



## Predicting stock returns<sup>☆</sup>

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### Abstract

This paper studies whether incorporating business cycle predictors benefits a real time optimizing investor who must allocate funds across 3,123 NYSE-AMEX stocks and cash. Realized returns are positive when adjusted by the Fama-French and momentum factors as well as by the size, book-to-market, and past return characteristics. The investor optimally holds small-cap, growth, and momentum stocks and loads less (more) heavily on momentum (small-cap) stocks during recessions. Returns on individual stocks are predictable out-of-sample due to alpha variation, whereas the equity premium predictability, the major focus of previous work, is questionable.

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## 1. Introduction

Previous work identifies business cycle and firm-level variables that predict future stock returns.<sup>1</sup> These predictive variables, when incorporated in studies that deal with the time-series and cross-sectional properties of expected returns, provide fresh insights into investment management and asset pricing. For example, Kandel and Stambaugh (1996) show that current values of predictive variables can exert substantial influence on asset allocation. Moreover, Lettau and Ludvigson (2001) and Avramov and Chordia (2006), among others, demonstrate that asset pricing models with time varying market premium or risk are reasonably successful relative to their unconditional counterparts. Notwithstanding, stock return predictability continues to be a subject of research controversy. Skepticism exists due to concerns relating to data mining, statistical biases, and weak out-of-sample performance of predictive regressions,<sup>2</sup> and moreover, if firm-level predictability indeed exists, it is not clear whether it is driven by time varying alpha, beta, or the equity premium.

This paper develops and applies an optimization framework within which to investigate the economic value and determinants of predictability at the stock level. Specifically, we seek to understand whether considering business cycle predictors benefits a real time investor who must allocate funds across 3,123 NYSE-AMEX stocks and the risk-free asset over the 1972–2003 period. The proposed framework is quite general as it allows alphas, betas, and the equity premium to vary with the predictive variables, it explicitly incorporates estimation risk, and it permits investments based on different stock return histories. This last aspect is desirable in analysis of real time investments because stocks enter (IPO) and leave (merger, acquisition, bankruptcy) the sample periodically. Relative to Stambaugh (1997), who develops an approach for analyzing investments whose histories differ in length, we allow expected returns, variances, and correlations to vary with the business cycle variables.

We implement several trading strategies and examine their ex post out-of-sample performance. If investment strategies that condition on macro variables outperform their unconditional counterparts, as well as common benchmarks, then they exploit superior information about the cross-section of future returns and the degree of superior performance provides a measure of the economic value of predictability. Moreover, our methodology allows one to explore whether the superior performance comes from time varying alphas, betas, and/or the risk premium. For instance, if investments based on time varying alphas outperform their constant alpha counterparts, then time varying alpha is a determinant of predictability.

Our investment universe consists of a risk-free asset, which we proxy by the short-term Treasury, and 3,123 individual stocks belonging to the three largest quartiles of NYSE- and AMEX-listed firms. Implementing a firm-level analysis is attractive because it addresses concerns about data snooping and loss of information that arise in procedures applied to equity portfolios sorted on firm characteristics or industry classifications, as

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<sup>1</sup>Keim and Stambaugh (1986) and Fama and French (1989) study business cycle variables such as the dividend yield on a market-wide index, the term spread, and the default spread. Basu (1977), Banz (1981), Jegadeesh (1990), Fama and French (1992), and Jegadeesh and Titman (1993) demonstrate the predictive ability of the firm-level size, book-to-market, and prior return measures.

<sup>2</sup>See Foster, Smith, and Whaley (1997), Bossaerts and Hillion (1999), and Stambaugh (1999).

noted by Litzenberger and Ramaswamy (1979) and Lo and MacKinlay (1990). In addition, Ferson and Harvey (1991) find that changes in risk premia explain most of the predictable variation of future returns, with changes in beta of second-order importance. However, Ferson and Harvey use portfolios and not individual stocks, and they suggest that the importance of beta variation for portfolios may be understated because much of the variation of individual firm's beta could be attenuated at the portfolio level.

The empirical evidence shows that over the 1972–2003 period, investment strategies that condition on business cycle variables, the dividend yield, the default spread, the term spread, and the Treasury bill yield deliver superior performance. To illustrate, the Sharpe ratio attributable to the market portfolio is 0.10 per month. The ex post out-of-sample Sharpe ratio is 0.10 when beta varies with business cycle variables, 0.17 under time varying alpha, and 0.20 under time varying equity premium. In addition, the Jensen's alpha that obtains by regressing investment excess returns produced by a no-predictability strategy on both the market index ( $\alpha_{cpm}$ ) and the Fama and French (1993) benchmarks ( $\alpha_{ff}$ ) is statistically indistinguishable from zero at conventional levels. In contrast, predictability-based strategies produce positive and significant  $\alpha_{cpm}$  and  $\alpha_{ff}$ . For example, when the equity premium varies with business cycle variables,  $\alpha_{cpm} = 2.10\%/month$  and  $\alpha_{ff} = 1.73\%/month$ . Both are statistically significant at conventional levels. For perspective, a 1% risk-adjusted monthly return obtains by implementing the momentum strategy of Jegadeesh and Titman (1993), which is one of the most prominent and intriguing puzzles documented in financial economics. Moreover, over the 1972–2003 investment period, taking a long position in a strategy that allows for alpha, beta, and equity premium variation with business conditions and taking a short position in the market portfolio generates a cumulative payoff of \$717 per \$1 investment. The corresponding figure is only \$18 when optimal portfolio strategies do not account for business conditions.

In related work, Schwert (2003) notes that the so-called financial market anomalies related to profit opportunities often disappear, reverse, or attenuate following their discovery. For example, he shows that the relation between the aggregate dividend yield and the equity premium is much weaker, based on both statistical and investment measures, after the discovery of that predictor by Keim and Stambaugh (1986) and Fama and French (1989). Consequently, we study investment performance before and after the discovery of the business cycle variables. Consistent with Schwert (2003), we demonstrate that strategies conditioned only on predictable equity premium are *much* stronger over the pre-discovery period than over the post-discovery period. Nevertheless, considering the business cycle variables is still beneficial in the post-discovery period because such variables drive stock-level alpha variation. To illustrate, in the post-discovery period, a strategy that allows alpha to vary with business cycle variables produces an out-of-sample Sharpe ratio of 0.25,  $\alpha_{cpm}$  of 2.55%, and  $\alpha_{ff}$  of 1.80%. Indeed, earlier work that documents no out-of-sample predictability does not consider variation in stock-level parameters. In particular, Bossaerts and Hillion (1999), Goyal and Welch (2003), and Schwert (2003) all analyze equity premium predictability only. By implementing firm-level analysis, we provide new evidence about stock return predictability: individual stock returns are predictable in real time, based on macro variables, while the equity premium predictability, the major focus of previous work, is questionable.

To understand the source of investment profitability, we relate our strategies to the size, book-to-market, and momentum effects. We show that investors who use business cycle

predictors hold small-cap, growth, and momentum stocks and they load less heavily (more heavily) on momentum (small-cap) stocks over the NBER-designated recession periods. Ultimately, these investors realize returns that are positive when adjusted by the Fama-French and momentum benchmarks as well as by the size, book-to-market, and prior return characteristics. To illustrate, the strategy that allows for predictability in alpha, beta, and the equity premium yields alpha of 1.32% (1.17%) per month when investment returns are adjusted by the Fama-French and momentum benchmarks with fixed (time varying) factor loadings, and it yields 1.14% per month when investment returns are adjusted by characteristics as in Daniel, Grinblatt, Titman, and Wermers (1997). All of these are significant at the 5% or 10% level.

In sum, earlier work that extensively studies equity premium predictability demonstrates only weak or even nonexistent out-of-sample predictability, especially in the period after the discovery of the macro variables. In this paper, we show that a focus on the equity premium fails to deliver evidence of predictability at the stock level. In particular, returns are predictable out-of-sample if one is willing to undertake a stock-level analysis allowing beta and especially alpha to vary with the dividend yield, the default spread, the term spread, and the Treasury bill yield. Our findings are based on a comprehensive set of performance evaluation measures and we are careful to analyze the determinants of predictability over the pre- and post-discovery periods as well as to associate our trading strategies with the size, book-to-market, and momentum effects. Indeed, a formal framework that uses macroeconomic variables generates trading strategies with long-only positions in individual stocks that outperform strategies that take long and short positions in the size, book-to-market, and momentum benchmarks as well as strategies that hold stocks with the same (potentially time varying) size, book-to-market, and momentum characteristics.

The remainder of the paper proceeds as follows. Section 2 develops an optimization framework for analyzing the profitability of predictability-based trading strategies that account for estimation risk. Section 3 describes the data. Section 4 presents the results. Section 5 offers conclusions and potential avenues for future research. Unless otherwise noted, all derivations are presented in the appendix.

## 2. Understanding firm-level predictability

This section sets forth a framework for studying the profitability of various portfolio strategies that incorporate macroeconomic variables to invest in individual stocks.

### 2.1. The statistical structure

The statistical structure extends that of Avramov (2004), who focuses on a constant beta and equity portfolios with equal return history. In particular, the underlying specifications for stock returns, factors, and predictors are

$$r_{t+1} = \alpha(z_t) + \beta(z_t)f_{t+1} + v_{t+1}, \quad (1)$$

$$\alpha(z_t) = \alpha_0 + \alpha_1 z_t, \quad (2)$$

$$\beta(z_t) = \beta_0 + \beta_1(I_K \otimes z_t), \quad (3)$$

$$f_{t+1} = a_f + A_f z_t + v_{ft+1}, \tag{4}$$

$$z_{t+1} = a_z + A_z z_t + v_{zt+1}, \tag{5}$$

where  $r_{t+1}$  is an  $N_t$ -vector of returns on stocks that are available for investment at time  $t$  in excess of the risk-free rate,  $f_{t+1}$  contains excess returns on  $K$  portfolio-based factors, and  $z_t$  is an  $M$ -vector of macroeconomic variables observed at time  $t$  that are potentially related to the probability distribution function of  $r_{t+1}$ . The intercepts  $\alpha_0$  and  $\alpha_1$  are an  $N_t$ -vector and  $N_t \times M$  matrix, respectively, reflecting fixed and time varying model mispricing,  $\beta(z_t)$  is an  $N_t \times K$  matrix of potentially time varying factor sensitivities, the symbol  $\otimes$  denotes the Kronecker product, and  $v_{t+1}$  is an  $N_t$  zero-mean vector of stock-specific events.

Modeling  $\alpha$  and  $\beta$  as linear functions of macroeconomic instruments goes back to Shanken (1990). The predictive regression characterization for factors allows for time varying risk premia. In particular, the time  $t$  vector of risk premia is  $\mathbb{E}(f_{t+1}|z_t) = a_f + A_f z_t$ . Risk premia vary with  $z_t$  if  $A_f \neq 0$ . The predictive variables follow vector AR(1), a characterization previously adopted in an investment context by Kandel and Stambaugh (1996), Stambaugh (1999), Barberis (2000), and Avramov (2002, 2004), among others, to model the evolution of the predictive variables. By construction, the covariance between  $v_{t+1}$  and  $v_{ft+1}$  is equal to zero. The covariances between  $v_{t+1}$  and  $v_{zt+1}$  and between  $v_{ft+1}$  and  $v_{zt+1}$  are denoted by  $\Sigma_{rz}$  and  $\Sigma_{fz}$ , respectively, and the variance-covariance matrices of  $v_{t+1}$ ,  $v_{ft+1}$ , and  $v_{zt+1}$  are denoted by  $\Psi$ ,  $\Sigma_{ff}$ , and  $\Sigma_{zz}$ , respectively.

Substituting Eqs. (2) and (3) into the right-hand side of Eq. (1) and then using the identity  $f_{t+1} \otimes z_t = (I_K \otimes z_t) f_{t+1}$  yields the following representation for excess stock returns:

$$r_{t+1} = \alpha_0 + \alpha_1 z_t + \beta_0 f_{t+1} + \beta_1 [f_{t+1} \otimes z_t] + v_{t+1}. \tag{6}$$

When  $\alpha_0 = \alpha_1 = 0$ , Eq. (6) stands for an unconditional representation of a conditional pricing model with  $K$  fundamental factors ( $f_{t+1}$ ) and  $KM$  scaled factors ( $f_{t+1} \otimes z_t$ ). We use restricted and unrestricted versions of Eqs. (4) and (6) to generate moments for asset allocation. In particular, the statistical structure implies three sources of predictability, namely, model misspecification ( $\alpha_1 \neq 0$ ), risk ( $\beta_1 \neq 0$ ), and risk premia ( $A_f \neq 0$ ).

We study various beliefs about the predictability structure. Agents either rule out predictability, setting  $\alpha_1$ ,  $\beta_1$ , and  $A_f$  to zero, or they recognize predictability, leaving all or subsets of these parameters unrestricted. Each such belief is examined under two polar views about pricing model accuracy. The first disregards pricing models, thereby leaving  $\alpha_0$  and  $\alpha_1$  unrestricted. The second imposes pricing restrictions. With unrestricted intercepts, factor models with predictability are merely conditional covariance models. With restricted intercepts, a risk-based asset pricing model is believed to capture the predictable variation in future stock returns.

Based on the underlying models for excess stock returns, asset pricing factors, and information variables, the mean and variance used to construct portfolio strategies in the presence of time varying alphas, betas, and the market premium are

$$\mu_t = \alpha(z_t) + \beta(z_t)(a_f + A_f z_t), \tag{7}$$

$$\Sigma_t = \beta(z_t) \Sigma_{ff} \beta(z_t)' + \Psi. \tag{8}$$

Of course, when predictability is disregarded, the mean and variances that obtain are such that  $\alpha(z_t) = \alpha_0$ ,  $\beta(z_t) = \beta_0$ , and  $A_f = 0$ . Observe from Eqs. (3) and (7) that under time

varying risk ( $\beta_1 \neq 0$ ) and risk premia ( $A_f \neq 0$ ),  $\mu_t$  is quadratic in  $z_t$ . Previous work studying predictability typically uses the predictive regression framework, thereby restricting the relation between conditional expected return and the predictive variables to be linear. Also, observe from Eq. (8) that under time varying risk,  $\Sigma_t$ , the covariance matrix of returns, is quadratic in  $z_t$ .

We assume that  $\Psi$  is diagonal, otherwise, the estimated covariance matrix may be singular. To illustrate, consider an investment universe that comprises 1,000 securities. Then, one has to estimate 1,000 variance and 499,500 covariance parameters. Our sample contains *at most* 497 return observations per stock. Thus, there are at most 497,000 return observations, less than the number of parameters in the covariance matrix. Indeed, Frost and Savarino (1986) and Best and Grauer (1991) note that mean-variance strategies formed based on a covariance matrix estimated from historical returns deliver poor performance even when the number of securities is considerably smaller than that we consider here. Poor performance is typically traced to an imprecise estimation of a large number of parameters. Note that since our empirical evidence is ultimately based on ex post out-of-sample portfolio analysis, the assumption of a diagonal covariance matrix should not drive our results regarding predictability and its determinants.

Our framework allows for investments with different return histories. Stambaugh (1997) analyzes such investments focusing on cases in which expected returns and the covariance matrix are constant over time. Here, we explicitly allow expected returns and the covariance matrix to vary with the macroeconomic variables. Observe from Eqs. (7) and (8) that  $\alpha$ ,  $\beta$ , and  $\Psi$  are based on a stock return history, which may be shorter than that of the factors and predictors. Longer histories of the factors and predictors are useful in estimating  $a_f$ ,  $A_f$ , and  $\Sigma_{ff}$ , and thereby improve the precision of means, variances, and covariances of the short-history stocks.

*2.2. Forming optimal portfolios*

The investment universe at time  $t$  comprises  $N_t$  stocks and a risk-free asset. The number of stocks varies over time as stocks enter (IPO) and leave (merger, acquisition, bankruptcy, etc.) the sample periodically. In this paper, a hypothetical investor maximizes the conditional expected value of the quadratic utility function

$$U(W_t, R_{p,t+1}, a_t, b_t) = a_t + W_t R_{p,t+1} - \frac{b_t}{2} W_t^2 R_{p,t+1}^2, \tag{9}$$

where  $a_t$  is some constant,  $W_t$  denotes time  $t$  wealth,  $b_t$  stands for the absolute risk aversion parameter, and  $R_{p,t+1}$  is the realized return on the optimal portfolio, computed as  $R_{p,t+1} = 1 + r_{ft} + w'_t r_{t+1}$ , with  $r_{ft}$  being the riskless rate and  $w_t$  being the set of optimal portfolio weights. Taking the conditional expectation of both sides of Eq. (9), letting  $\gamma_t = (b_t W_t)/(1 - b_t W_t)$  be the relative risk aversion parameter, and letting  $A_t = [\Sigma_t + \mu_t \mu'_t]^{-1}$  yields the following formulation for the expected utility maximization:

$$w_t = \arg \max_{w_t} \left\{ w'_t \mu_t - \frac{1}{2(1/\gamma_t - r_{ft})} w'_t A_t^{-1} w_t \right\}. \tag{10}$$

We derive optimal portfolios by maximizing Eq. (10) precluding short selling of stocks as well as constraining the overall investment in stocks to 200%/Regulation T of the

Federal Reserve Board.<sup>3</sup> For illustration, the unconstrained conditionally efficient mean-variance strategy is given by (suppressing the time subscript)

$$w = (1/\gamma - r_f)A\mu, \quad (11)$$

$$= (1/\gamma - r_f) \frac{\Sigma^{-1}\mu}{1 + \mu'\Sigma^{-1}\mu}, \quad (12)$$

where Eq. (12) follows by applying the inverse matrix theorem (e.g., Seber, 1984, p. 520) to  $A$ . Inverting the large scaled matrix  $\Sigma$  is straightforward by applying the inverse matrix theorem to Eq. (8), which yields

$$\Sigma^{-1} = \Psi^{-1} - \Psi^{-1}\beta[\beta'\Psi^{-1}\beta + \Sigma_{ff}^{-1}]^{-1}\beta'\Psi^{-1}. \quad (13)$$

Hence, the inversion of  $\Sigma$  involves the inversion of the diagonal matrix  $\Psi$  and the low-dimensional ( $K \times K$ ) matrices  $\Sigma_{ff}$  and  $\beta'\Psi^{-1}\beta + \Sigma_{ff}^{-1}$ .

Note that Eqs. (7) and (8) describe the mean and variance of future returns under the case in which predictability is due to time varying alpha, beta, and risk premia. Below, we consider portfolio selection under several nested scenarios, including the no-predictability case in which  $\mu_t$  and  $\Sigma_t$  are based on constant alpha, beta, and risk premia, as well as the case in which asset pricing restrictions are imposed.

In forming optimal portfolios, we replace  $\mu_t$  and  $\Sigma_t$  in Eq. (10) by the mean and variance of the Bayesian predictive distribution<sup>4</sup>

$$p(r_{t+1}|\mathcal{D}_t) = \int_{\Theta} p(r_{t+1}|\mathcal{D}_t, \Theta)p(\Theta|\mathcal{D}_t) d\Theta, \quad (14)$$

where  $\mathcal{D}_t$  denotes the data (excess stock returns, factors, and predictors) observed up to and including time  $t$ ,  $\Theta$  is the set of parameters underlying the processes in Eqs. (1)–(5), and  $p(\Theta|\mathcal{D}_t)$  is the posterior density of  $\Theta$ . Specific details are below.

### 2.3. Asset allocation with estimation risk

Academics and practitioners often approach asset allocation by first specifying a model for stock returns, which in the context of this paper amounts to determining the collection of information variables and factors, and then replacing the parameter values in the mean and variance expressions by maximum likelihood estimates. This common practice implies that the specified return generating model is undoubtedly correct and that the maximum likelihood estimates are the true parameter values.

<sup>3</sup>Regulation T requires a margin of 50% for purchasing securities using a loan given by the broker/dealers. The portfolio constraint in the presence of margin and no short selling is  $\sum_{i=1}^N \omega_i < \frac{1}{\phi}$ , where  $\omega_i$  is the allocation to stock  $i$  and  $\phi$  is the margin, both in fractions. Although Regulation T permits limited short positions, we prohibit short selling because, as Sharpe (1991) notes, many institutional investors are prohibited from taking short positions either through explicit rules or via the implicit threat of lawsuit for violating fiduciary standards. Other investors often voluntarily impose constraints on portfolio holdings since short sales are costly.

<sup>4</sup>Our expected utility maximization is a version of the general Bayesian control problem developed by Zellner and Chetty (1965). Frost and Savarino (1986), Pastor (2000), and Pastor and Stambaugh (2000) compute optimal portfolios in a framework in which returns are assumed independently and identically distributed. Kandel and Stambaugh (1996), Barberis (2000), Avramov (2002, 2004), and Avramov and Wermers (2006) analyze a portfolio decision when returns are potentially predictable.

Portfolio selection based on this practice typically yields extreme long and short positions (see, e.g., Best and Grauer, 1991), which are often due to imprecise estimation of the moments. Incorporating estimation risk in a Bayesian framework could help improve performance of mean-variance strategies. Indeed, Frost and Savarino (1986) use Bayes methods to reduce estimation errors and improve portfolio performance. Recent advances in Bayesian asset allocation include Kandel and Stambaugh (1996), Barberis (2000), Pastor (2000), Pastor and Stambaugh (2000, 2002), Avramov (2002, 2004), Tu and Zhou (2004), and Avramov and Wermers (2006). The Bayesian approach explicitly addresses estimation risk because the predictive distribution in (14) integrates over the posterior distribution, which summarizes the state of uncertainty (based on the prior beliefs and data) about the unknown parameters  $\Theta$ .

We form optimal portfolios that account for estimation risk. Prior beliefs about  $\Theta$  are assumed to be noninformative, and the residuals in regressions (1), (4), and (5) are assumed to be both i.i.d. over time and normally distributed. Indeed, normality has often been rejected as returns display fat tails. However, Tu and Zhou (2004) show that normality works well in evaluating portfolio performance for a mean-variance investor. Next, let  $\hat{\alpha}(z_T)$ ,  $\hat{\beta}(z_T)$ ,  $\hat{a}_f$ ,  $\hat{A}_f$ ,  $\hat{\Sigma}_{ff}$ , and  $\hat{\Psi}$  be the maximum likelihood estimates of  $\alpha(z_T)$ ,  $\beta(z_T)$ ,  $a_f$ ,  $A_f$ ,  $\Sigma_{ff}$ , and  $\Psi$ , respectively. When predictability is attributable to time varying alphas, betas, and risk premia, the mean and variance that go into portfolio optimization, say at time  $T$ , obey the expressions

$$\mu_T = \hat{\alpha}(z_T) + \hat{\beta}(z_T)(\hat{a}_f + \hat{A}_f z_T), \quad (15)$$

$$\Sigma_T = \mathcal{P}_1 \hat{\beta}(z_T) \hat{\Sigma}_{ff} \hat{\beta}(z_T)' + \mathcal{P}_2 \hat{\Psi}. \quad (16)$$

The predictive variance in Eq. (16) is larger than its maximum likelihood analog as it incorporates the factors  $\mathcal{P}_1$  and  $\mathcal{P}_2$ . In particular,  $\mathcal{P}_1$  is a scalar greater than one and  $\mathcal{P}_2$  is a diagonal matrix such that each diagonal entry is greater than one. The Appendix derives explicit expressions for  $\mathcal{P}_1$  and  $\mathcal{P}_2$  as well as for the maximum likelihood estimators. Ultimately,  $\mu_T$  and  $\Sigma_T$  have analytic closed-form expressions.

### 3. Data

The data consists of monthly excess returns, size, book-to-market, turnover, and lagged 1-year returns for a sample of NYSE- and AMEX-listed common stocks as well as the four macroeconomic variables we describe below. The sample period is from July 1962 through December 2003. To be included in the investment universe, a stock has to satisfy the following criteria. First, its return through the investment date and over the past 60 months must be available from CRSP. Second, there must be sufficient data available to calculate the size, as measured by market capitalization, and trading volume. Third, there must be sufficient data available on the COMPUSTAT tapes to calculate the book-to-market ratio as of December of the previous year. We focus on the largest firms by deleting the smallest quartile of firms from the sample. This screening process yields a total of 3,123 stocks and an average (median) of 973 (988) stocks/month.

Table 1 presents some summary statistics. Panel A presents the time-series averages of the cross-sectional means, medians, and standard deviations of returns and the firm characteristics. The mean (median) excess return of the sample stocks is 0.69% (0.31%) per month. The mean firm market capitalization is \$2.96 billion, while the median is \$0.72

Table 1

## Summary statistics

This table presents summary statistics for returns, individual firm characteristics, and macroeconomic variables. Panel A presents the time-series averages of the cross-sectional means, medians, and standard deviations for an average (total) of 973 (3,123) NYSE-AMEX stocks over the investment period July 1972 through November 2003. Size represents the market capitalization in billions of dollars. Turnover is the monthly share trading volume divided by shares outstanding in percent. RET12 is the cumulative return over the 12 months prior to the current month. To be included in the sample each stock has to satisfy the following criteria: (i) its return in the investment month  $t$  and in the previous 60 months is available from CRSP, (ii) sufficient data are available to calculate the size and turnover as of the month  $t - 2$ , (iii) sufficient data are available on the COMPUSTAT tapes to calculate the book-to-market ratio as of December of the previous year. Panel B presents the time-series averages of the macroeconomic variables, the dividend yield, the term spread, the default spread, and the 3-month T-bill yield over the period July 1962–November 2003.

	Mean	Median	Standard deviation
<i>Panel A: Firm characteristics</i>			
Excess return (%)	0.69	0.31	8.91
Firm size (\$ billions)	2.96	0.72	8.25
Book-to-market ratio	1.25	0.83	4.21
Turnover	5.28	3.96	5.58
RET12 (%)	12.88	9.95	31.59
<i>Panel B: Macroeconomic variables</i>			
Dividend yield (%)	2.84	2.82	0.97
Default spread (%)	1.01	0.89	0.44
Term spread (%)	0.83	0.78	1.12
T-bill yield (%)	5.90	5.37	2.71

billion. The average monthly turnover is 5.28%. As in Fama and French (1992), we compute the book-to-market ratio for July of year  $t$  to June of year  $t + 1$  using accounting data at the end of year  $t - 1$ . The mean (median) book-to-market ratio is 1.25 (0.83). The mean (median) lagged 12-month return is 12.88% (9.95%). Note that the firm-level variables display considerable skewness.

Panel B of Table 1 presents the time-series averages of the macroeconomic variables. We use the dividend yield, the default spread, the term spread, and the 3-month T-bill yield. The dividend yield is the total dividend payments on the value-weighted CRSP index over the previous 12 months divided by the current level of the index. The default spread is the yield differential between bonds rated BAA by Moodys and bonds with a Moodys rating of AAA. The term spread is the yield differential between Treasury bonds with more than ten years to maturity and T-bills that mature in three months. The mean dividend yield over the sample period is 2.84%, the mean term spread is 0.83%, the mean default spread is 1.01%, and the mean 3-month T-bill yield is 5.90%.

#### 4. Results

We form portfolio strategies as in Eq. (10) subject to (i) no short selling of stocks and (ii) the overall investment in stocks cannot exceed 200% per Regulation T of the Federal Reserve Board, which requires an initial margin of 50%. In forming optimal portfolios, we take  $f_t$  in Eq. (1) to be the excess return on the market portfolio. In addition, we replace  $\mu_t$

and  $\Sigma_t$  by the first and the second moments of the Bayesian predictive distribution in Eq. (14). The difference  $(1/\gamma_t) - r_{ft}$  in Eq. (10) is set such that  $r_{ft}$  is the actual risk-free rate prevailing at the time the investment is made and  $\gamma_t$  is determined in every investment period such that if an agent could invest her wealth only in the market index (excluding predictability) and the risk-free asset, the entire wealth will be allocated to the market index. Over the investment period, the mean and standard deviation of  $\gamma_t$  are 1.97 and 0.59, respectively. Over the subperiod January 1988 through December 2003, which is the period after the discovery of the predictive variables, the corresponding numbers are 2.29 and 0.35.

Optimal portfolios are first derived using the first  $P = 120$  monthly observations, are then rebalanced using the first  $P + 1$  monthly observations, and so on, ..., and are finally rebalanced using the first  $T - 1$  monthly observations, with  $T = 497$  denoting the sample size. Hence, the first investment is made in July 1972, the second in August 1972, and so on, ..., and the last in November 2003. The month  $t$  realized excess return on an investment is obtained by multiplying the month  $t - 1$  portfolio weights by the month  $t$  realized excess returns. This recursive scheme produces  $T - P = 377$  payoffs on various investment strategies. Strategies differ with respect to the predictive moments used in the optimization. Part A (B) of the Appendix derives closed-form expressions for the first two predictive moments when alphas, betas, and the market premium may (may not) vary with the predictive variables.

#### 4.1. Conditioning investments on business cycle variables

##### 4.1.1. Is predictability beneficial in real time?

Table 2 reports various measures for evaluating the performance of unconditionally efficient strategies (**IID**) as well as conditional strategies formed based on four predictors, the dividend yield, the default spread, the term spread, and the Treasury bill yield. Specifically, **TV- $\beta$**  allows for time varying betas, **TV-RP** allows for time varying market premium, **TV- $\alpha$**  allows for time varying alphas, **TV-All** allows for time varying alphas, betas, and market premium, and **MKT** denotes a strategy that optimally invests in the value-weighted market portfolio (between 0% and 200%) and the risk-free asset, incorporating the four predictors. We study portfolio strategies (excluding **TV- $\alpha$**  and **MKT**) when the CAPM restrictions ( $\alpha_0 = \alpha_1 = 0$ ) are first disregarded (denoted **UN**) and then imposed (denoted **RE**).

Note that short positions in the market portfolio and individual stocks are not permissible. Indeed, many institutional investors are prohibited from taking short positions either through explicit rules or via the implicit threat of lawsuit for violating fiduciary standards. Other investors often voluntarily impose constraints on portfolio holdings since short sales are costly. Of course, if expected returns, variances, and correlations were known with certainty, then imposing no short selling would hamper ex post performance. In practice, however, these quantities are unknown. Thus, whether imposing portfolio constraints hurts or helps is an empirical issue.

Panel A of Table 2 evaluates performance of investments that realize returns over the August 1972–December 2003 period. Panel B focuses on the August 1972 through December 1987 subperiod, and Panel C focuses on the January 1988 through December 2003 subperiod. This last subperiod reflects a purely out-of-sample experiment because

Table 2

Performance measures of optimal portfolio strategies

The table reports several measures for evaluating performance of various optimal portfolio strategies. The **IID** strategy is based on stock return moments that disregard conditioning information altogether. All other strategies are conditioned on macroeconomic predictors. **TV- $\beta$**  permits time varying beta, **TV-RP** permits time varying equity premium, **TV- $\alpha$**  permits time varying alpha, **TV-All** permits time varying alpha, beta, and the equity premium, and **MKT** invests only in the market index and a risk-free asset that exploits predictability. We study all strategies (excluding **TV- $\alpha$**  and **MKT**) when CAPM restrictions are first disregarded (denoted *UN*) and then imposed (denoted *RE*). Investment strategies are formed on a monthly basis using a recursive scheme. Short positions in stocks are not permissible and the overall investment in stocks cannot exceed 200% per Regulation T of the Federal Reserve Board. The risk aversion parameter is determined in every investment period such that an agent who could invest only in a risk-free asset and the market index (disregarding predictability) would allocate her entire wealth into the market index. Panel A evaluates performance of investments whose returns have been realized over the August 1972–December 2003 period using 377 ex post excess returns. Panel B focuses on the August 1972–December 1987 subperiod. Panel C focuses on the January 1988–December 2003 subperiod. The evaluation measures are as follows.  $\mu$  is the average excess realized returns, SR is the monthly Sharpe ratio,  $\beta$  is the market beta,  $\alpha_{cpm}$  ( $\tilde{\alpha}_{cpm}$ ) is the intercept in a regression of excess realized returns on the market factor when beta is constant (beta varies with the macroeconomic variables), IR is the information ratio obtained by dividing  $\alpha_{cpm}$  by the standard deviation of the residuals,  $\Delta Wealth$  denotes the cumulative return of an investment that takes a long position in any of the portfolio strategies and a short position in the market portfolio, starting with \$1, Equities denotes the average allocation to stocks throughout the sample period, and  $SE(x)$  is the standard error of  $x$ . The measures  $\mu$  as well as alphas and their corresponding standard errors are in percent per month.

	IID		TV- $\beta$		TV-RP		TV- $\alpha$	TV-All		MKT
	UN	RE	UN	RE	UN	RE	UN	UN	RE	
<i>Panel A: 08/72-12/03</i>										
$\mu$	1.90	0.64	1.33	0.51	3.16	1.65	2.90	3.16	3.04	0.98
$SE(\mu)$	0.76	0.22	0.72	0.21	0.80	0.61	0.87	0.84	0.77	0.28
SR	0.13	0.15	0.10	0.13	0.20	0.14	0.17	0.19	0.20	0.18
$SE(SR)$	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$\beta$	2.51	0.84	2.40	0.78	2.23	1.27	2.10	2.17	1.82	0.69
$\alpha_{cpm}$	0.70	0.24	0.18	0.14	2.10	1.04	1.90	2.12	2.17	0.65
$SE(\alpha_{cpm})$	0.46	0.09	0.43	0.08	0.60	0.53	0.71	0.66	0.64	0.22
IR	0.08	0.13	0.02	0.09	0.18	0.10	0.14	0.17	0.18	0.15
$\tilde{\alpha}_{cpm}$	0.71	0.16	0.28	0.10	1.38	-0.08	2.18	2.24	1.55	0.07
$SE(\tilde{\alpha}_{cpm})$	0.45	0.09	0.42	0.08	0.56	0.40	0.72	0.67	0.61	0.12
$\Delta Wealth$	18.10	1.73	2.90	1.06	1247.29	13.32	188.02	716.69	802.99	4.48
Equities	200.00	94.34	200.00	84.36	195.36	74.97	199.91	200.00	179.42	64.43
<i>Panel B: 08/72-12/87</i>										
$\mu$	0.92	0.45	0.00	0.33	3.87	2.05	1.98	2.72	3.36	1.39
$SE(\mu)$	1.29	0.35	1.19	0.31	1.38	1.05	1.36	1.35	1.08	0.44
SR	0.05	0.10	0.00	0.08	0.21	0.14	0.11	0.15	0.23	0.23
$SE(SR)$	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07
$\beta$	2.92	0.87	2.72	0.79	2.50	1.39	2.25	2.33	1.66	0.73
$\alpha_{cpm}$	0.08	0.20	-0.78	0.10	3.15	1.65	1.33	2.05	2.88	1.18
$SE(\alpha_{cpm})$	0.68	0.12	0.60	0.10	1.02	0.91	1.07	1.03	0.89	0.35
IR	0.01	0.13	-0.10	0.07	0.23	0.13	0.09	0.15	0.24	0.25
$\tilde{\alpha}_{cpm}$	0.15	0.19	-0.48	0.13	1.39	-0.50	1.97	2.51	2.17	0.12
$SE(\tilde{\alpha}_{cpm})$	0.70	0.11	0.62	0.10	0.98	0.73	1.12	1.07	0.88	0.21
$\Delta Wealth$	0.58	1.32	0.15	1.05	88.14	7.13	2.58	10.56	73.93	6.16
Equities	200.00	90.11	200.00	79.27	190.54	80.73	199.82	200.00	170.10	74.32

Table 2 (continued)

	IID		TV- $\beta$		TV-RP		TV- $\alpha$	TV-All		MKT
	UN	RE	UN	RE	UN	RE	UN	UN	RE	
<i>Panel C: 01/88-12/03</i>										
$\mu$	2.85	0.83	2.61	0.69	2.48	1.27	3.79	3.58	2.74	0.57
$SE(\mu)$	0.83	0.28	0.82	0.27	0.84	0.66	1.09	1.01	1.11	0.33
SR	0.25	0.21	0.23	0.19	0.21	0.14	0.25	0.25	0.18	0.12
$SE(\text{SR})$	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.08	0.07	0.08
$\beta$	1.95	0.79	1.95	0.76	1.87	1.13	1.87	1.94	2.04	0.65
$\alpha_{cpm}$	1.56	0.30	1.32	0.19	1.24	0.52	2.55	2.30	1.39	0.15
$SE(\alpha_{cpm})$	0.57	0.15	0.57	0.13	0.62	0.56	0.94	0.83	0.92	0.27
IR	0.20	0.15	0.17	0.11	0.15	0.07	0.20	0.20	0.11	0.04
$\tilde{\alpha}_{cpm}$	1.77	0.08	1.51	0.04	1.34	0.41	2.58	2.15	1.15	0.17
$SE(\tilde{\alpha}_{cpm})$	0.59	0.13	0.59	0.13	0.63	0.35	0.98	0.86	0.84	0.11
$\Delta Wealth$	31.07	1.31	19.45	1.01	14.15	1.87	72.89	67.87	10.86	0.73
Equities	200.00	98.41	200.00	89.26	200.00	69.42	200.00	200.00	188.40	54.89

Keim and Stambaugh (1986) and Fama and French (1989) discover the macroeconomic predictors using data realized up to December 1987.

Performance measures reported in Table 2 are as follows:  $\mu$  is the average excess realized returns, SR is the monthly Sharpe ratio,  $\beta$  is the market beta,  $\alpha_{cpm}$  ( $\tilde{\alpha}_{cpm}$ ) is the intercept in a regression of excess realized returns on the market factor when beta is constant (beta varies with the four macro variables), IR is the information ratio obtained by dividing  $\alpha_{cpm}$  by the standard deviation of the residuals,  $\Delta Wealth$  denotes the cumulative return of an investment that takes a long position in any of the portfolio strategies and a short position in the market portfolio, starting with \$1, Equities denotes the average allocation to stocks throughout the investment period, and  $SE(x)$  is the standard error of  $x$ . We compute the standard error of the Sharpe ratio using the asymptotic tests derived by Lo (2002) under i.i.d. stock returns and under the case where returns display dependence through time. Only the latter is presented, as both computations deliver virtually the same inferences. The measures  $\mu$ ,  $\alpha_{cpm}$ , and  $\tilde{\alpha}_{cpm}$  and their corresponding standard errors are in percent per month.

Let us first analyze the case in which asset pricing restrictions are not imposed. Panel A of Table 2 provides strong support for the predictive power of the dividend yield, the term spread, the default spread, and the Treasury bill yield over the entire sample period. Portfolio strategies conditioned on these macroeconomic predictors outperform the IID strategy, **IID**, as well as passive and dynamic investments in the market index, **MKT**. The monthly excess return generated by **IID** is 1.90%. This figure is 1.33% under time varying betas, **TV- $\beta$** , 3.16% under time varying market premium, **TV-RP**, 2.90% under time varying alphas, **TV- $\alpha$** , and 3.16% when alphas, betas, and the market premium vary with the macroeconomic predictors, **TV-All**. Note that all the strategies that do not impose pricing restrictions (excluding **MKT**) hold either 200% or close to this amount in equities, leading to relatively high betas.

Indeed, predictability-based strategies typically record higher volatilities. Nevertheless, these strategies provide the investor a superior risk-return tradeoff. In particular, the monthly Sharpe ratio generated by the **IID** strategy is 0.13. It is 0.10 under **TV- $\beta$** , 0.20 under **TV-RP**, 0.17 under **TV- $\alpha$** , and 0.19 under **TV-All**. The Jensen's alphas obtained by

regressing realized excess returns produced by the **IID** strategy on the market index with fixed ( $\alpha_{cpm}$ ) as well as time varying ( $\tilde{\alpha}_{cpm}$ ) betas are statistically indistinguishable from zero under conventional levels. In contrast, except for time varying beta strategies, the measures  $\alpha_{cpm}$  and  $\tilde{\alpha}_{cpm}$  are positive and statistically and economically significant for all other predictability-based strategies. The same evidence emerges based on the information ratio (IR). The IR for the predictability-based strategies (except **TV- $\beta$** ) are all higher than that for the **IID** case.

Over the investment period July 1972 through November 2003, taking a long position in the **TV-All** strategy and a short position in the market portfolio generates a cumulative payoff (starting with \$1) of  $\Delta Wealth = \$717$ . The corresponding figure is only \$18 when the aggregate predictors are disregarded altogether. The scenario in which the investment universe consists of the market portfolio and the risk-free asset, **MKT**, seems to outperform the **IID** strategy as well as a pure investment in the market index, generating a larger Sharpe ratio. Still, the gain from implementing market timing strategies is not as large as that attributable to investments in individual stocks, exploiting predictability in asset mispricing, risk, and the equity premium.

What about the profitability of trading strategies over the post-discovery period? Schwert (2003) notes that financial market anomalies related to profit opportunities often disappear, reverse, or attenuate following their discovery. We examine whether the abnormal performance of predictability-based investment strategies disappear, reverse, or attenuate after December 1987, the end of sample period in Fama and French (1989). As noted earlier, Panel B reflects the performance of investments made in the pre-discovery period, whereas Panel C focuses on the post-discovery period.

Comparing the performance evaluation measures from Panels B and C of Table 2 reveals that the *absolute* performance of the portfolio strategies **IID**, **TV- $\beta$** , **TV- $\alpha$** , and **TV-All** has improved over the post-discovery period. Focusing on **IID**,  $\mu$  rises from 0.92% (Panel B) to 2.85% (Panel C) even though risk, measured by volatility and beta, has declined. Likewise, predictability-based strategies generate larger Sharpe ratios. For example, under **TV-All**, the Sharpe ratio rises from 0.15 to 0.25.

Next, we examine performance *relative* to the market. Starting with the **IID** strategy,  $\alpha_{cpm}$  increases from 0.08% (Panel B) to 1.56% (Panel C). Hence, the performance of the **IID** strategy relative to the market does improve. Moving to strategies that exploit the four macroeconomic predictors, Sharpe ratios based on **TV- $\alpha$** , **TV- $\beta$** , **TV-RP**, and **TV-All** are larger than those generated by passively investing in the market index and  $\alpha_{cpm}$  is significant at the 5% level. Focusing on **TV- $\alpha$**  and **TV-All**, we demonstrate that  $\alpha_{cpm}$  and  $\tilde{\alpha}_{cpm}$  are higher than those obtained under the **IID** strategies. In addition, the  $\Delta Wealth$  measures indicate that predictability-based strategies have outperformed the market index as well as the **IID** strategy. Specifically,  $\Delta Wealth = \$31.07$  under **IID**, \$72.89 under **TV- $\alpha$** , and \$67.87 under **TV-All**.

Let us now analyze the case in which CAPM restrictions on predictability are imposed. Incorporating such restrictions typically hurts the performance of trading strategies in the pre- and post-discovery periods. Focusing on Panel A,  $\tilde{\alpha}_{cpm}$  declines from 0.71% to 0.16% (**IID**) and from 2.24% to 1.55% (**TV-All**). Nevertheless, strategies that permit variations in the market premium and beta with the business cycle still outperform the market index and outperform the **IID** strategy. Under **TV-All**,  $\alpha_{cpm}$  and  $\tilde{\alpha}_{cpm}$  are all positive and statistically and economically significant. The overall evidence in favor of predictability is thus robust to imposing asset pricing restrictions.

The diminishing performance under CAPM restrictions suggests that, based on our “test statistic,” that is the ex post out-of-sample performance of strategies that invest in single stocks, conditional and unconditional versions of the CAPM do not explain the cross section variation in average returns. Imposing CAPM restrictions makes average return as well as volatility smaller both ex ante (not reported) and ex post. The reduction in average return dominates portfolio selection, as the overall investment in stocks, exhibited in Table 2, is much smaller under restricted intercepts.

#### 4.1.2. Performance sensitivity to investment timing

Table 2 reports performance evaluation measures based on the entire sample period as well as the pre- and post-discovery subperiods. Fig. 1 provides further insights about performance sensitivity to investment timing. Alphas, Sharpe ratios, and cumulative returns are computed on the basis of 318 investment subperiods that differ with respect to  $P$ , which ranges between 120 and 437 in a 1-month interval. In Fig. 1, “7208” corresponds to investments that realize returns over the August 1972–December 2003 period ( $P = 120$ ), similar to Panel A of Table 2. Similarly, “9812” corresponds to investments that realize returns over the December 1998 through December 2003 period ( $P = 437$ ). The left (right) plots display performance measures under unrestricted (restricted) intercepts.

Plots A1 and A2 describe  $\alpha_{cpm}$ . Plots B1 and B2 exhibit Sharpe ratios. Plots C1 and C2 show the evolution of \$1 invested in July 1972 in the **IID** and **TV-All** strategies as well as in the market portfolio (100%), wherein cash dividends are reinvested. The dashed–dotted (solid) lines in plots A1, A2, B1, and B2 correspond to **IID** (**TV-All**). The same symbols apply to plots C1 and C2, where, in addition, the dashed lines describe a passive investment in the market index. The evolutions of investments in the **IID** strategy and the market index are hard to see in Plots C1 and C2 because the value of \$1 invested in these two strategies is extremely small compared to that invested in strategies that exploit stock return predictability.

Fig. 1 shows that portfolio strategies based on **TV-All** uniformly generate larger alphas and larger cumulative returns over the investment horizon. When pricing restrictions are disregarded, the (time-series) mean difference in  $\alpha_{cpm}$  corresponding to **TV-All** and **IID** is 88 basis point per month. When we restrict the intercepts, the mean difference is 126 basis points. The Sharpe ratio is often higher in the **IID** case, especially during the late 1980s and the 1990s when we impose the CAPM restrictions. However, it should be noted that in those periods the Sharpe ratio for the **IID** strategy is higher because the overall volatility (both systematic and idiosyncratic) of the **TV-All** strategy is higher. Focusing on the systematic component only would provide more supportive evidence for return predictability. For instance, the Treynor ratio (not reported) of predictability-based strategies is uniformly higher.

Remarkably, observe from plot C1 that the December 2003 value of the \$1 invested in **TV-All** starting from July 1972 with monthly rebalancing is about \$6,537. The corresponding value is only \$27.60 (\$118.10) when invested in the market index (**IID**). Incorporating CAPM restrictions, the December 2003 value of \$1 invested in July 1972 with monthly rebalancing is about \$11,273 (\$54.40) under **TV-All** (**IID**).

#### 4.1.3. The determinants of predictability

Strategies based on macroeconomic predictors typically generate superior performance relative to strategies that disregard predictability and relative to static and dynamic

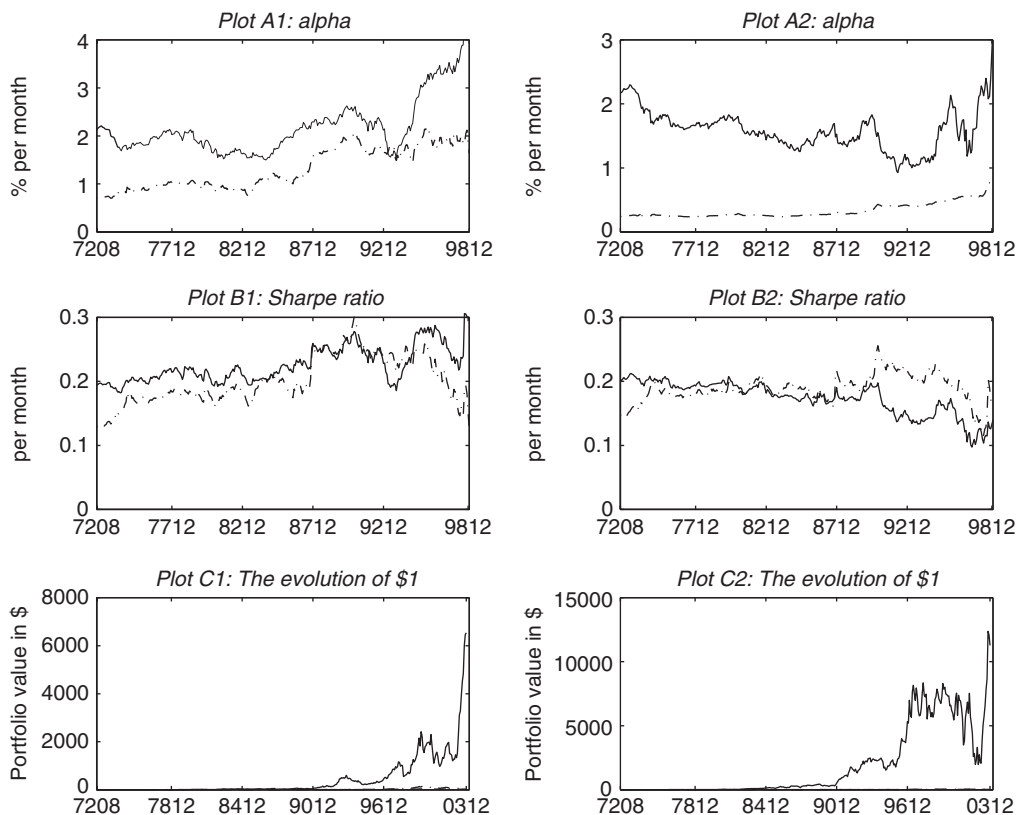


Fig. 1. Ex post out-of-sample performance of portfolio strategies: The figure displays alphas, Sharpe ratios, and cumulative returns computed based on 318 investment subperiods. 7208 corresponds to investments that realize returns over the August 1972–December 2003 period ( $P = 120$ ), similar to Panel A in Table 2. In the same vein, 9812 corresponds to investments that realize returns over the December 1990–December 2003 period ( $P = 316$ ). The left (right) plots display performance measures under unrestricted (restricted) intercepts. Plots A1 and A2 describe alphas. Plots B1 and B2 exhibit Sharpe ratios. Plots C1 and C2 show the evolution of \$1 invested in July 1972 in the IID and TV-All strategies as well as in the market portfolio, in which cash dividends are reinvested. The dashed-dotted (solid) lines in plots A1, A2, B1, and B2 correspond to IID (TV-All). The same symbols apply to plots C1 and C2, where, in addition, the dashed lines describe an investment in the market index.

investments in the market portfolio. In this subsection, we closely examine the drivers of this superior performance. We ask: Is predictability due to time varying alphas, and/or betas, and/or the market premium? We start by analyzing the market premium variation, which is the focus of Bossaerts and Hillion (1999) and Schwert (2003).

In the pre-discovery period, the TV-RP strategy generates the largest Sharpe ratio (0.21 under unrestricted intercepts) and significant and positive  $\alpha_{cpm}$  and  $\tilde{\alpha}_{cpm}$ . In the post-discovery period, performance of investments based on TV-RP deteriorates. The Sharpe ratio remains unchanged even though stock prices have substantially advanced over the period and  $\alpha_{cpm}$  declines from 3.15% (Panel B) to 1.24% (Panel C). TV-RP even underperforms IID. Also, the MKT strategy, which captures potential predictability in the equity premium, produces relatively weak performance measures.

In the post-discovery period, time varying alphas, or time varying alphas and betas combined, account for the profitability of trading strategies that use macro predictors. First, the largest Sharpe ratio (0.25) is attributed to the **TV- $\alpha$**  and the **TV-ALL** strategy. Moreover, the largest  $\alpha_{cpm}$  (2.55%) and  $\tilde{\alpha}_{cpm}$  (2.58%) are attributed to the **TV- $\alpha$**  strategy, and the second-largest to the **TV-ALL** strategy, with  $\alpha_{cpm} = 2.30\%$  and  $\tilde{\alpha}_{cpm} = 2.15\%$ /month. This evidence is important because recent work (e.g., Lettau and Ludvigson, 2001) argues that versions of the CAPM with time varying risk premia are reasonably robust in explaining the cross-sectional dispersion in expected returns. Here, we show that time varying alphas are economically significant, thereby providing evidence against the conditional CAPM.

#### 4.2. Associating investments that condition on macro variables with size, value, and momentum effects

This section examines the source of investment profitability by associating strategies that condition on macro variables with the size, value, and momentum effects. In particular, we ask two questions:

1. Do strategies with time varying parameters perform better than static and dynamic investments in the size, book-to-market, and momentum benchmarks?
2. Do such strategies perform better than portfolios with the same (potentially time varying) size, book-to-market, and momentum characteristics?

By answering these questions we hope to provide insights about the stock-picking ability of the trading strategies over the different cycles of the economy. In asking the first question, we take the view that size, book-to-market, and momentum are common factors. Indeed, while the size and book-to-market effects are empirically driven, the Fama and French (1993) model has become a standard in empirical asset pricing. We also control for the winner-minus-loser (WML) factor because momentum adjustment has been widely used in performance evaluation and we would like to make sure our trading strategies do not merely rediscover momentum in stock returns.<sup>5</sup> In asking the second question, we take the view that equity returns may not be driven by risk factors but instead by the size, book-to-market, and momentum characteristics. Here, we follow Daniel, Grinblatt, Titman, and Wermers (1997) who adjust mutual fund returns by the same equity characteristics.

Table 3 sheds light on these two questions. The parameter  $\alpha_{ff}$  ( $\tilde{\alpha}_{ff}$ ) is the intercept obtained by regressing excess realized investment returns on the three Fama-French benchmarks when factor loadings are constant (when factor loadings vary with the dividend yield, the term spread, the default spread, and the Treasury bill yield),  $\alpha_{ffwml}$  and  $\tilde{\alpha}_{ffwml}$  are the corresponding intercepts in the regressions of excess realized returns on the Fama-French benchmarks as well as WML, and CS is the Daniel, Grinblatt, Titman, and Wermers (1997) selectivity measure. A CS measure of zero indicates that the performance of the underlying strategy could have been replicated by simply purchasing stocks with the same size, book-to-market, and momentum characteristics.

<sup>5</sup>That the WML adequately controls for momentum has been documented by Carhart (1997), who shows that the Hendricks, Patel, and Zeckhauser (1993) persistence among winners completely disappears upon controlling for the past 1-year returns.

Table 3

Exploring size, value, and momentum effects in trading strategies that condition on macro variables

The table reports performance measures that adjust the investment returns, using both risk-based and characteristics-based methods, to potential size, value, and momentum effects in stock returns. The measures are as follows.  $\alpha_{ff}$  ( $\tilde{\alpha}_{ff}$ ) is the intercept that obtains by regressing excess realized investment returns on the three Fama-French benchmarks when benchmark loadings are constant (benchmark loadings vary with the dividend yield, the term spread, the default spread, and the T-bill yield).  $\alpha_{ffwml}$  and  $\tilde{\alpha}_{ffwml}$  are the corresponding intercepts in regressions of excess investment returns on both the Fama-French and WML benchmarks.  $CS$  is the time-series average of  $CS_t$ , where  $CS_t$  is the difference between the return realized on an active investment strategy made at time  $t - 1$  and a return on a passive portfolio that holds stocks with the same size, book-to-market, and momentum characteristics as the active investment.  $SE(x)$  reports the standard error of  $x$ .

	IID		TV- $\beta$		TV-RP		TV- $\alpha$	TV-All	
	UN	RE	UN	RE	UN	RE	UN	UN	RE
<i>Panel A: 08/72-12/03</i>									
$\alpha_{ff}$	0.75	-0.08	0.32	-0.11	1.73	0.23	1.57	1.88	1.52
$SE(\alpha_{ff})$	0.46	0.07	0.42	0.07	0.61	0.52	0.71	0.65	0.64
$\alpha_{ffwml}$	0.48	0.05	0.12	0.02	1.62	0.93	1.01	1.32	2.08
$SE(\alpha_{ffwml})$	0.47	0.06	0.44	0.06	0.62	0.51	0.72	0.66	0.65
$\tilde{\alpha}_{ff}$	0.70	-0.03	0.34	-0.06	1.24	-0.35	1.77	1.89	1.03
$SE(\tilde{\alpha}_{ff})$	0.43	0.05	0.40	0.05	0.56	0.36	0.71	0.65	0.60
$\tilde{\alpha}_{ffwml}$	0.41	0.09	0.03	0.07	0.71	-0.08	1.15	1.17	1.13
$SE(\tilde{\alpha}_{ffwml})$	0.44	0.05	0.41	0.05	0.57	0.35	0.73	0.66	0.61
$CS$	0.71	0.01	0.11	-0.04	1.30	0.31	1.17	1.14	1.36
$SE(CS)$	0.44	0.13	0.43	0.11	0.53	0.22	0.71	0.63	0.59
<i>Panel B: 08/72-12/87</i>									
$\alpha_{ff}$	0.39	-0.06	-0.17	-0.09	2.43	0.30	1.49	2.34	2.11
$SE(\alpha_{ff})$	0.69	0.09	0.59	0.09	1.04	0.88	1.11	1.05	0.90
$\alpha_{ffwml}$	-0.16	0.03	-0.55	-0.02	1.72	0.88	0.56	1.63	1.85
$SE(\alpha_{ffwml})$	0.70	0.09	0.60	0.09	1.06	0.90	1.12	1.07	0.93
$\tilde{\alpha}_{ff}$	0.74	0.03	-0.26	0.01	0.84	-0.91	1.32	1.87	1.22
$SE(\tilde{\alpha}_{ff})$	0.68	0.07	0.62	0.07	1.01	0.67	1.19	1.12	0.93
$\tilde{\alpha}_{ffwml}$	0.19	0.10	-0.29	0.09	0.31	-0.59	0.52	1.21	0.79
$SE(\tilde{\alpha}_{ffwml})$	0.66	0.07	0.63	0.07	0.99	0.68	1.22	1.14	0.95
$CS$	0.30	0.04	-0.65	0.00	1.50	0.29	0.47	0.60	1.77
$SE(CS)$	0.63	0.10	0.60	0.09	0.83	0.32	1.04	0.95	0.76
<i>Panel C: 01/88-12/03</i>									
$\alpha_{ff}$	1.58	-0.05	1.23	-0.10	1.17	0.07	1.80	1.65	0.65
$SE(\alpha_{ff})$	0.57	0.09	0.56	0.09	0.62	0.55	0.87	0.76	0.89
$\alpha_{ffwml}$	1.42	0.11	1.25	0.07	1.63	1.01	1.51	1.18	1.85
$SE(\alpha_{ffwml})$	0.59	0.08	0.58	0.08	0.63	0.50	0.90	0.77	0.86
$\tilde{\alpha}_{ff}$	1.67	-0.08	1.49	-0.08	1.39	0.24	2.11	1.42	0.67
$SE(\tilde{\alpha}_{ff})$	0.56	0.06	0.54	0.07	0.60	0.32	0.89	0.79	0.82
$\tilde{\alpha}_{ffwml}$	1.58	0.05	1.26	0.08	1.44	0.34	1.98	0.82	1.27
$SE(\tilde{\alpha}_{ffwml})$	0.57	0.05	0.55	0.06	0.60	0.27	0.92	0.80	0.83
$CS$	1.16	-0.02	0.79	-0.08	1.12	0.32	1.81	1.63	0.99
$SE(CS)$	0.61	0.23	0.60	0.20	0.68	0.30	0.96	0.85	0.90

Observe from Panel A of Table 3 that the strategies **TV-RP**, **TV- $\alpha$** , and **TV-All** generate positive and significant alphas when investment returns are adjusted by the Fama-French benchmarks with constant factor loadings. The  $\alpha_{ff}$  measures of these strategies are 1.73%,

1.57%, and 1.88%/month, respectively, all of which are significant at conventional levels. Adding WML to adjust investment returns substantially diminishes performance of the **TV- $\alpha$**  and **TV-All** strategies, suggesting that optimal portfolios that incorporate alpha variations pick higher prior-year return stocks. However, momentum adjustment does not eliminate the profitability of strategies with time varying parameters. Note that the  $\alpha_{ffwml}$  [ $SE(\alpha_{ffwml})$ ] measures for the **TV-RP**, **TV- $\alpha$** , and **TV-All** strategies are 1.62% [0.62], 1.01% [0.72], and 1.32% [0.66], respectively. Thus, the strategies that use macroeconomic variables provide predictions about the cross-section of expected returns over and above those already contained in the size, book-to-market, and momentum benchmarks.

Next, we adjust the investment returns by scaling factor loadings by the macroeconomic predictors. Such a beta scaling procedure is economically meaningful in that it captures benchmark timing strategies. In particular, the performance regressions are run not only on the  $K$  factors, as is the case with constant betas, but on the  $K$  factors as well as  $KM$  scaled factors ( $f_t \otimes z_{t-1}$ ). Essentially, we compare the performance of our proposed optimal portfolio strategies with a dynamic investment that times the benchmark assets based on the most recent realizations of the predictors. Then, an optimal portfolio strategy produces a positive alpha if and only if it outperforms a dynamic investment in the benchmarks with the same time varying risk exposures.

Such a beta scaling procedure has been implemented in evaluating the performance of mutual funds, following the findings of Dybvig and Ross (1985) and Grinblatt and Titman (1989). Both studies recognize that identifying skilled managers in an unconditional framework may be problematic; for example, negative performance may be inferred even when a manager successfully implements market timing strategies. In the context of our paper, we note that a priori one cannot tell what would be the impact of scaling beta on the ex post alpha. That is, when the true return generating process is unknown, as is the case in empirical research, implementing benchmark timing strategies may appear appealing ex ante even when doing so is the wrong path to follow ex post. Below, we show that scaling beta can either increase or decrease alpha.

Specifically, the profitability of **TV-RP**, **TV- $\alpha$** , and **TV-All** is typically robust to adjusting risk by the Fama-French benchmarks with time varying benchmark loadings. For these strategies, the  $\tilde{\alpha}_{ff}$  measures are, respectively, 1.24%, 1.77% and 1.89%/month over the August 1972–December 2003 investment period (Panel A), 0.84%, 1.32% and 1.87% over the August 1972–December 1987 period (Panel B), and 1.39%, 2.11%, and 1.42% over the January 1988–December 2003 period (Panel C). All estimates in Panels A and C are significant at conventional levels. That is, portfolio strategies with time varying parameters outperform timing strategies among the Fama-French benchmarks. At the same time, based on Panel A, none of the trading strategies with time varying parameters outperform (at the 5% level) dynamic investments in the Fama-French and WML benchmarks. In particular, the **TV-RP**, **TV- $\alpha$** , and **TV-All** strategies generate  $t$ -statistics on  $\tilde{\alpha}_{ffwml}$  of 1.24, 1.58, and 1.77, respectively.

Still, portfolio strategies with time varying parameters have a lot to offer. First, focusing on the January 1988 through December 2003 out-of-sample period, the **TV- $\alpha$**  strategy generates remarkable performance, summarized by  $\tilde{\alpha}_{ffwml} = 1.98\%$ /month, which is significant at the 1% level. Second, the strategies **TV-RP**, **TV- $\alpha$** , and **TV-All** consistently produce positive alphas even when they need not be statistically significant at the 5% level for all subperiods. Third, strategies based on time varying parameters perform significantly better than portfolios with the same time varying size, book-to-market, and momentum

Table 4

Time-series average of value-weighted quintile scores for size, book-to-market, and past 1-year return of trading strategies

The table reports the time-series average of the cross-section-weighted (based on portfolio weights) quintile scores (1 is the lowest, 5 is the highest) for the size, book-to-market, and past 1-year return for all portfolio strategies described earlier. The last column (EW) reports the equal-weighted average numerical value of the corresponding characteristics with size in billions of dollars and momentum, measured by the past 12-month returns, in percent. Panel A presents scores for portfolios that realize returns over the August 1972–December 2003 period. Panel B focuses on the August 1972–December 1987 subperiod. Panel C focuses on the January 1988–December 2003 subperiod. Panels D and E report the same characteristics under NBER-designated expansions and recessions and Panel F reports the difference (in italics when statistically significant at the 1% level) between the scores in Panels D and E.

	IID		TV- $\beta$		TV-RP		TV- $\alpha$	TV-All		EW
	UN	RE	UN	RE	UN	RE	UN	UN	RE	
<i>Panel A: 08/72-12/03</i>										
Size	3.31	3.32	3.08	3.36	2.97	2.63	2.36	2.33	2.23	2.96
Book-to-market ratio	1.55	2.95	1.57	2.91	1.84	2.97	2.87	2.86	3.15	2.25
Momentum	4.04	3.04	3.83	3.05	3.92	2.93	4.23	4.17	2.92	12.88
<i>Panel B: 08/72-12/87</i>										
Size	3.31	3.23	3.03	3.30	2.85	2.74	2.40	2.35	2.41	1.13
Book-to-market ratio	1.44	3.01	1.43	2.97	1.87	2.85	2.74	2.69	3.08	1.19
Momentum	4.14	3.03	3.77	3.03	4.15	3.26	4.07	4.13	3.28	14.61
<i>Panel C: 01/88-12/03</i>										
Size	3.32	3.40	3.12	3.43	3.08	2.53	2.33	2.32	2.06	4.72
Book-to-market ratio	1.65	2.90	1.70	2.85	1.80	3.08	3.00	3.02	3.22	1.30
Momentum	3.96	3.04	3.89	3.06	3.71	2.61	4.38	4.20	2.58	11.21
<i>Panel D: NBER-designated expansions</i>										
Size	3.29	3.32	3.07	3.38	2.96	2.60	2.40	2.37	2.19	3.09
Book-to-market ratio	1.57	2.95	1.59	2.90	1.82	3.02	2.91	2.90	3.20	1.25
Momentum	4.08	3.04	3.92	3.06	3.95	2.90	4.34	4.27	2.93	15.19
<i>Panel E: NBER-designated recessions</i>										
Size	3.43	3.28	3.12	3.27	3.05	2.81	2.14	2.11	2.45	2.15
Book-to-market ratio	1.43	2.96	1.48	2.97	1.91	2.62	2.63	2.61	2.85	2.22
Momentum	3.83	3.04	3.29	2.95	3.75	3.13	3.55	3.55	2.90	-0.93
<i>Panel F: Difference</i>										
Size	-0.14	0.04	-0.05	<i>0.11</i>	-0.09	-0.21	<i>0.26</i>	<i>0.26</i>	<i>-0.26</i>	
Book-to-market ratio	0.14	-0.01	0.11	<i>-0.07</i>	-0.09	<i>0.40</i>	0.28	0.29	<i>0.35</i>	
Momentum	0.25	0.00	<i>0.63</i>	<i>0.11</i>	0.20	-0.23	<i>0.79</i>	<i>0.72</i>	0.03	

characteristics. For instance, focusing on the January 1988 through December 2003 out-of-sample period, the TV- $\alpha$  and TV-All strategies produce large and statistically significant CS measures of 1.81% and 1.63%/month, respectively.

The evidence shows that strategies that condition on macro variables outperform passive investments in stocks with the same (potentially time varying) characteristics. Still, it is still interesting to examine whether such strategies load on investment styles and whether they switch styles over the business cycle. We turn to this question next.

Table 4 reports the time-series averages of the cross-section-weighted (based on optimal portfolio weights) quintile scores (1 is the lowest, 5 is the highest) for the size,

book-to-market, and past 1-year return. To provide some perspective about the numerical values of size, book-to-market, and momentum, in the last column we report the equal-weighted average of these characteristics across all investable stocks. Panel A presents scores for portfolios that realize returns over the August 1972–December 2003 period. Panel B focuses on the August 1972–December 1987 subperiod, and Panel C focuses on the January 1988–December 2003 subperiod. Panels D and E report the same characteristics under NBER-designated expansions and recessions. Panel F reports the difference (in bold when statistically significant at the 1% level) between the scores in Panels D and E.

Of course, a quintile score higher (lower) than 3 suggests that the underlying strategy over (under) weights towards the characteristic being examined. The **TV-RP**, **TV- $\alpha$** , and **TV-All** strategies invest in higher prior year return stocks but less so over recessionary periods, at least when alpha is allowed to vary. This is consistent with momentum timing because Chordia and Shivakumar (2002) find that the strategy of buying winners and selling losers is especially profitable over expansions. In addition, the **TV-RP**, **TV- $\alpha$** , and **TV-All** strategies tend to hold lower-than-average book-to-market and size stocks. When alpha may vary, the underlying strategy invests in even smaller stocks moving from expansions to recessions. Moving to the **IID** strategy, the evidence shows no style rotation over the business cycle.

To further understand the association between the predictability-based strategies and the size, value, and momentum effects, we study the ex ante alpha and beta functions of the return generating process. This experiment may explain why the strategies select the styles they do and why they switch styles over the business cycle. Specifically, we examine the cross-section dependence of the stock-level alpha and beta functions on size, book-to-market, and past 1-year return.

To illustrate, consider the **TV-All** strategy. For each of the 377 investment months we consider, we first run time-series regressions of the excess return ( $r_{it}$ ) on every stock belonging to the investment universe on the market factor ( $r_{mt}$ ) as follows:

$$\begin{aligned}
 r_{it} = & \underbrace{\alpha_{i0} + \alpha_{it-1,Div}Div_{t-1} + \alpha_{it-1,Def}Def_{t-1} + \alpha_{it-1,Trm}Trm_{t-1} + \alpha_{it-1,Tbl}Tbl_{t-1}}_{\alpha_{it-1}} \\
 & + \underbrace{(\beta_{i0} + \beta_{it-1,Div}Div_{t-1} + \beta_{it-1,Def}Def_{t-1} + \beta_{it-1,Trm}Trm_{t-1} + \beta_{it-1,Tbl}Tbl_{t-1})}_{\beta_{it-1}} r_{mt},
 \end{aligned}
 \tag{17}$$

where *Div*, *Def*, *Trm*, and *Tbl* are the dividend yield, the default spread, the term spread, and the Treasury yield, respectively. This regression is the same as that used to generate the predictive moments for the **TV-ALL** strategy. We then run cross-section regressions of the estimated  $\alpha_{it-1}$  and  $\beta_{it-1}$  on the time  $t - 1$  size, book-to-market, and past 1-year return. Table 5 reports the Fama and MacBeth (1973) regression coefficients along with standard errors that are corrected for serial correlation. We multiply coefficient estimates of size by 10,000, and those of book-to-market and momentum by 100. Before we pursue the analysis let us emphasize that the estimates in Table 5 are based on the entire universe of investable stocks whereas Table 4 focuses on the stocks ultimately picked by the trading strategy.

Table 5 shows that the coefficients on size and book-to-market are insignificant (significant) when the dependent variable is alpha (beta), while the reverse is true for the past 1-year return. This suggests that size and book-to-market capture the impact of risk

Table 5

Understanding the ex ante alpha and beta functions

Panel A reports the time-series average and standard error (adjusted for serial correlation) of coefficients in monthly cross-section regressions of  $\alpha$  and  $\beta$  of all investable stocks on the stock-level size (SIZE), book-to-market (BM), and past one-year return (MOM). Panel B (C) displays the same quantities for NBER-designated expansions (recessions). Panel D examines the significance of the difference between expansions and recessions. The SIZE, BM, and MOM coefficients are multiplied by 10,000, 100, and 100, respectively.

	TV-RP			TV- $\alpha$			TV-All		
	SIZE	BM	MOM	SIZE	BM	MOM	SIZE	BM	MOM
<i>Panel A: 08/72-12/03</i>									
$\alpha$	-0.21	-0.06	0.62	-0.14	-0.01	3.11	-0.80	-0.04	3.07
	0.28	0.04	0.05	0.75	0.01	0.25	0.78	0.03	0.28
$\beta$	-101.44	-0.84	-6.50	-95.00	-0.82	-5.90	-33.30	-1.30	-7.21
	34.18	0.35	5.35	34.53	0.46	5.42	27.70	0.93	9.23
<i>Panel B: NBER-designated expansions</i>									
$\alpha$	-0.15	-0.05	0.62	0.57	-0.01	3.14	-0.06	-0.03	3.13
	0.33	0.02	0.05	0.59	0.01	0.23	0.69	0.03	0.25
$\beta$	-87.02	-0.80	-2.40	-80.92	-0.84	-2.61	-25.91	-1.42	-1.59
	29.77	0.37	5.03	30.79	0.48	5.42	21.08	1.00	8.50
<i>Panel C: NBER-designated recessions</i>									
$\alpha$	-0.55	-0.13	0.57	-4.38	0.00	2.97	-5.26	-0.06	2.70
	0.37	0.06	0.10	2.50	0.02	0.99	2.32	0.06	0.94
$\beta$	-187.67	-1.07	-31.00	-179.22	-0.70	-25.60	-77.50	-0.59	-40.83
	59.88	0.59	7.47	65.06	0.87	8.85	49.89	1.81	15.53
<i>Panel D: Difference</i>									
$\alpha$	0.40	0.08	0.06	4.95	-0.01	0.17	4.95	0.03	0.42
	0.12	0.02	0.04	0.60	0.02	0.22	0.60	0.02	0.23
$\beta$	100.65	0.27	28.59	98.31	-0.14	22.99	98.31	-0.83	39.24
	13.44	0.22	4.14	13.60	0.24	4.07	13.60	0.82	6.58

and thus are insignificantly related to the risk-adjusted return, while momentum is unrelated to risk and is significantly related to the risk-adjusted return. In addition, the magnitude and significance with which these characteristics impact investment opportunities display nontrivial sensitivity to the state of the economy. For instance, alpha and beta are both significantly negatively related to size only during recessions, which may explain why the strategies that use macroeconomic variables load on even smaller stocks over recessions. The book-to-market impact on alpha and beta does not seem to differ in moving from expansions to recessions, which may explain why the investment strategies do not switch the book-to-market style over the economic cycles.

The past 1-year return positively (negatively) affects  $\alpha$  ( $\beta$ ) across the three models we examine. In expansionary periods, a higher past-year return translates into a higher expected return with no additional risk ( $\alpha$  is positive and significant and  $\beta$  is insignificant). In recessionary periods, the momentum effect on  $\alpha$  seems to decline, as the coefficient estimate is smaller and its standard error is larger, and a higher past-year return translates into smaller  $\beta$ . Ultimately, conditioning investments on the past 1-year return seems less desirable in recessions, which explains the diminishing momentum score in moving from expansions to recessions, as Table 4 documents.

In sum, modeling stock returns as in Eq. (6) is ex ante statistically appealing and ex post economically profitable. This specification delivers ex ante predictions about risk and return that guide optimizing investors to hold small-cap, growth, and momentum stocks and to load less heavily (more heavily) on momentum (small cap) stocks over recessions. Such predictions turn out to be robust ex post, as indicated by our comprehensive set of performance measures. Indeed, some of the investment profitability of strategies that use macro predictors is attributable to the size, value, and momentum effects. Take, for example, the **TV-All** strategy. Over the entire investment period, it generates an excess realized return ( $\mu$ ) of 3.16%/month. When investment returns are adjusted by the Fama-French and momentum factors, the residual performance ( $\alpha_{ffwmt}$ ) is 1.32%. The characteristic-based residual performance ( $CS$ ) is 1.14%. Still, some residual performance is unexplained by the size, book-to-market, and momentum effects. This suggests that the return generating process in (6) may capture missing or unknown determinants of returns, either risk factors or equity characteristics. For instance, alpha variation implies that the conditional CAPM is inadequate to explain the cross-section of returns. In addition, in unreported tests, we confirm alpha variation when the Fama-French and momentum factors are considered in forming optimal portfolios. Thus, either there is systematic asset mispricing that varies with the business cycle,<sup>6</sup> or there is no such mispricing if one considers the true, yet unknown, set of determinants.

#### 4.3. The impact of transaction costs and survivorship and delisting biases

Incorporating business cycle predictors enhances investment performance in a framework that is reasonably robust to data mining, finite sample biases, and weak out-of-sample performance of predictive regressions. In pre-discovery periods, the profitability of predictability-based strategies is primarily attributable to the time varying market premium. In post-discovery periods, the relation between the market premium and business cycle variables attenuates or even disappears. Even so, investment strategies that exploit predictability still outperform because alpha and beta vary with business cycle variables. These findings reinforce the merits of studying firm-level predictability: returns on individual stocks are predictable in real time even when market premium predictability, which is the major focus of previous work, is questionable.

*Transaction costs:* Thus far, we do not consider the impact of transaction costs. From a practical investment perspective, however, such an impact could be important. Berkowitz, Logue, and Noser (1988) estimate the one-way transaction cost to be around 23 basis points for institutional investors in NYSE stocks. This cost could be even smaller in the context of our work since we eliminate the smallest quartile of stocks from the investment universe. To gauge the economic gain of predictability under reasonable transaction costs, we compute turnover measures for all dynamic trading strategies we consider here. We find that transaction costs associated with predictability-based strategies are much smaller than the alphas. Consider for example, the strategy that incorporates time varying alpha, beta, and market premium in Panel C of Table 2. The average turnover associated with this strategy is about 70%. Using 50 basis points as a roundtrip transaction cost, which is an inflated measure, the turnover generates a 35 basis-point deduction from risk-adjusted returns, considerably less than  $\alpha_{cpm}$  (2.30%),  $\tilde{\alpha}_{cpm}$  (2.15%), and  $\alpha_{ff}$  (1.65%).

<sup>6</sup>See Modigliani and Cohn (1979) for a possible source of such mispricing.

*Potential survivorship and delisting biases:* The analysis requires 61-month returns/stock, otherwise, the stock is excluded from the investment universe. Does this requirement induce survival bias? The issue of survivorship is extensively studied in the context of evaluating and investing in mutual funds. A relevant question in the context of our work is: Should performance measures be adjusted to reflect the 61-month return requirement? Below we explain why the answer is no. In particular, in the spirit of Baks, Metrick, and Wachter (2001) and Pastor and Stambaugh (2002), first we assume that conditional on the realized stock returns the probability of survival is unaffected by conditioning on the true values of the parameters that govern the evolution of stock returns. Then, by implementing Bayes rule and by assuming that the residual in Eq. (1) is uncorrelated across stocks, we find that conditioning on survival has no effect on the posterior distribution of the model parameters. Hence, any survivorship-based adjustment to the reported performance measures is not needed.

Note, however, that in a mutual fund context, Stambaugh (2003) and Jones and Shanken (2004) explore survival issues in a framework that accounts for prior dependence across funds. Under prior dependence, conditioning on survival can affect the posterior distribution of the parameters, and hence can affect the predictive distribution of future stock returns. In the context of our study one can entertain prior dependence across stocks. For instance, stocks that belong to the same industry or stocks that have the same size, book-to-market, and lagged 1-year return may be assumed a priori to have some commonalities. While we do not examine this relation here, modeling and examining prior dependence across stocks in an investment context represents an opportunity for future work (see also Jones and Shanken, 2004).

Delisting bias also does not effect any of our results. Shumway (1997) suggests that the main problem associated with delistings occurs with respect to performance-related delistings, with delisting returns almost always collected for other types of delistings. Performance-related delistings are not problematic for our study for two reasons. First, subsequent to the Shumway (1997) study, CRSP has added many delisting returns. [See also footnote 7 in Shumway (1997).] Second, and more importantly, we restrict our sample to the largest 75% of NYSE- and AMEX-listed stocks by market capitalization, which eliminates all performance-related delistings in our sample because it is the smaller stocks that delist due to performance reasons.

#### 4.4. Comparison with related work

Our findings are fully consistent with Schwert (2003), who shows that the relation between the aggregate dividend yield and future return is much weaker statistically and economically after the discovery of the business cycle variables by Keim and Stambaugh (1986) and Fama and French (1989). As noted earlier, we demonstrate that the relation between the four business cycle predictors and the equity premium is strong over the pre-discovery period (see **TV-RP** in Panel B of Table 2), but substantially weaker over the post-discovery period (see **TV-RP** in Panel C of Table 2). Moreover, related work studying predictability, including Bossaerts and Hillion (1999), Goyal and Welch (2003), and Schwert (2003), does not consider firm-level predictability. Rather, these studies attempt to forecast the equity premium only, that is, they do not account for potential time varying alpha and beta. In contrast, we study firm-level predictability, whereby alpha and beta could vary with the predictive variables, and show that even when equity premium

predictability is questionable in the post-discovery period, returns on individual stocks are predictable in real time.

Our paper also relates to Cooper, Gutierrez, and Marcum (2001), who demonstrate that a hypothetical investor using firm size, book-to-market, and the 1-year prior return does not beat the market in real time. We do not explicitly condition on such variables. However, we do show that integrating alpha variation with macroeconomic predictors generates outperforming portfolios that invest in small-market cap, growth, and momentum stocks. Thus, firm-level variables potentially give guidance on profitable trading strategies. Indeed, one could formally account for firm-level size, book-to-market, and momentum in forming optimal portfolios. Potentially, the outcome could support real-time predictability based on such variables.

Note that a decision making-based metric in economics and finance has been suggested, among others, by McCulloch and Rossi (1990), Leitch and Tanner (1991), and West, Edison, and Cho (1993). McCulloch and Rossi (1990) show that a utility-based metric used to test the arbitrage pricing theory provides markedly different conclusions relative to those that follow from studies that employ traditional significant measures. Leitch and Tanner (1991) demonstrate that summary statistics including mean-squared errors may not be closely related to a forecast's profits. This may explain why profit-maximizing firms buy professional forecasts even when statistics such as the root-mean-squared error often indicate that a naive model performs about as well. West, Edison, and Cho (1993) note that a utility-based criterion is fundamentally different from statistical ones based on mean-squared errors.

Last, it is worth noting that Kandel and Stambaugh (1996) recognize that weak statistical evidence on predictability based on R-squared is consistent with the ability of the current values of predictive variables to exert substantial influence on asset allocation. While the Kandel-Stambaugh point is made from an *ex ante* perspective, here we show that this point is valid from an out-of-sample perspective as well.

## 5. Conclusions

A long-standing question in financial economics is whether future stock returns can be predicted based on public information. Keim and Stambaugh (1986) and Fama and French (1989) identify macroeconomic variables such as the aggregate dividend yield, the term spread, and the default spread that explain a substantial portion of future return variations. Basu (1977), Banz (1981), Jegadeesh (1990), Fama and French (1992), and Jegadeesh and Titman (1993) demonstrate the predictive power of the firm-level size, book-to-market, and prior return measures.

However, predictability remains a focus of research controversy because of (i) concerns about data mining, (ii) concerns that the statistical methods implemented to explore predictability indicate spurious results, (iii) concerns about the large gap between the in-sample evidence of predictability and the real-time performance of active investment management, and (iv) concerns about the poor out-of-sample performance of predictive regressions.

This paper studies whether incorporating business cycle predictors benefits a real time optimizing investor who must allocate funds across 3,123 NYSE- and AMEX-listed stocks and the risk-free asset over the 1972–2003 period. The proposed framework is quite general in that it allows for predictability in stock-level alphas and betas as well as the

equity premium, it allows for investments whose histories differ in length, and it derives closed-form expressions for moments used in portfolio selection with estimation risk. We implement several investment strategies and examine their ex post out-of-sample performance using a comprehensive set of evaluation measures.

We find that returns are predictable out-of-sample by the dividend yield, the term spread, the default spread, and the Treasury bill yield. Predictability-based investments outperform static and dynamic investments in the market, the Fama-French, and the Fama-French plus momentum factors, as well as strategies that invest in stocks with the same (potentially) time varying size, book-to-market, and momentum characteristics.

The outperforming strategies hold small-cap, growth, and momentum stocks. Moreover, such strategies change their investment styles over the business cycle. Over recessions they load less heavily on momentum stocks and more heavily on small-cap stocks. In contrast, portfolio strategies that rule out predictability do not switch investment styles over the business cycle and ultimately deliver dismal performance.

In the period prior to the discovery of the business cycle variables by Keim and Stambaugh (1986) and Fama and French (1989), the predictable equity premium is the major determinant of investment profitability. In the post-discovery period, performance of asset allocation conditioned on predictability in the equity premium deteriorates. Even so, incorporating business cycle variables is beneficial to a real time investor because firm-level alpha varies with such variables. Hence, returns on individual stocks are predictable in real time even when equity premium predictability, the major focus of previous work, is weak over the post-discovery period.

To summarize, earlier work extensively studies equity premium predictability and demonstrates virtually no out-of-sample predictability. This paper shows that the focus on the equity premium fails to demonstrate the evidence on stock-level predictability. In particular, returns are predictable out-of-sample in a framework that incorporates alpha, beta, and risk premium variations with the dividend yield, the default spread, the term spread, and the Treasury bill yield, and that allows long-only positions in stocks. Our findings are based on a comprehensive set of performance evaluation measures. The bottom line is that modeling stock returns with time varying parameters is ex ante statistically appealing and ex post economically profitable.

## Appendix A. Asset allocation conditioned on business cycle variables

Here we derive moments for asset allocation under the case in which alphas, betas, and market premia are all allowed to vary with the macroeconomic variables.

### A.1. The likelihood function

The sample contains  $T_i$  monthly excess return observations on stock  $i$  (there are  $N = 3,123$  NYSE- and AMEX-traded stocks over the August 1962–December 2003 period),  $T$  excess returns on  $K$  benchmark assets, and  $T$  realizations of  $M$  forecasting variables. Stock  $i$  enters the sample at time  $t_i$  (IPO) and leaves at time  $t_i + T_i - 1$  (merger, acquisition, bankruptcy, etc.) or remains till the end-of-sample period. Let us fix some notation. Let  $r_i$  denote the  $T_i$ -vector of excess returns on stock  $i$ , let  $G_i = [G'_{t_i}, \dots, G'_{t_i+T_i-1}]'$ , where  $G_t = [1, z'_{t-1}, f'_{t-1}, f'_{t-1} \otimes z'_{t-1}]'$ , let  $\Gamma_i = [\alpha_{i0}, \alpha'_{i1}, \beta'_{i0}, \beta'_{i1}]'$ , where, for example,  $\beta_{i0}$  is the  $i$ th security version of  $\beta_0$ , let  $Z = [z'_1, \dots, z'_T]'$ , let  $F = [f'_1, \dots, f'_T]'$ , let  $X = [x'_0, \dots, x'_{T-1}]'$ ,

where  $x_0 = [1, z_0]'$ , with  $z_0$  being the first observation of the forecasting variables, let  $V_f = [v'_{f1}, \dots, v'_{fT}]'$ , let  $V_z = [v'_{z1}, \dots, v'_{zT}]'$ , let  $V_{rz}$  be a  $T \times N$  matrix whose  $i$ th column contains  $T_i$  values of  $v_{it}$ , the security-specific residual in regression (1), when returns on security  $i$  are recorded and  $T - T_i$  zeros when such returns are missing, let  $A_Z = [a_z, A_z]'$ , and let  $A_F = [a_f, A_f]'$ . In addition, let  $Q_{G_i} = I_{T_i} - G_i(G'_i G_i)^{-1} G'_i$ , let  $Q_X = I_T - X(X'X)^{-1}X'$ , let  $W_Z = [X, V_f, V_{rz}]$ , and let  $Q_Z = I_T - W_Z(W'_Z W_Z)^{-1}W'_Z$ . The stochastic processes for excess returns, factors, and predictors can be rewritten as  $r_i = G_i \Gamma_i + v_i$ ,  $F = X A_F + v_f$ , and  $Z = X A_Z + v_z$ .

Denoting the set of parameters by  $\Theta$ , the likelihood function can be factored as

$$\mathcal{L}(r_i, \dots, r_N, Z, F | \Theta, z_0) = \left[ \prod_{i=1}^N p(r_i | F, Z, \Gamma_i, \psi_i, z_0) \right] \times p(F | Z, A_F, \Sigma_{ff}, z_0) p(Z | \xi, \Sigma_{zz,rf}, z_0), \tag{18}$$

where  $\xi = [A'_Z, \Sigma_{zf} \Sigma_{ff}^{-1}, \Sigma_{zr} \Psi^{-1}]'$  and  $\Sigma_{zz,rf} = \Sigma_{zz} - \Sigma_{zr} \Psi^{-1} \Sigma_{rz} - \Sigma_{zf} \Sigma_{ff}^{-1} \Sigma_{fz}$ . The likelihood function can then be reexpressed as

$$\begin{aligned} \mathcal{L} \propto & \left[ \prod_{i=1}^N (\psi_i)^{-T_i/2} \exp \left\{ -\frac{1}{2} \psi_i^{-1} (r'_i Q_{G_i} r_i + (\Gamma_i - \hat{\Gamma}_i)' G'_i G_i (\Gamma_i - \hat{\Gamma}_i)) \right\} \right] \\ & \times |\Sigma_{ff}|^{-T/2} \exp \left\{ -\frac{1}{2} \text{tr}[\Sigma_{ff}^{-1} (F' Q_X F + (A_F - \hat{A}_F)' X' X (A_F - \hat{A}_F))] \right\} \\ & \times |\Sigma_{zz,rf}|^{-T/2} \exp \left\{ -\frac{1}{2} \text{tr}[\Sigma_{zz,rf}^{-1} (Z' Q_Z Z + (\xi - \hat{\xi})' W'_Z W_Z (\xi - \hat{\xi}))] \right\}, \end{aligned} \tag{19}$$

where  $\hat{\Gamma}_i = (G'_i G_i)^{-1} G'_i r_i$ ,  $\hat{A}_F = (X'X)^{-1}X'F$ , and  $\hat{\xi} = (W'_Z W_Z)^{-1}W'_Z Z$ . Next, let  $\hat{\Psi}$  be a diagonal matrix whose  $(i, i)$  element is  $\hat{\psi}_i = r'_i Q_{G_i} r_i / T_i$ , let  $\hat{\Sigma}_{ff} = F' Q_X F / T$ , and let  $\hat{\Gamma}$  be the  $N$ -stock version of  $\hat{\Gamma}_i$ . The maximum likelihood estimates of  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_0$ , and  $\beta_1$  are the first column, the next  $M$  columns, the next  $K$  columns, and the last  $KM$  columns of  $\hat{\Gamma}$ , respectively. When estimation and model risks are disregarded in asset allocation decisions, the above-derived maximum likelihood estimates replace the true parameters in Eqs. (7) and (8) to yield the mean and covariance of stock returns. In this paper, all portfolio strategies account for estimation risk (and often for model uncertainty), thus the following analysis presents the impact of estimation risk and model uncertainty on those inputs used for portfolio optimization.

### A.2. The mean and variance of predicted stock returns

In the presence of estimation risk the predictive moments are

$$\mu_T = \hat{\Gamma} [I_{M+1}, \hat{A}_F, \hat{A}_F (I_K \otimes z'_T)]' x_T, \tag{20}$$

$$\Sigma_T = A_T + (1 + \delta_T) \hat{\beta}(z_T) \hat{\Sigma}_{ff} \hat{\beta}(z_T)', \tag{21}$$

where  $\hat{\Sigma}_{ff} = T / (T - M - 2K - 3) \hat{\Sigma}_{ff}$ ,  $A_T$  is an  $N \times N$  diagonal matrix whose  $i$ th entry is  $\hat{\psi}_i (1 + \delta_{iT})$ , and

$$\delta_T = \frac{1}{T} [1 + (\bar{z} - z_T)' \hat{V}_z^{-1} (\bar{z} - z_T)], \tag{22}$$

$$\delta_{iT} = x'_T \Omega_i^{11} x_T + 2x'_T [\Omega_i^{12} + \Omega_i^{13} (I_K \otimes z_T)] \hat{A}'_F x_T + \text{tr}[\hat{A}'_F x_T x'_T \hat{A}_F \tilde{\Omega}_i] + (1 + \delta_T) \text{tr}[\tilde{\Sigma}'_{ff} \tilde{\Omega}_i], \tag{23}$$

$$\tilde{\Omega}_i = \Omega_i^{22} + \Omega_i^{23} (I_K \otimes z_T) + (I_K \otimes z'_T) \Omega_i^{32} + (I_K \otimes z'_T) \Omega_i^{33} (I_K \otimes z_T), \tag{24}$$

$$\tilde{\psi}_i = \frac{T_i}{T_i - M - K - KM - 3} \hat{\psi}_i, \tag{25}$$

with  $\Omega_i^{mm}$  being the following partitions based on  $G_t = [1, z'_{t-1}; f'_t; f'_t \otimes z'_{t-1}]'$ :

$$(G'_i G_i)^{-1} = \begin{pmatrix} \Omega_i^{11} & \Omega_i^{12} & \Omega_i^{13} \\ \Omega_i^{21} & \Omega_i^{22} & \Omega_i^{23} \\ \Omega_i^{31} & \Omega_i^{32} & \Omega_i^{33} \end{pmatrix}. \tag{26}$$

The two additional variance components  $\delta_T$  and  $\delta_{iT}$  are attributed to estimation risk.

**Appendix B. Asset allocation when stock returns are identically and independently distributed**

To describe the maximum likelihood estimates in the i.i.d. case, let the process for excess returns and factors be  $r_t = \alpha + \beta f_t + u_t$  and  $f_t = \mu_f + u_{ft}$ . The corresponding unconditional moments are  $\mu = \alpha + \beta \mu_f$  and  $\Sigma = \beta V_{ff} \beta' + V_{rr}$ , where  $V_{ff}$  and  $V_{rr}$  are the covariance matrices of  $u_{ft}$  and  $u_t$ , respectively. ( $V_{rr}$  is assumed diagonal.) Let  $H_i = [H'_i, \dots, H'_{i+T_i-1}]'$ , where  $H_t = [1, f'_t]'$ , and let  $\Gamma_i = [\alpha_i, \beta'_i]'$ . The parameters  $\Gamma_i$ ,  $\mu_f$ ,  $V_{ff}$ , and  $V^i_{rr}$  are estimated by  $(H'_i H_i)^{-1} H'_i r_i$ ,  $F' \iota_T / T$ ,  $F' Q_Y F / T$ , and  $r'_i Q_{H_i} r_i / T_i$ , respectively, where  $Q_Y = I_T - \iota_T \iota'_T / T$ ,  $Q_{H_i} = I_{T_i} - H_i (H'_i H_i)^{-1} H'_i$ , and  $\iota_T$  is a  $T$ -vector of ones. In addition, let  $\tilde{V}_{ff} = T / (T - 2K - 3) \hat{V}_{ff}$ , and let  $\tilde{V}^i_{rr} = T_i / (T_i - K - 3) \hat{V}^i_{rr}$ . Some algebra shows that the Bayesian predictive variance in the no predictability case is equal to

$$\Sigma_T = B_T + \left(1 + \frac{1}{T}\right) \hat{\beta} \tilde{V}_{ff} \hat{\beta}', \tag{27}$$

where  $B_T$  is an  $N \times N$  diagonal matrix whose  $i$ th entry is  $\tilde{V}^i_{rr} (1 + \tilde{\delta}_{iT})$ , and

$$\tilde{\delta}_{iT} = \Omega_i^{11} + 2\Omega_i^{12} \hat{\mu}_f + \text{tr}[\hat{\mu}_f \hat{\mu}'_f \Omega_i^{22}] + \left(1 + \frac{1}{T}\right) \text{tr}[\tilde{V}_{ff} (\Omega_i^{22})], \tag{28}$$

with  $\Omega_i^{mm}$  being the following partitions:

$$(H'_i H_i)^{-1} = \begin{pmatrix} \Omega_i^{11} & \Omega_i^{12} \\ \Omega_i^{21} & \Omega_i^{22} \end{pmatrix}. \tag{29}$$

**References**

Avramov, D., 2002. Stock return predictability and model uncertainty. *Journal of Financial Economics* 64, 423–458.  
 Avramov, D., 2004. Stock return predictability and asset pricing models. *The Review of Financial Studies* 17, 699–738.

- Avramov, D., Chordia, T., 2006. Asset pricing models and financial model anomalies. *The Review of Financial Studies* 19, 1001–1040.
- Avramov, D., Wermers, R., 2006. Investing in mutual funds when returns are predictable. *Journal of Financial Economics* 81, 339–377.
- Baks, K.P., Metrick, A., Wachter, J., 2001. Should investors avoid all actively managed mutual funds? A study in Bayesian performance evaluation. *Journal of Finance* 56, 45–85.
- Banz, R.W., 1981. The relative efficiency of various portfolios: some further evidence: discussion. *Journal of Finance* 32, 663–682.
- Barberis, N., 2000. Investing for the long run when returns are predictable. *Journal of Finance* 55, 225–264.
- Basu, S., 1977. Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis. *Journal of Finance* 32, 663–682.
- Berkowitz, S.A., Logue, D.E., Noser, E.A., 1988. The total costs of transactions on the NYSE. *Journal of Finance* 43, 97–112.
- Best, M.J., Grauer, R.R., 1991. On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results. *The Review of Financial Studies* 4, 315–342.
- Bossaerts, P., Hillion, P., 1999. Implementing statistical criteria to select return forecasting models: what do we learn? *Review of Financial Studies* 12, 405–428.
- Carhart, M., 1997. On persistence in mutual fund performance. *Journal of Finance* 52, 57–82.
- Chordia, T., Shivakumar, L., 2002. Momentum, business cycle and time-varying expected returns. *Journal of Finance* 57, 985–1019.
- Cooper, M., Gutierrez, R.C., Marcum, W., 2001. On the predictability of stock returns in real time. Unpublished working paper, Purdue University.
- Daniel, K., Grinblatt, M., Titman, S., Wermers, R., 1997. Measuring mutual fund performance with characteristic-based benchmarks. *Journal of Finance* 52, 1035–1058.
- Dybvig, P.H., Ross, S.A., 1985. Differential information and performance measurement using a security market line. *Journal of Finance* 40, 383–399.
- Fama, E.F., French, K.R., 1989. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics* 25, 23–49.
- Fama, E.F., French, K.R., 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427–465.
- Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3–56.
- Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: empirical tests. *Journal of Political Economy* 81, 607–636.
- Ferson, W., Harvey, C., 1991. The variation in economic risk premia. *Journal of Political Economy* 99, 385–415.
- Foster, F.D., Smith, T., Whaley, R., 1997. Assessing the goodness-of-fit of asset pricing models: the distribution of maximal  $R^2$ . *Journal of Finance* 52, 591–607.
- Frost, P., Savarino, J., 1986. An empirical Bayes approach to efficient portfolio selection. *Journal of Financial and Quantitative Analysis* 21, 293–305.
- Goyal, A., Welch, I., 2003. Predicting the equity premium with dividend ratios. *Management Science* 49, 639–654.
- Grinblatt, M., Titman, S., 1989. Mutual fund performance: an analysis of quarterly portfolio holdings. *Journal of Business* 62, 394–415.
- Hendricks, D., Patel, J., Zeckhauser, R., 1993. Hot hands in mutual funds: the persistence of performance, 1974–88. *Journal of Finance* 48, 93–130.
- Jegadeesh, N., 1990. Evidence of predictable behavior in security returns. *Journal of Finance* 45, 881–898.
- Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance* 48, 65–91.
- Jones, C.S., Shanken, J., 2004. Mutual fund performance with learning across assets. *Journal of Financial Economics* 78, 507–552.
- Kandel, S., Stambaugh, R., 1996. On the predictability of stock returns: an asset allocation perspective. *Journal of Finance* 51, 385–424.
- Keim, D., Stambaugh, R.F., 1986. Predicting returns in the stock and the bond markets. *Journal of Financial Economics* 17, 357–390.
- Leitch, G., Tanner, J.E., 1991. Economic forecast evaluation: profits versus the conventional error measures. *American Economic Review* 81, 580–590.
- Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109, 1238–1286.

- Litzenberger, R., Ramaswamy, K., 1979. The effect of personal taxes and dividends on capital asset prices: theory and empirical evidence. *Journal of Financial Economics* 7, 163–196.
- Lo, A.W., 2002. The statistics of Sharpe ratios. *Financial Analyst Journal* 58, 36–52.
- Lo, A.W., MacKinlay, A.C., 1990. Data-snooping biases in tests of financial asset pricing models. *Review of Financial Studies* 3, 431–467.
- McCulloch, R., Rossi, P.E., 1990. Posterior, predictive, and utility-based approaches to testing the arbitrage pricing theory. *Journal of Financial Economics* 28, 7–38.
- Modigliani, F., Cohn, R.A., 1979. Inflation, rational valuation and the market. *Financial Analyst Journal* 35, 24–44.
- Pastor, L., 2000. Portfolio selection and asset pricing models. *Journal of Finance* 55, 179–223.
- Pastor, L., Stambaugh, R., 2000. Comparing asset pricing models: an investment perspective. *Journal of Financial Economics* 56, 335–381.
- Pastor, L., Stambaugh, R., 2002. Investing in equity mutual funds. *Journal of Financial Economics* 63, 351–380.
- Schwert, G.W., 2003. Anomalies and market efficiency. In: G.M. Constantinides, M. Harris, R. Stulz, *Handbook in Economic and Finance*.
- Seber, G.A.F., 1984. *Multivariate Observations*. Wiley, New York.
- Shanken, J., 1990. Intertemporal asset pricing: an empirical investigation. *Journal of Econometrics* 45, 99–120.
- Sharpe, W., 1991. Capital asset prices with and without negative holdings. *Journal of Finance* 46, 489–509.
- Shumway, T., 1997. The delisting bias in CRSP data. *Journal of Finance* 52, 327–340.
- Stambaugh, R., 1997. Analyzing investments whose histories differ in length. *Journal of Financial Economics* 45, 285–331.
- Stambaugh, R., 1999. Predictive regressions. *Journal of Financial Economics* 54, 375–421.
- Stambaugh, R., 2003. Inference about survivors. Unpublished working paper, Wharton.
- Tu, J., Zhou, G., 2004. Data-generating process uncertainty: what difference does it make in portfolio decisions? *Journal of Financial Economics* 72, 385–421.
- West, D.K., Edison, H.J., Cho, D., 1993. A utility-based comparison of some models of exchange rate volatility. *Journal of International Economics* 35, 23–45.
- Zellner, A., Chetty, V.K., 1965. Prediction and decision problem in regression models from the Bayesian point of view. *Journal of the American Statistical Association* 60, 608–615.

## Further Reading

- Chan, K.C., Chen, N.F., 1988. An unconditional asset pricing test and the role of firm size as an instrumental variable for risk. *Journal of Finance* 43, 309–325.
- Jensen, M.C., 1969. Risk, the pricing of capital assets, and the evaluation of investment portfolios. *Journal of Business* 42, 167–247.