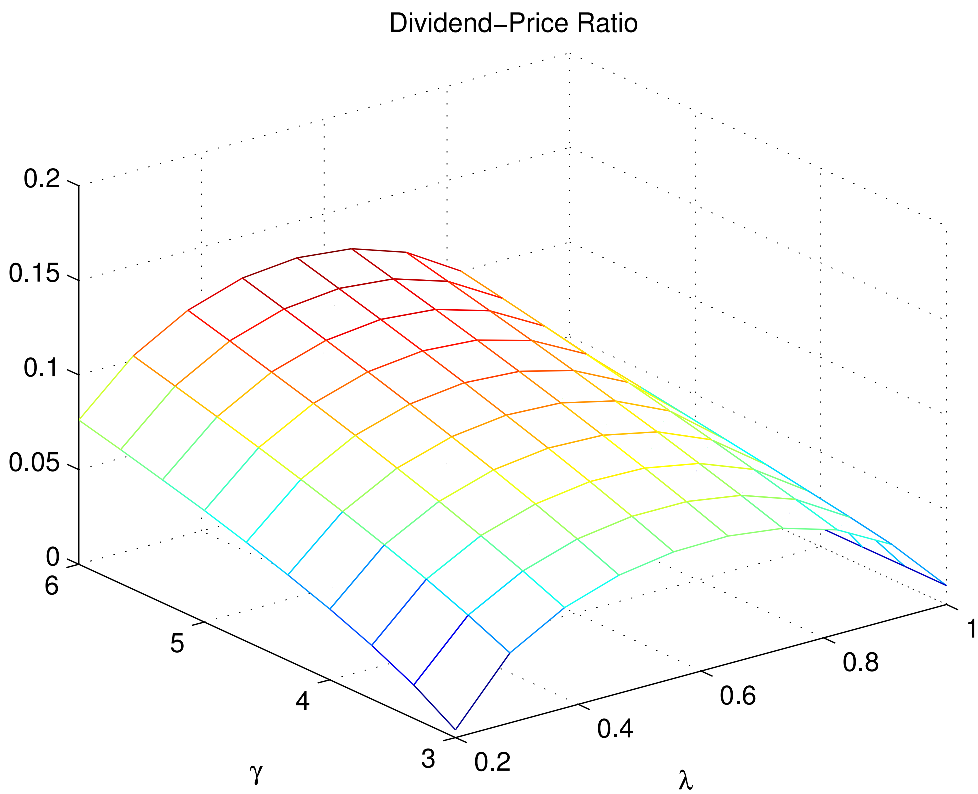

Figure 1: *Dividend-Price ratios implied by the model for* $\beta = 0.01$, $\kappa = 0.1$, $\bar{X} = 0.05$, $\sigma_D = .05$, $\sigma_x = .035$, $\rho = .35$. *The state is set to* $\bar{X}$, *i.e.* $X_0 = \bar{X}$
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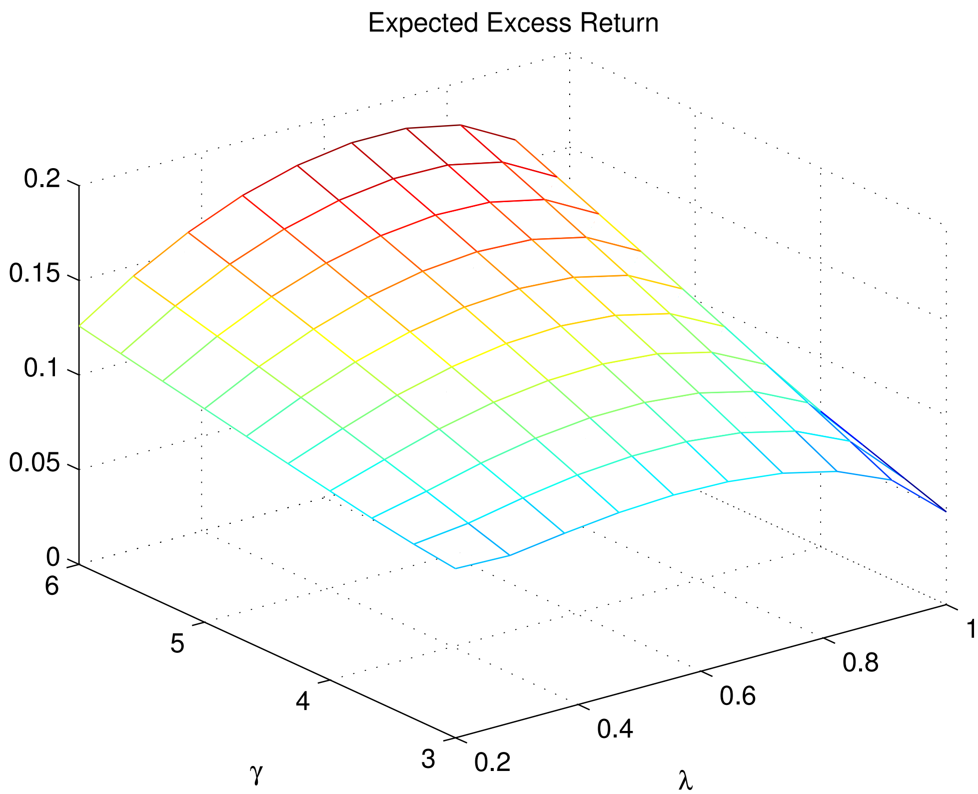

Figure 2: *Expected excess return implied by the model for* $\beta = 0.01$, $\kappa = 0.1$, $\bar{X} = 0.05$, $\sigma_D = .05$, $\sigma_x = .035$, $\rho = .35$. *The state is set to* $\bar{X}$, *i.e.*$X_0 = \bar{X}$
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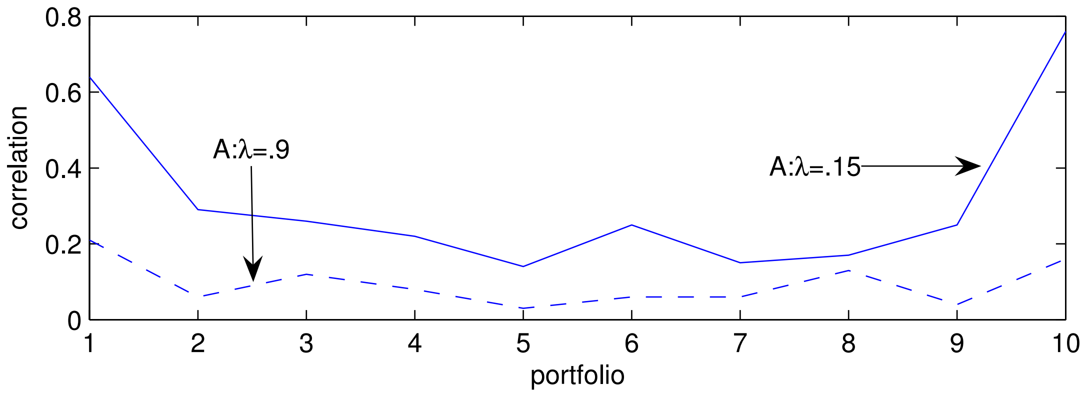

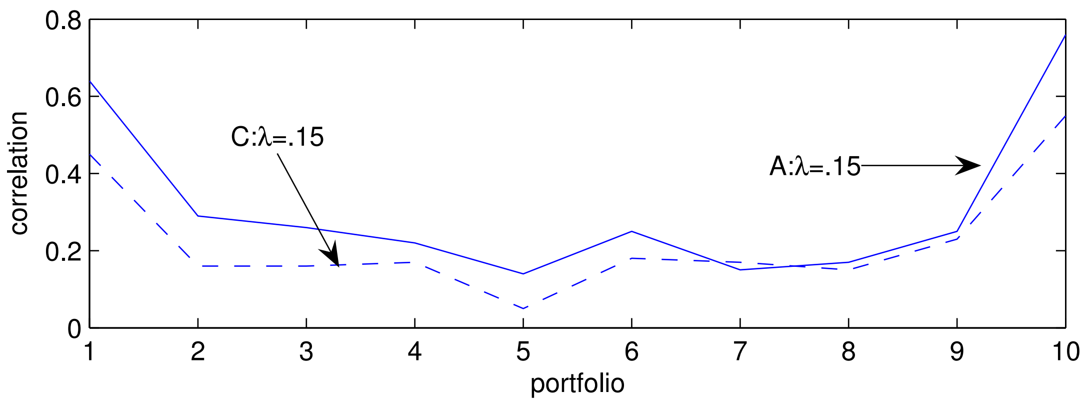

Figure 3: *Both figures display the correlation between past return and future return. A formation period of one year is chosen. Stocks are sorted into ten portfolios based on cumulative one year returns. Subsequently each stock is tracked for a period of 6 months. Then the correlation of future cumulative returns (6 months subsequent to formation) with the cumulative return at the beginning of the period is computed at the portfolio level. The x-axis marks the portfolios - 1 being the losing and 10 the winning portfolio. The y-axis plots the correlation of future cumulative returns with past cumulative return for each portfolio.*

Table 1: This table lists the set of parameters we are considering. A and B are cases with high volatility of dividend growth and expected dividend growth. Case B is the same as A with higher risk-aversion. Cases C and D are with low volatilities with D having higher risk-aversion than C. E and F have moderate volatilities and higher correlation where F has lower volatility of expected dividend growth. G is similar to E, except with lower autocorrelation.

	β	γ	κ	$\bar{X}$	σ_D	σ_x	ρ
A	0.01	5.0	0.05	0.05	0.08	0.07	0.2
B	0.01	8.0	0.05	0.05	0.08	0.07	0.2
C	0.01	5.0	0.05	0.05	0.04	0.03	0.1
D	0.01	8.0	0.05	0.05	0.04	0.03	0.1
E	0.01	5.0	0.05	0.05	0.06	0.05	0.2
F	0.01	5.0	0.05	0.05	0.06	0.02	0.2
G	0.01	10.0	0.20	0.05	0.06	0.05	0.2

Table 2: This table shows the average instantaneous expected excess return (annualized percentage) under the different parameter settings subsequent to one year in which the cumulative return has fallen into 1 of the 10 deciles labeled 1-10. Column 1 is the expected excess return for the lowest decile and Column 10 is for the highest. The system is simulated forward for 5000 different paths for one year, and each path here depicts one security over the year. For each path, we compute the observed return in (21) and the corresponding expected return. At the end of one year, we sort the 5000 paths based on observed return and assign them into the ten portfolios. The average of the expected return for each portfolio (equally-weighted) is then reported. In all cases, $X_0 = \bar{X}$.

	λ	1	2	3	4	5	6	7	8	9	10
A	0.15	15.02	17.13	18.25	18.83	19.48	20.04	20.67	21.39	22.99	26.51
	0.25	18.75	20.32	21.08	21.50	21.91	22.26	22.68	23.12	24.09	26.13
	0.30	20.37	21.79	22.43	22.85	23.22	23.52	23.86	24.28	25.11	26.88
	0.40	22.94	24.11	24.65	24.98	25.26	25.50	25.76	26.13	26.73	28.12
	0.70	25.77	26.21	26.43	26.55	26.65	26.74	26.83	26.97	27.22	27.71
	0.90	21.61	21.68	21.74	21.76	21.78	21.79	21.81	21.84	21.90	21.98
B	0.15	20.13	21.91	22.81	23.28	23.76	24.15	24.66	25.18	26.32	33.29
	0.20	23.45	24.98	25.73	26.21	26.65	26.99	27.37	27.88	28.83	30.89
	0.30	28.56	29.92	30.56	30.97	31.25	31.54	31.88	32.26	33.00	34.64
	0.40	32.43	33.55	34.05	34.43	34.61	34.81	35.14	35.41	35.99	37.27
	0.70	34.25	34.78	35.12	35.77	36.12	36.88	37.22	37.45	37.89	38.14
	0.90	37.30	37.66	37.89	37.98	38.08	38.15	38.24	38.36	38.62	39.01
C	0.16	12.06	13.07	13.53	13.71	13.87	14.01	14.16	14.30	14.58	15.01
	0.25	7.88	8.51	8.85	9.00	9.14	9.30	9.44	9.61	9.95	10.62
	0.30	8.17	8.72	9.01	9.14	9.27	9.40	9.53	9.69	9.97	10.57
	0.40	8.71	9.15	9.38	9.48	9.58	9.69	9.79	9.90	10.14	10.62
	0.70	8.57	8.76	8.84	8.88	8.93	8.96	8.99	9.04	9.13	9.30
	0.90	5.96	5.98	6.00	6.01	6.02	6.02	6.02	6.03	6.05	6.07
D	0.15	10.56	11.94	12.63	12.93	13.20	13.46	13.72	13.99	14.52	15.42
	0.20	9.69	10.32	10.67	10.81	10.97	11.11	11.27	11.43	11.79	12.52
	0.30	10.83	11.38	11.66	11.79	11.92	12.06	12.19	12.33	12.63	13.24
	0.40	11.83	12.31	12.52	12.64	12.76	12.85	12.96	13.07	13.32	13.83
	0.70	12.05	12.47	12.33	12.39	12.42	12.46	12.50	12.55	12.64	12.82
	0.90	8.77	8.79	8.81	8.82	8.82	8.83	8.83	8.84	8.86	8.88

Table 2 (Continued):

	λ	1	2	3	4	5	6	7	8	9	10
E	0.15	11.54	13.39	14.47	14.98	15.50	16.03	16.57	17.21	18.57	21.41
	0.20	13.15	14.28	14.84	15.13	15.43	15.68	15.97	16.27	16.96	18.38
	0.30	14.11	15.13	15.64	15.89	16.15	16.37	16.62	16.90	17.48	18.72
	0.40	15.68	16.51	16.89	17.11	17.33	17.49	17.69	17.91	18.37	19.30
	0.70	16.94	17.27	17.42	17.53	17.58	17.65	17.74	17.82	17.99	18.37
	0.90	13.49	13.55	13.59	13.60	13.61	13.62	13.64	13.66	13.69	13.75
F	0.20	7.16	7.47	7.62	7.68	7.74	7.79	7.85	7.90	8.01	8.21
	0.25	6.30	6.64	6.79	6.86	6.94	7.00	7.07	7.14	7.28	7.57
	0.30	6.34	6.63	6.78	6.84	6.91	6.96	7.03	7.09	7.23	7.50
	0.40	6.72	6.95	7.05	7.11	7.17	7.21	7.25	7.31	7.42	7.63
	0.70	7.09	7.17	7.20	7.22	7.24	7.25	7.27	7.28	7.32	7.40
	0.90	5.74	5.75	5.75	5.76	5.76	5.77	5.77	5.78	5.78	5.79
G	0.15	7.25	7.34	7.38	7.39	7.41	7.42	7.44	7.45	7.48	7.54
	0.20	8.33	8.48	8.54	8.57	8.60	8.63	8.66	8.68	8.74	8.85
	0.30	10.02	10.21	10.30	10.34	10.34	10.41	10.45	10.49	10.57	10.71
	0.40	11.22	11.34	11.48	11.52	11.56	11.59	11.62	11.69	11.74	11.90
	0.70	12.07	12.14	12.17	12.19	12.20	12.21	12.22	12.24	12.27	12.33
	0.90	9.54	9.54	9.55	9.55	9.55	9.55	9.55	9.55	9.55	9.56

Table 3: This table repeats the exercise in Table 3 with different time preference parameter β. Certain sub-cases are taken and they are replicated with a higher β. The first line for each sub-case is copied from the corresponding line on Table 3, and the following line is the repeat of the same simulation with higher beta, such that the transversality condition is still satisfied. This table shows that increasing β lowers expected return at every decile but still maintains a healthy difference between the highest and lowest deciles.

	β	1	2	3	4	5	6	7	8	9	10
$A(\lambda = 0.25)$	0.01	18.75	20.32	21.08	21.50	21.91	22.26	22.68	23.12	24.09	26.13
	0.08	10.88	12.23	12.96	13.34	13.76	14.11	14.53	14.99	16.05	18.42
$B(\lambda = 0.15)$	0.01	20.13	21.91	22.81	23.28	23.76	24.15	24.66	25.18	26.32	33.29
	0.08	11.80	13.44	14.33	14.82	15.37	15.81	16.36	16.98	18.40	21.76
$B(\lambda = 0.90)$	0.01	37.30	37.66	37.89	37.98	38.08	38.15	38.24	38.36	38.62	39.01
	0.08	18.86	18.91	18.96	18.97	18.99	18.99	19.01	19.02	19.07	19.13
$E(\lambda = 0.20)$	0.01	13.15	14.28	14.84	15.13	15.43	15.68	15.97	16.27	16.96	18.38
	0.055	7.64	8.69	9.29	9.59	9.89	10.20	10.54	10.89	11.71	13.51

Table 4: This table shows the holding period return differential based on investment horizons of 3-12 months. The formation period is one year. We use 5000 different paths where each path denotes one stock. At the end we sort the observed return into ten equally weighted portfolios. The average difference between the top and bottom decile is reported in this table. In all cases, $X_0 = \bar{X}$.

	λ	3	6	9	12
A	0.15	2.78	5.52	7.87	10.35
	0.25	1.78	3.49	5.05	6.65
	0.30	1.58	3.11	4.46	5.87
	0.40	1.24	2.46	3.54	4.66
	0.70	0.46	0.89	1.29	1.82
	0.90	0.09	0.18	0.26	0.37
D	0.15	3.21	3.25	3.28	3.20
	0.20	2.63	2.65	2.71	2.63
	0.30	2.25	2.27	2.32	2.23
	0.40	1.85	1.85	1.91	1.85
	0.70	0.72	0.67	0.73	0.72
	0.90	0.11	0.12	0.14	0.13
E	0.15	2.38	4.75	6.75	8.98
	0.20	1.48	2.96	4.23	5.62
	0.30	1.10	2.18	3.13	4.14
	0.40	0.87	1.72	2.48	3.27
	0.70	0.33	0.65	0.95	1.30
	0.90	0.06	0.13	0.18	0.26
G	0.15	1.41	2.78	4.00	3.96
	0.20	1.26	2.47	3.57	3.49
	0.30	1.04	2.03	2.97	2.83
	0.40	0.83	1.63	2.36	2.27
	0.70	0.31	0.61	0.86	0.87
	0.90	0.06	0.12	0.19	0.18

6 Appendix

Preferences

In continuous time, the recursive utility function takes the form of stochastic differential utility. The stochastic differential utility $U : \mathcal{L}^2 \to \mathcal{R}$ is a mapping from a square integrable space to the real line and is defined by two primitive functions: (f, A) where $f : \mathcal{R}^+ \times \mathcal{R} \to \mathcal{R}$ and $A : \mathcal{R} \to \mathcal{R}$. For any consumption process $C \in \mathcal{L}^2$, the utility process J is the unique SDE

$$dJ_t = \left[-f(C_t, J_t) - \frac{1}{2} A(J_t) \sigma_v \sigma_v' \right] dt + \sigma_v dB_t$$

with boundary condition $J_T = 0$. The different components are - J_t, a continuation utility for the agent given consumption C_t, $f(C_t, J_t)$ is an ordinal map of date $t's$ consumption and continuation utility, and $A(J_t)$ is a measure of local risk-aversion. If given an initial consumption C_t and as long as the solution of the above SDE is well-defined, the utility at time t is defined as $U(C_t) = J_t$. Under certain conditions, the above SDE is well-defined and hence the utility exists. The function U is monotonic and risk-averse for $A \leq 0$. Given an f and two functions A^* and A, let U^* and U be the two utilities corresponding to the aggregators (f, A^*) and (f, A). If $A^* \leq A$, then U^* is more risk-averse than U, i.e. any consumption stream rejected by a deterministic consumption path by one will also be rejected by another. A convenient normalization that produces an ordinally equivalent utility function is achieved by setting $A = 0$, which means the above SDE solves $E_t[dJ_t] + f(C, J) = 0$ for normalized aggregator $(f, 0)$. The normalization is useful because it produces a much simpler Bellman equation to be solved than if $A \neq 0$. Fortunately, there exists a transformation from $(\bar{f}, A)$ to $(f, 0)$ such that the utilities generated from both will be ordinally equivalent. Further discussion of the aggregators and the normalization that leads to an ordinally equivalent representation of the aggregators is given in Duffie and Epstein (1992).

Proof of Proposition 1: The Bellman equation in (10) can be written as

$$J_C C\lambda\left[X_t + \frac{1}{2}(\lambda-1)\sigma_D^2\right] + J_X\kappa(\bar{X}-X_t) +$$
$$\frac{1}{2}J_{CC}C^2\lambda^2\sigma_D^2 + \frac{1}{2}J_{XX}\sigma_X^2 + J_{XC}C\lambda\sigma_D\sigma_x\rho + f(C,J) = 0$$

The continuation utility J has a solution of the form

$$(1-\gamma)J = \exp(u_0 \ln C_t + u_1 X_t + u_2)$$

Substituting it in and collecting terms, reduces the above equation to a system of ODEs that can be solved recursively

$$\begin{aligned}
u_0 &= (1-\gamma) \\
u_1 &= \frac{(1-\gamma)\lambda}{\kappa+\beta} \\
u_2 &= \frac{(1-\gamma)\lambda}{\beta}\left[\frac{(\lambda-1-\lambda\gamma)\sigma_D^2}{2} + \frac{\kappa\bar{X}}{\kappa+\beta} + \frac{(1-\gamma)\lambda}{\kappa+\beta}\left[\frac{\sigma_x^2}{2(\kappa+\beta)} + \sigma_D\sigma_x\rho\right]\right]
\end{aligned}$$

Thus, the continuation utility function reduces to $J(C_t, X_t) = \frac{C_t^{1-\gamma}}{1-\gamma}\exp(u_1 X_t + u_2)$.

Proof of Proposition 2: The pricing kernel for stochastic differential utility can be written as

$$\frac{d\Lambda}{\Lambda} = \frac{df_C}{f_C} + f_J dt$$

Using the above utility function, define $g = f_C = \frac{\beta(1-\gamma)J}{C}$ and $f_J = -\beta(1 + u_1 X + u_2)$. Use Ito's Lemma on g and (5) and (7) one can rewrite the pricing kernel as

$$\begin{aligned}
\frac{d\Lambda}{\Lambda} &= -r_t^f dt - \lambda\gamma\sigma_D dW_1 + u_1\sigma_x dW_2 \\
r_t^f &= \lambda X_t + u_1\lambda\gamma\sigma_D\sigma_x\rho + \beta(u_2+1) - u_1\kappa\bar{X} - \frac{1}{2}\lambda\gamma\sigma_D^2(\lambda\gamma+1) - \frac{1}{2}\sigma_x^2 u_1^2
\end{aligned}$$

Proof of Proposition 3: The firm stock price is given by

$$\begin{aligned}
P_t &= \frac{1}{\Lambda_t} E_t \int_t^\infty \Lambda_s D_s ds \\
&= \frac{1}{\Lambda_t} \int_t^\infty E_t \Lambda_s D_s ds
\end{aligned}$$

Applying Feynman-Kac, we know $E_t\left[\Lambda_s D_s\right] = f(\Lambda_t D_t, X_t, s-t)$. Applying Ito's Lemma,

$$P_t = D_t G(X_t)$$

where $G(X_t) = \int_t^\infty \exp(P_1(s-t)X_t + P_2(s-t))ds$. Making a change of variable $\tau = s - t$, $G(X_t) = \int_0^\infty \exp(P_1(\tau)X_t + P_2(\tau))d\tau$. $P_1(\tau)$ and $P_2(\tau)$ satisfy a set of ODEs that can be solved recursively with initial conditions $P_1(0) = P_2(0) = 0$

$$P_1'(\tau) = -(\lambda - 1) - \kappa P_1(\tau)$$

$$P_2'(\tau) = u_1\kappa\bar{X} + \frac{1}{2}\sigma_D^2(\lambda\gamma - 1) - \beta(u_2+1) + P_1(\tau)\left[\kappa\bar{X} + u_1\sigma_x^2 - (\lambda\gamma-1)\sigma_D\sigma_x\rho\right]$$
$$+\frac{1}{2}\left[(\lambda\gamma-1)^2\sigma_D^2 + u_1^2\sigma_x^2 - 2u_1(\lambda\gamma-1)\sigma_D\sigma_x\rho + P_1^2(\tau)\sigma_x^2\right]$$

The solution of $P_1(\tau)$ is given by $P_1(\tau) = \frac{1-\lambda}{\kappa}\left(1 - e^{-\kappa\tau}\right)$ and then that can be used to solve for $P_2(\tau)$. Plugging in $P_1(\tau)$, $P_2(\tau)$ becomes

$$P_2(\tau) = a\tau + b(e^{-\kappa\tau} - 1) + c(1 - e^{-2\kappa\tau})$$

where

$$\begin{aligned}
a &= \left(u_1 + \frac{1-\lambda}{\kappa}\right)\left(\bar{X}\kappa - (\lambda\gamma-1)\sigma_D\sigma_x\rho + \frac{1}{2}\sigma_x^2\left(u_1 + \frac{1-\lambda}{\kappa}\right)\right) + \frac{1}{2}\sigma_D^2\lambda\gamma(\lambda\gamma-1) \\
&\quad -\beta(u_2+1) \\
b &= \frac{1-\lambda}{\kappa^2}\left[\frac{\sigma_x^2(1-\lambda)}{\kappa} + \kappa\bar{X} + u_1\sigma_x^2 - (\lambda\gamma-1)\sigma_D\sigma_x\rho\right] \\
c &= \frac{\sigma_x^2}{4}\frac{(1-\lambda)^2}{\kappa^3}
\end{aligned}$$

The transversality condition holds for $a < 0$, which holds for believable parameter values. Applying Ito's lemma to $P_t = D_t G(X_t)$, we derive the process for cumulative excess return $dR_t = \frac{D_t + dP}{P_t} - r_t^f$

$$\begin{aligned}
dR_t &= \mu_t^R dt + \sigma_D dW_1 + \frac{G_X}{G}\sigma_x dW_2 \\
d\mu_t^R &= (\cdot)dt + \left(\frac{G_X}{G}\right)_X (\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2)\sigma_x dW_2
\end{aligned}$$

where $\mu_t^R = (\lambda\gamma\sigma_D^2 - u_1\sigma_D\sigma_x\rho) + \frac{G_X}{G}(\lambda\gamma\sigma_D\sigma_x\rho - u_1\sigma_x^2)$. The latter is derived from the equilibrium argument that the expected excess return is given by $\mu_t^R = -Cov_t\left(\frac{d\Lambda_t}{\Lambda_t}, dP_t\right)$.