Presupposition Projection, Trivalence and Relevance[1]
Danny Fox

1. Goals

1 To argue that facts about presuppositions projection out of quantificational sentences follow from:
   a. trivalent theories of projection (of Peters 1979, Beaver and Krahmer 2001, George 2008, Fox 2008)
   b. two auxiliary mechanisms (local accommodation, presupposition strengthening)

[And on some other occasion maybe
2. To suggest a bivalent method of deriving the trivalent predictions, via a new assertability condition (Relevance, hinted at in Fox 2008).]

2. Projection from the nuclear scope – An Empirical Debate

(1) Some student_1 talked about both of his_1 papers.
Some student $[\text{x talked about both of x's paper}]_{\text{x has (exactly) two papers}}$

(2) Every student_1 talked about both of his_1 papers.
Some student $[\text{x taked about both of x's paper}]_{\text{x has (exactly) two papers}}$

(3) No student_1 talked about both of his_1 papers.
No student $[\text{x taked about both of x's paper}]_{\text{x has (exactly) two papers}}$

(4) **Competing Empirical Claims**:

Universal Projection (Heim 1983): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\forall x(A(x)\rightarrow p(x))$
Existential Projection (Beaver 1992): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\exists x(A(x)\wedge p(x))$
Nuanced Projection (Peters, George, Chemla): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes different things depending on various properties of Q.

[1] This work owes on obvious debt to Schlenker's work on presupposition projection (see Fox 2008). It has also been modified as a result of ongoing work with Yasu Sudo, Jacopo Romoli and Martin Hackl (Sudo, et. al.). Many thanks to Emmanuel Chemla, Paul Egre, Kai von Fintel, Ben George, Irene Heim, Ofra Magidor, Alejandro Pérez Carballo, Raj Singh, Benjamin Spector, Steve Yablo, and especially to Alexandre Cremers and Philippe Schlenker.

3. Some (rather nuanced) Evidence for Nuanced Projection

For existential sentences very few speakers (if any) report a universal inference:

Beaver's test

(5) Half of the ten boys wrote two papers. Furthermore, one of the ten boys is proud of t both of his papers.

By contrast negative existentials (at least for some speakers) lead to a universal inference. (see Chemla for relevant experimental evidence).

Applying Beaver's test:

(6) Half of the ten boys wrote two papers. (#) And/but none of the 10 boys is proud of both of his papers.

The plot thickens in that existential questions seem to lead to a universal inference when embedded in a polar question (Schlenker 2009) – again, at least for some speakers.

(7) Is one of these 10 students [t_1 proud of both his_1 papers].
Leads to a universal inference (for some speakers)

(8) Half of the ten boys wrote two papers. (#) is one/any of the 10 boys proud of both of his papers.

Important Issue to Address: there seem to be quite a bit of speaker variation in judgment.

Desiderata:
(a) to ascertain the range of variation across speakers and to explain what is ascertained.
(b) to ascertain the range of variation across quantificational sentences and to explain what is ascertained.

In any event, there does seem to be some evidence for differences among quantifiers (for some speakers).

But…

4. Charlow's evidence in favor of universal presuppositions

(9) Just five of these 100 boys have smoked in the past. They have all smoked Nelson #Unfortunately, some/at least two of these 100 boys have also smoke Marlboro_F.

Charlow's Conclusion: *also* is a "strong trigger" and reveals the true projection properties which are universal.

For other presupposition triggers there is a method of cancelling presuppositions (called, local accommodation).

Remaining Questions:

-What explains the difference between quantifiers when soft triggers are involved (assuming that such a difference exists)?
-Why is cancellation easier with weak triggers in quantificational sentences than elsewhere (say under negation)?

5. The Tivalent Presuppositions

(10) Trivalent denotation of the nuclear scope in (1)a,b,c:

$$\lambda x.\begin{cases} 1 & \text{if x has a (unique) car and x drives it to school} \\ 0 & \text{if x has a (unique) car and x doesn't drive it to school} \\ \# & \text{if x has no car} \end{cases}$$

(11) **Strong Kleene:**
The denotation of S in w is
(a) 1 if its denotation (in a bivalent system) would be 1 under every bivalent correction of sub-constituents.
(b) 0 if its denotation would be 0 under every bivalent correction of sub-constituents.
(c) # if neither (a) nor (b) hold

(12) a function $g{:}X\rightarrow\{0,1\}$ is a *bivalent correction* of a function $f{:}X\rightarrow\{0,1,\#\}$ if $\forall x[(f(x)=0 \vee f(x)=1)\rightarrow g(x)=f(x)]$

(13) **Stalnaker's Bridging Principle**:
A sentence S is assertable given a context set C only if
$\forall w \in C$ [the denotation of S in w is either 0 or 1].

(1)' Some student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and talked about them] or
[Every student has two papers]

(2)' Every student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and didn't talk about one of them] or

[Every student has two papers]

(3)' No student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and talked about them] or
[Every student has two papers]

6. Arguments for the Trivalent presuppositions

6.1. Collapsing assertion and presupposition

If we were allowed to think of the inference of a sentence as following from the conjunction of assertion and presupposition, we would predict the difference between *some* and *none*, which some speakers report.

(1)' Some student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and talked about them] or
[Every student has two papers]

Asserts:
Some student has two papers and talked about them.

Hence: no universal inference

(3)' No student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and talked about them] or
[Every student has two papers]

Asserts:
It's not the case that some student has two papers and talked about them.

Hence: yes universal inference

Of course, also the universal inference of a universal statement is predicted (though perhaps less surprising)

(2)' Every student [x taked about both of x's paper]$_{\text{x has (exactly) two papers}}$
Presupposes:
Either [Some student has two papers and didn't talk about one of them] or
[Every student has two papers]

Asserts:

Ever student has two papers and talked about them.

Perhaps more surprising: the negation of a universal statement will not be associated with a universal inference

(14) A: There are many students around, hence many cars.
B: No, half of the students don't have a car.
Furthermore, some don't drive their car to school.
Furthermore, not every student drives his car to school.
\# Furthermore, every student leaves his car at home

But:

1. Is it right to collapse assertion and presupposition?
2. What explains variations in judgments?
3. What explains the universal inferences with strong triggers?
4. What explains the universal inference that some speakers get for polar questions?

6.2. The oddness of the presupposition – qua presupposition

(15) Q student [x talked about both of x's papers]$_{\text{x has two papers}}$
Presupposes:
Either [QP_2 has two papers and talked about them] or
[Every student has a car]
(where QP_2 can, though need not, be identical to *Q student*)

Equivalently:
$\neg$[QP_2 has a car and drives it to school] $\rightarrow$
[Every student has a car]

Believing this disjunction prior to assertion would involve believing the disjunction without believing one of the disjuncts. This is odd: it suggests that there is a connection between the two (if one is false, the other is true).

So (as in the discussion of the *proviso problem* to which we will return) this might require **additional mechanisms** (e.g. strengthening)

Hope: discussion of the additional mechanisms would allow us to address are various questions.

Look ahead:

1. Is it right to collapse assertion and presupposition?

There is a strategy of dealing with oddness which (when applied in a particular way) ends up collapsing assertion and presupposition.

2. What explains variations in judgments?
   Different speakers employ different strategies to deal with oddness. Collapsing assertion with presupposition is only one instantiation of one strategy.

3. What explains the universal inferences with strong triggers?
   Strong triggers are limited in the strategies that they allow (in particular they do not allow collapsing of assertion and presupposition).

4. What explains the universal inference that some speakers get for polar questions?
   There is no assertive component (at the matrix level). Hence strategies are limited. In particular collapsing assertion with presupposition is impossible (at the matrix level).

But first we will present on argument based on cases where strategies are not needed, where believing the presupposition (the disjunction) without believing one of the disjuncts is not odd.

6.3. Cases that do not require additional mechanisms.

The disjunctive presupposition seems to be sufficient whenever it is not odd (that is to say, whenever there is a connection between the disjuncts that makes it reasonable to believe the disjunctive presupposition without believing one of the disjuncts)

This observation holds independent of the choice of quantifier or trigger

6.3.1. Existential sentences embedded under polar questions

(16) John and Bill meet for a game of poker. The rules they set for their engagement are the following. They each give Jane 100 dollar and get chips in return. The game will continue until one of them has no more chips left. The moment this happens, the winner (the player that has 200 chips) goes to Jane and cashes his chips.

Fred (who knows the rules of engagement) is responsible for cleaning the room the moment the game is over. He calls Jane and asks one of the following questions:

Did one of the two players cash his chips?

(17) Did anyone of these bankers acquire his fortune by wiping out one of the others?
Presupposition: if none of these bankers acquired his fortune by wiping out one of the others, they all have a fortune.

Confound (Ben George p.c.): nominals can receive temporal interpretations independent of tense, Hence it is not clear that a universal presupposition will be wrong here.

Can be addressed by explicating the temporal interpretation of the nominal:

(18) Did anyone of these bankers acquire the fortune he deposited in the bank last week by wiping out one of the others?
Presupposition: if no banker acquired the fortune he deposited in the bank last week by wiping out one of the others, they each deposited a fortune last week.

(19) Is any one of the two players allowed to cash the chips that he now has in his possession?

Or by cases where the universal presupposition fails to hold independent of time:

(20) Five people started a new company based on a new algorithm that they developed. If none of the partners reveals the algorithm they will each earn millions once the company goes public. If, however, one of the five partners shares the algorithm with Tom – a well known English businessman – before the company goes public, he will be getting millions from Tom but then some of the other four partners will remain very poor.

Will one of the partners get his millions from Tom?

6.3.2. Strong triggers

(21) TV game “diamonds are not enough”: Every week, there are ten contestants and one million dollars to be spent on prizes for the contestants. As in many TV games there are all sorts of ways of scoring points – irrelevant for our purposes.

What is important is that there are two possible outcomes

1. If everyone scores less than 1000 point, the million dollars will be used to purchase 10 diamonds (each for 100K) and each contestant will receive a diamond.
2. Otherwise, the top scoring contestant (the winner) will receive 500K and the 5 highest scoring contestants (including the winner) will each receive a (100K) diamond.

Every week at least 5 of the ten contestants gets a diamond. I bet that this week one of the 10 contestant will also get 500 K.

Every week at least 5 of the ten contestants gets a diamond. I wonder if this week one of the 10 contestant will also get 500 K.

6.3.3. Universal sentences embedded under polar questions

(22) John got money gambling in the race track. If he invests his money wisely he will be able to pay Bill (who will have no money otherwise.) If Bill has money and intvests it wisely he will be able to pay Fred (who will have no money otherwise). If Fred has money and invests it wisely he will be able to pay me and otherwise I will have no money at all.

So it matters to me quite a bit whether each of the three fellows will invest his money wisely.

6.4. Interim Summary

- The disjunctive presupposition predicted by trivalent theories is odd in most cases.
-One way to get rid of oddness is to collapse assertion and presupposition. Then we get a distinction between quantifiers in whether or not a universal inference is predicted – a distinction that conforms to the intuitions of some speakers (or at least so it seems informally).
- In the few cases where the disjunctive presupposition is not odd, it seems appropriate for all speakers and all determiners (or at least that's my hope).

Remaining Question: what explains variation in judgments?
Answer we will be giving: there are other strategies to deal with oddness.

7. Additional Mechanisms

Consider again the disjunctive presupposition?

(15) Q student [x talked about both of x's papers]$_{\text{x has two papers}}$
Presupposes:
Either [QP_2 has two papers and talked about them] or
[Every student has a car]
(where QP_2 can, though need not, be identical to *Q student*)

Equivalently:
¬[QP_2 has a car and drives it to school] →
[Every student has a car]

Believing this disjunction without believing one of the disjuncts is odd.

Proposal (in a sense a generalization of Charlow's idea within a trivalent setup): there are two possible mechanisms that can save us from an odd presupposition:

1. Presupposition strengthening (sometimes called global accommodation)
2. Collapsing assertion and presuppositions at various levels (local accommodation).

But as a build up for this, I will begin by introducing the assumptions about architecture that would allow us to collapse assertion and presuppositions at the matrix level (as we did in 6.1.)

7.1. Collapsing Assertion and Presupposition at the matrix level

If we collapse assertion and presupposition at the matrix level, we will get what we described in section 6.1., i.e. a difference between different determiners: *none* and *all* will yield universal inferences and *some* and *not all* will not.

My goal in this sub-section is to show how this could come about from a particular architecture for accommodation – one that we will reject to accommodate strong triggers and some experimental results we will describe.

A secondary goal is to show a case where this strategy fails, namely polar questions (where there is no assertive component) and to use this case to introduce the second mechanism, namely presupposition strengthening.

7.1.1. Indicative *some*

(1)' Some student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

It is odd for a speaker to believe this disjunction without believing one of the disjuncts.

But what conditions on use does this entail?

Four possible scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of the common ground, C, at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual tautology).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic scenario here in contrast to the cases we investigated in 7.3.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required and we need a model of accommodation.

Two possible architectures to consider

A. Minimal Accomodation→ Assertion→ plausibility check→ possible strengthening of presupposition:
   Accommodation is minimal and is followed by update of the context by the assertion. If the resulting context is plausible speaker and hearer are happy. If not, one needs to consider different accommodations (i.e. strengthening).

B. Minimal Accomodation→ plausibility check→ possible strengthening of presupposition → Assertion:

   Accommodation is followed by update of the context by the assertion. But prior to update, one must make sure that the result of accommodation is plausible.

For now we will assume architecture A. If architecture A is correct, the resulting context (after minimal accommodation and assertion) is one in which the first disjunct is part of the common ground, hence a plausible context.

Conclusion: there is a scenario (scenario 4) in which the sentence is acceptable without a resulting context that entails the universal statement (the second disjunct). Hence, speakers should not report a universal inference.

7.1.2. Architecture A involves collapsing of assertion and presuppositions

Architecture A in greater detail

Assertability Condition: When a sentence S is asserted in a context C it is associated with a formal presupposition p. When p is entailed by (the common ground in) C, the sentence is assertable. When p is not entailed by C, a repair strategy might come into play.

Accommodation: When p is not entailed by C, it would either be judged as unacceptable or C might be modified minimally so that p is satisfied

Accommodation (C,p) = C∩p

I.e. accommodation is always minimal.

Update: After S is asserted, the context will be updated
(Update (C, S) = C∩{w: S is true in w} if S is indicative)

Is this architecture plausible?

Since under architure A, the interface with context and plausibility takes place not immediately after we update the context with the presupposition but rather after we update the context with the assertion as well, we are collapsing assertion and presupposition here. This is not the way Stalnaker taught us to think about presuppositions, but it might nevertheless be correct.

Look ahead: to suggest that Architecure B is, at the end of the day, the correct architecture and that it applies whenever strong triggers are involved. Architecture A is a special case of local accommodation (allowed more easily with weak triggers) – special case in that it involves local accommodation at the matrix level.

7.1.3. Indicative *every*

(2)' Every student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and doesn't drive it to school] or
[Every student has a car]

Again, it is odd for a speaker to believe the disjunction without believing one of these disjuncts.

Four possible scenarios to consider:

Scenario 1: The first disjunct *some student has a car and doesn't drive it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual contradiction).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required. Under architecture A, it is minimal and is followed by update of the context by the assertion. In our particular case, the resulting context is one in which the second conjunct is part of the common ground, hence a stable state.

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers do report a universal inference.

7.1.4. Indicative *no*

(3)' No student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

It is very odd for a speaker to believe the disjunction without believing one of these disjuncts.

Four possible scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual contradiction).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic Scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required. Under architecture A, it is minimal and is followed by update of the context by the assertion. In our particular case, the resulting context is one in which the second conjunct is part of the common ground, hence a plausible context.

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers report a universal inference.

7.1.5. Negated Universals

As we've seen:

(23) A: There are many students around, hence many cars.
B: No, half of the students don't have a car.
Furthermore, some don't drive their car to school.
Furthermore, not every student drives his car to school.
\# Furthermore, every student leaves his car at home

7.1.5. Questions

(24) Does one of these 10 students [t_1 drive his_1 car to school].
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the question is not assertable (the answer is already part of the common ground).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the question is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic Scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required. Under architecture A, it is minimal and is followed by update of the context by the question. In this particular case, the resulting context is not affected, hence we are left in implausible context and strengthening is required.

Clearly the first disjunct is not a possible strengthening, but the second disjunct is.[2]

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers do report a universal inference.

Prediction: A yes/no question will reveal weaker presuppositions if we make it plausible to believe the disjunction without believing one of the disjuncts. (We've seen evidence that this is correct in 6.3.)

[2] There are well known problems for this line of reasoning that we will bring up in section y and attempt to address in section x. In particular we will discuss reasons to believe that the set of possible strengthenings of a presupposition is formally defined. Using a definition based on Schlenker's recent work, we will argue that the second disjunct is indeed available as a potential strengthening.

7.2. Back to Strong Triggers

(25) Just five of these 100 boys have smoked in the past. They have all smoked Nelson #Unfortunately, some/at least two of these 100 boys have also smoke Marlboro$_F$.

Charlow's Conclusion: *also* is a "strong trigger" and reveals the true projection properties which are universal.

Hypothesis: Charlow is correct that "strong triggers" are different, but they don't have universal presuppositions as we've seen in 6.3.

What is special about strong triggers is that they do not allow assertions and presuppositions to be collapsed. Specifically architecture B is true after all:

> Minimal Accomodation→ plausibility check→ possible strengthening of presupposition → Assertion:
>
> Accommodation is followed by update of the context by the assertion. But prior to update, one must make sure that the result of accommodation is plausible.

The disjunctive presupposition alone simply cannot be accommodated if it is not plausible. If it is not plausible it is strengthened to the universal inference as discussed in the case of polar questions.

But what is the property of weak triggers.

Hyopthesis: Architecture B is correct but there is also the mechanism of local accommodation that is allowed to be introduced in response to odd presuppositions.

$$(26) [\![A]\!](p_t) = \begin{cases} 1 \text{ if } p=1 \\ 0\text{, otherwise (i.e., if } p = 0 \text{ or } p=\#) \end{cases}$$

What we called architecture A results from applying A at the matrix level, which might be a preference for some speakers.

Recall: Under Charlow we didn't understand what is special about quantificational environments, as apposed to say negation in that local accommodation is easy only in the former.

Possible answers:
-local accommodation is not local, it is only possible at the matrix (but then we won't explain speakers that never get universal inferences, and apparently they exist).

-local accommodation is a rescue strategy for cases in which minimal accommodation yields an implausible information state.

7.3. Two Strategies

Possible Conclusion: when one encounters an odd presupposition, i.e. a presupposition that by minimal accommodation would take the context set to an implausible information state there are two strategies to consider:

A. Local accommodation: applying A at various scope positions.
B. Strengthening to a universal presupposition.

Speakers might vary in which strategy they employ, and if they employ strategy A, they might vary in where they employ it. Speakers that would distinguish quantifiers along the lines we discussed in section 3 would be speakers that at least sometimes employ A at the matrix level.

So back to our questions:

1. Is it right to collapse assertion and presupposition?
   No. But there is a way to eliminate the presuppositions of weak triggers when they are odd (by the introduction of *A*) and this amounts to the elimination of the presuppositions

2. What explains variations in judgments?
   Different speakers might employ different strategies to deal with oddness. Some speakers might use A and then they might differ in their preferences vis. the syntactic location of A.

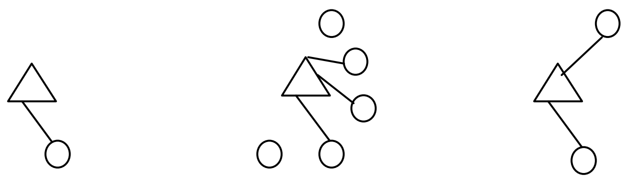
3. What explains the universal inferences with strong triggers?
   Strong triggers (following Charlow) do not allow for local accommodation.

4. What explains the universal inference that some speakers get for polar questions?
   Speakers that prefer to employ A at the matrix level can't do so with polar questions. Hence they must use local accommodation. (There is a more nuanced answer in Sudo et. al.)

7.4. Experimental Results (Sudo, Romoli, Hackl, Fox)

Item 1

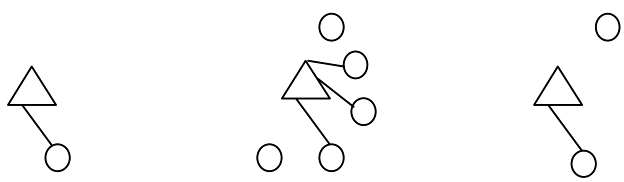
One of the following three triangles is connected to both of the circles in its vicinity.



Options: TRUE, FALSE/STRANGE

Item 2

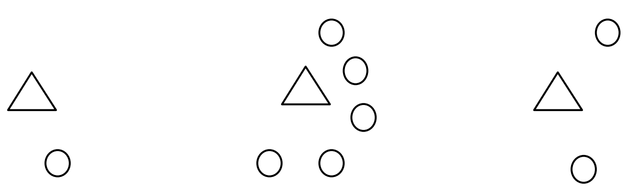
None of the following three triangles is connected to both of the circles in its vicinity.



Options: TRUE, FALSE/STRANGE

Item 3

The triangles below were connected to some of the circles by lines that have been deleted. Can you help me out? Was one of the three triangles connected to both of the circles in its vicinity?



Options: NATURAL, STRANGE

Results: If a speaker rejects the sentence in the *some* case, s/he rejects it all other cases (and not the other way around).

Interpretation: A speaker who rejects the sentence in the *some* case is a speaker that dislikes an introduction of the A operator and thus would reject the sentence in all other cases.

8. Challenges for the Trivalent Setup

8.1. The Proviso Challenge

The type of explanation we gave for the presuppositions of questions (4.5.) is familiar from Karttunen and Heim, and much subsequent work.

(27) a. If John is a scuba diver, he will bring his wetsuit.
Appears to presuppose: If John is a scuba diver, he has a wetsuit.
b. If John flies to London, his sister will pick him up.
Appears to presuppose: John has a sister.

The Heim/Karttunen claim: Both sentences in (27) have a conditional presupposition. It is not plausible to believe the conditional *If John flies to London, he has a sister* without believing that he has a sister. Hence, one would tend to infer that John has a sister (*pragmatic strengthening*).

Criticism by Geurts (1997): By parity of reasoning, we would expect the presupposition of (28) to be strengthened, but it isn't.

(28) Bill knows that if John flies to London, he has a sister.

Conclusion reached by Singh (2008, 2010) and Schlenker (2010): if we want a mechanism that strengthens presuppositions, we need to say something that would predict when strengthening is possible.

The trivalent system would have to face the same challenge:

(29) Bill knows that either some student drives his car to school or every student has a car.

And an evern more dramatic challenge:

(30) a. Does one of your two sons drive his car to school?
b. #Does one of your two sons have a car and drive it to school or do both your sons have a car and neither drives it to school?

If trivalent presuppositions are correct, the two sentences in (30) have the same presupposition: Either (p) one of your 2 children has a car and drives it to school or (q) both of your children have a car and neither drives it to school. Furthermore, they ask for exactly the same information: they have {p,q} as their Hamblin denotation. But they feel different.

A way to approach the problem: Believing *p or q* without believing one of the disjunction is odd and thus motivates pragmatic strengthening. But such strengthening is only available in (30)a.

Note, when strengthening is not required to avoid oddity, the two questions do seem equivalent.

(31) a. Did one of the 10 bankers make his fortune by whipping out one of the others?
b. Did one of the 10 bankers make a fortune by whipping out one of the others or did they all make a fortune in some other way?

What we seem to need: a theory that would derive for each sentence the set of possible pragmatic strengthenings of its presupposition.

But at this point, the trivalent system doesn't provide us with such a theory

8.2. Presupposition of non-truth-denoting expressions (thanks to A. Cremers)

The trivalent system might work for describing presupposition projection in indicative sentences which have a truth value. But how do we extend it to deal with the presupposition of non indicative sentences, e.g. questions? (For a different answer than my own, see George 2008, in progress.)

My Goal for a future talk: to develop a new way of deriving the trivalent predictions in a bivalent system which will deal with the challenges mentioned in this section.

But I should at least say something minimal about the proviso problem

9. Using Schlenker on Proviso

Schlenker: p+ is a possible strengthening of the formal presupposition p of a sentence S(X) if
$p+ = \cap\{p': \exists X'$ X' has no presupposition trigger and p' is the presupposition of S(X'))

Intuition: strengthening involves ignoring the identity of subconstituents of the sentence in computing the presupposition (treating them like the system in general treats material that follows a trigger).

Key observation:
Let Q be a generalized Quantifier and A a total function of type et:
Every (A) $(\lambda x Q(x)) = \cap\{p: \exists Z$ p is the trivalent presupposition of $Q(A)(\lambda x.Z_{Q(x)})\}$

The setup

First Ingredient (classical bivalent semantics):

Certain lexical items will have a two dimensional entry (presupposition triggers). However semantics is not two dimensional or trivalent. Only the smallest sentences that dominate a presupposition trigger will have a two dimensional representation.

Notation: The minimal clausal node that dominates a p-trigger, S, will be annotated as S_p, where p expresses the presupposition (possibly assignment dependent). Since the system is bivalent, the semantics will behave as if p was not there.

Second Ingredient: An assertability condition

Presuppositions of complex sentences will be predicted (following Schlenker) by a pragmatic condition on an utterance of a sentence φ that has S_p as a constituent. The condition, again following Schlenker, will have a global version (that will have no left right asymmetry) that we will then incrementalize (to derive the asymmetries).

However, the pragmatic condition will be different from Schlenker's. It will bear some resemblance to Stalnaker's bridging principle in (13).

Let's start with the "propositional case" in which S_p has no free variables in it (which are not in the domain of the contextually given assignment function).

9. The Propositional Case

9.1. The Global Version

Let $\varphi(S_p)$ be a sentence dominating (or identical to) S_p.

(32) $\varphi(S_p)$ is assertable in C only if
$\forall w \in C$: Relevant(S_p, $\varphi(S_p)$, w) $\rightarrow$ p is true in w.[3]

(33) Rel(S_p, $\varphi(S_p)$, w) $\Leftrightarrow_{def}$ $((\llbracket\varphi(T)\rrbracket^w \neq \llbracket\varphi(\bot)\rrbracket^w)$
Where $\llbracket T\rrbracket^w = 1$ for all w and $\llbracket\bot\rrbracket^w = 0$ for all w

9.1.1. Negation

$\varphi(S_p)$: $\neg S_p$

[3] Henceforth: 'Rel(S, φ(S), w)'. This should be read as the value of S is relevant for the value of φ in w.

$\forall w\ \forall S$: Rel($S, \neg S, w$).

Hence, $\neg S_p$ is assertable in C, by (32), only if $\forall w \in C$: p is true in w.

9.1.2. Symmetric theory of disjunction, conjunction

$\varphi(S_p)$: $S_1 \vee S_p$

$\forall w \in C$: S_1 is false in w → Rel(S_p, $\varphi(S_p)$, w).
$\forall w \in C$: S_1 is true in w → $\neg$Rel(S_p, $\varphi(S_p)$, w).

Hence $S_1 \vee S_p$ is assertable in C, by (32), only if
$\forall w \in C$: S_1 is false in w → p is true in w.

$\varphi(S_p)$: $S_1 \wedge S_p$

$\forall w \in C$: S_1 is true in w → Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w → $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \wedge S_p$ is assertable in C, by (32), only if
$\forall w \in C$: S_1 is true in w → p is true in w.

9.1.3. (Material-)Conditionals

$\varphi(S_p)$: $S_1 \rightarrow S_p$

$\forall w \in C$: S_1 is true in w → Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w → $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \rightarrow S_p$ is assertable in C, by (32), only if
$\forall w \in C$: S_1 is true in w → p is true in w.

9.2. The Incremental Version

(34) $\varphi(S_p)$ is assertable in C only if
$\forall w \in C$: Rel_{inc} (S_p, $\varphi(S_p)$, w) → p is true in w.

(35) Rel_{inc} (S_p, $\varphi(S_p)$, w) $\Leftrightarrow_{def}$ $\exists \varphi' \in$GOOD-FINAL(S, φ) s.t. Rel(S_p, $\varphi'(S_p)$, w)

(36) GOOD-FINAL(S, φ) =
{φ': φ' can be derived from φ by replacing constituents in φ that follow S}

For more general statements, see Appendix A

10. Generalizing to an Extensional System with one Free Variable

Again, we will start with a global version which can then be incrementalized

(37) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha\ [Rel(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \rightarrow [\![p(x)]\!]^{w,x\to a} = 1)]$[4]

(38) $Rel(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}\ \exists T_a, F_a$
a. $\langle T_a, F_a \rangle$ is a-differing-extension of $S(x)_{p(x)}$ (an a-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

(39) $\langle T_a, F_a \rangle$ is an a-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\forall w\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&$
$\forall \alpha \neq a\ [([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&\ [([\![p(x)]\!]^{w,x\to\alpha} = 1) \rightarrow ([\![T_a]\!]^{w,x\to\alpha} = [\![S]\!]^{w,x\to\alpha})]]$

Equivalently:

(39)' $\langle T_a, F_a \rangle$ is an a-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\exists\psi\exists T_a\exists F_a$
$\forall \alpha \neq a\ [([\![p(x)]\!]^{w,x\to\alpha} = 1) \rightarrow ([\![\psi]\!]^{w,x\to\alpha} = [\![S]\!]^{w,x\to\alpha})]\ \&$
$T_a = [x = a \vee \psi]$ and $F_a = [x \neq a \wedge \psi]$

Below we state results without proofs. For proofs, see appendix B:

10.1. Binding by an expression of type e

(40) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
S_p (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

$\forall w \forall a [Rel(S_p, \varphi, w, a) \leftrightarrow\ a = John]$

Hence (40) presupposes that John has a unique mother.

[4] 'Rel(S(x), φ(S(x)), w, a)' should be read as the value of S(x) is relevant for the value of φ in w given an individual a (or under an assignment function g, s.t. g(x)=a).

10.2. Quantification

φ: Every(NP)(λx (S(x)$_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:

$Rel(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$

$a \in [\![NP]\!]^w \ \& \ \neg\exists b \neq a{:}b \in [\![NP]\!]^w \ \& \ [\![p(x)]\!]^{w,\, x \to b} = 1 \ \& \ [\![S(x)_{p(x)}]\!]^{w,\, x \to b} = 0$

Hence *Every(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:

$Rel(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$

$a \in [\![NP]\!]^w$ and $\neg\exists x \neq a{:}x \in [\![NP]\!]^w \ \& \ [\![p(x)]\!]^{w,\, 1 \to x} = 1 \ \& \ [\![S(x)_{p(x)}]\!]^{w,\, 1 \to x} = 1$

Hence *Some(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

11. Incremental Version

(41) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if

$\forall w \in C \ \forall a \in D_\alpha \ [Rel_{inc}(S\,(x)_{p(x)}, \varphi, w, a) \rightarrow [\![p(x)]\!]^{w, x \to a} = 1]$

(42) $Rel_{inc}(S, \varphi(S), w, a) \quad \Leftrightarrow_{def} \quad \exists\varphi' \in \text{GOOD-FINAL}(S, \varphi)$ s.t., $Rel(S, \varphi'(S), w, a)$

More Radical Incrementalization

(43) $Rel_{r\text{-}inc}(S, \varphi(S), w, a) \ \Leftrightarrow_{def} \ \exists S'$ s.t. $Rel_{inc}(S', \varphi(S'), w, a)$

(44) φ is assertable in C only if

$\forall w \in C \ \forall a \in D_\alpha \ [Rel_{r\text{-}inc}(S\,(x)_{p(x)}, \varphi, w, a) \rightarrow [\![p(x)]\!]^{w, x \to a} = 1]$

More constituents will be r-incrementally relevant than those that are incrementally relevant (which are in turn more than those that are globally relevant). Hence, the more we incrementalize the stronger the presuppositions.

In particular, (44) will give us the Heim/Schlenker predictions (see appendix C).

12. Proviso and Formal Alternatives

Schlenker's (2010) solution to the proviso problem: the set of possible strengthening of the presupposition of a sentence φ come from various forms of radical incrementalization, in particular by treating all sorts of constituents that do not follow the relevant presupposition trigger, as if they followed the trigger.

Since we get the classical (Heim/Schlenker) predictions by considering substitutions of the nuclear scope (which does not follow the trigger), we understand why the Heim/Schlenker presuppositions are possible strengthenings of the trivalent presuppositions.

13. Generalizing to an extensional system with any number of free variables

The global version

(45) Let $\varphi(S([x_i])_{p([xi])})$ be a sentence that dominates $S([x_i])_{p([xi])}$ where $x_1 \ldots x_n$ are all the to-be-bound-variable in $S([x_i])_{p([xi])}$.

φ is assertable in C only if
$\forall w \in C\ \forall [a_i]\ Rel(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \rightarrow [\![p([xi])]\!]^{w,[xi] \rightarrow [ai]} = 1)$

(46) $Rel(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \Leftrightarrow_{def}$

$\exists \langle T_{[ai]}, F_{[ai]} \rangle$ s.t. $\langle T_{[ai]}, F_{[ai]} \rangle$ is an $[a_i]$-DE of $S([x_i])_{p([xi])}$ and
$[\![\varphi(T_{[ai]}))]\!]^{w,g} \neq [\![\varphi(F_{[ai]}))]\!]^{w,g}$

(47) $\langle T_{[ai]}, F_{[ai]} \rangle$ is an a-DE of $S([x_i])_{p([xi])} \Leftrightarrow_{def}$
$\forall w$
a. $\forall x \neq [ai]: [\![T_{[ai]}]\!]^{w, [xi] \rightarrow [ai]} = [\![F_{[ai]}]\!]^{w, [xi] \rightarrow [ai]}$
b. $\forall x \neq [ai]: [\![p([x_i])]\!]^{w, [xi] \rightarrow [ai]} = 1 \rightarrow [\![T_{[ai]}]\!]^{w, [xi] \rightarrow [ai]} = [\![S([x_i])]\!]^{w, [xi] \rightarrow [ai]}$
c. $T_{[ai]}([a_i])=1$ and $F_{[ai]}([a_i])=0$

Equivalently:
(48) $\langle T_{[ai]}, F_{[ai]} \rangle$ is an a-DE of $S([x_i])_{p([xi])}$ if $\exists \psi$
a. $\forall x \neq [ai]: [\![p([x_i])]\!]^{w, [xi] \rightarrow [ai]} = 1 \rightarrow [\![\psi]\!]^{w, [xi] \rightarrow [ai]} = [\![S([x_i])]\!]^{w, [xi] \rightarrow [ai]}$
b. $T_{[ai]} = ([x_i]=[a_i] \vee \psi)$
c. $F_{[ai]} = ([x_i] \neq [a_i] \wedge \psi)$

The incremental version
As above

14. Generalizing to an intensional system with any number of free variables

The lazy thing to do at this stage it to assume that world variables are always represented in the syntax and to hope that this reduces to what we have in section 13.

Do we get the right predictions?

In particular, how do intensional operators project?

The predictions here seem different from what is stated by Kartunnen (1973, 1974) and Heim (1992). So I have serious homework to do.

Possibly relevant:

(49)
a. I think it's possible that John has a job. But it's also possible that his job pays very little.
b. I think it's possible that John has a job. But I'm not certain his job pays that much.
c. #I think it's possible that John has a job. But I'm certain his job pays very little.

(50)
a. I think it's possible that John has a job. And it's possible that his wife has a job, as well.
b. I think it's possible that John has a job. But I'm not certain his wife has a job, as well.
c. #I think it's possible that John has a job. And I'm certain his wife has a job as well.

Appendices

A. More General Statements (for the propositional case of section 9)

Compositionality of Relevance (R-compositionality): Let φ(S(A)) be a sentence that dominates S which, in turn, dominates A.

a. If A is (inc-)relevant for the value of S in w, and S is (inc-)relevant for the value of φ in w, then A is (inc-)relevant for the value of φ in w.
b. If A is not (inc-)relevant for the value of S in w, A is not (inc-)relevant for the value of φ in w.
c. If S is not (inc-)relevant for the value of φ in w, A is not (inc-)relevant for the value of φ in w.

Proof: trivial.

Terminology:

If a sentence φ obeys the incremental assertability condition in (34) in every context that entails p and fails to obey the condition in every context that does not entail p, we will say that φ *presupposes p*. It will turn that for every sentence φ, there is a unique proposition that φ presupposes. Hence we can write *Presup(φ)* for this unique presupposition.

In the proofs below, we assume for simplicity that (34) is an iff condition. (It is easy to restate the proofs without this assumption.)

A.1. Negation

Claim: Presup(¬φ) = Presup(φ)

Proof:
Let C be a context that does not entail Presup(φ)
Let w∈C be a world s.t. Presup(φ)(w)=0.
φ is not assertable in any C, s.t. w∈C. by definition
∃S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (34)
S_p is inc-relevant for ¬φ in w. φ is always relevant for ¬φ + R-compositionality

Hence ¬φ is unassertable in C.

Let C be a context that does entail Presup(φ)
∀w∈C: Presup(φ)(w)=1.
φ is assertable in C. by definition
¬∃w∈C, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (34)
¬∃w∈C , S_p dominated by ¬φ, s.t. S_p is inc-relevant for ¬φ in w and p(w)=0. R-compositionality

Hence ¬φ is assertable in C.
Hence: Presup(¬φ) = Presup(φ)

A.2. disjunction

Presup(φ∨ψ) = Presup(φ)∧(¬φ→Presup(ψ))

Proof:
Let C be a context that does not entail Presup(φ)∧(¬φ→Presup(ψ)).
Let w∈C be a world in which Presup(φ)∧(¬φ→Presup(ψ)) is false.

First Possibility -- Presup(φ) is false in w:

∃S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (34)
S_p is incrementally relevant for φ ∨ ψ in w. choose contradiction for ψ
Hence φ∨ψ is not assertable in C by (34).

Second Possitivlity -- (¬φ→Presup(ψ)) is false in :,

¬φ is true in w and Presup(ψ) is false in w. by (34)

Since $\neg\varphi$ is true in w, ψ is relevant for the truth value of $\varphi \vee \psi$
and the rest is just as above

Hence $\varphi\vee\psi$ is not assertable in C

Under both possibilities $\varphi\vee\psi$ is unassertable in C.

Let C be a context that entails $Presup(\varphi)\wedge(\neg\varphi\rightarrow Presup(\psi))$.

$\forall w\in C$: $Presup(\varphi)(w)=1$.
φ is assertable in C. by definition
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and $p(w)=0$. by (34)
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and $p(w)=0$.
R-compositionality

$\forall w\in C$
if $\varphi(w) = 1$, ψ is irrelevant for the value of $\varphi\vee\psi$, and so is any S_p dominated by ψ
R-compositionality
if $\varphi(w) = 0$, then $Presup(\psi)(w)=1$ $C \Rightarrow \neg\varphi\rightarrow Presup(\psi)$
So, there will be no S_p dominated by ψ, which is both inc. relevant for ψ and $p(w)=0$.
Hence:
$\neg\exists w\in C$, S_p dominated by ψ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and $p(w)=0$.
R-compositionality

Hence
$\neg\exists w\in C$, S_p dominated by $\varphi\vee\psi$, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and $p(w)=0$.
Hence, $\varphi\vee\psi$ is assertable in C.

Hence: $Presup(\varphi\vee\psi) = Presup(\varphi)\wedge(\neg\varphi\rightarrow Presup(\psi))$

A.3…

B. Missing Proofs from section 10

B.1. Binding by an expression of type e

(51) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
$S(x)_{p(x)}$ (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

$\forall w\forall a[Rel(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow a{=}John]$

Proof (trivial):
$Rel(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{John } \lambda x T_a]\!]^w \neq [\![\text{John } \lambda x F_a]\!]$ $\leftrightarrow$ by lambda conversion

$\exists\langle T_a,F_a\rangle \ldots [\![T_a]\!]^{w,x\to John} \neq [\![F_a]\!]^{w,x\to John}$ $\leftrightarrow$ by definition of a-DE

$a = John$

Hence (40) presupposes that John has a unique mother.

B.2. Quantification

φ: Every(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$:

$Rel(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$

$a \in [\![NP]\!]^w \;\&\; \neg\exists b\neq a{:}b \in [\![NP]\!]^w \;\&\; [\![p(x)]\!]^{w,\, x\to b} = 1 \;\&\; [\![S(x)_{p(x)}]\!]^{w,\, x\to b} = 0$

Proof:
$Rel(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{every NP } 1T_a]\!]^w \neq [\![\text{every NP } 1F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![NP]\!]^w \subseteq [\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^w \subseteq [\![F_a]\!]^{w,x\to a})$ $\leftrightarrow$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w \;\&\; \exists\langle T_a,F_a\rangle \forall b\neq a[\, b \in [\![NP]\!]^w \rightarrow (x \in [\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w \;\&\; \forall b\neq a[\, b \in [\![NP]\!]^w \rightarrow ([\![p(x)]\!]^{w,\, x\to b}=0 \text{ or } [\![S(x)_{p(x)}]\!]^{w,\, x\to b}=1]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards
$a \in [\![NP]\!]^w \;\&\; \neg\exists b\neq a{:}b \in [\![NP]\!]^w \;\&\; [\![p]\!]^{w,\, x\to b}=1 \;\&\; [\![S(x)_{p(x)}]\!]^{w,\, x\to b} = 0$

Hence *Every(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$: Rel(S(x)$_{p(x)}$, φ, w, a) $\leftrightarrow$

$a \in [\![NP]\!]^w$ and
$\neg\exists x{\neq}a{:}x \in [\![NP]\!]^w$ & $[\![p(x)]\!]^{w,\, 1\to x} = 1$ & $[\![S(x)_{p(x)}]\!]^{w,\, 1\to x} = 1$

Proof:
Rel(S(x)$_{p(x)}$, φ, w, a) $\leftrightarrow$ by definition of relevance

$\exists\langle T_a, F_a\rangle$ an a-DE of S(x)$_{p(x)}$ s.t.
$[\![$some NP $\lambda x T_a]\!]^w \neq [\![$some NP $\lambda x F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\varnothing \neq [\![NP]\!]^w \cap [\![T_a]\!]^{w,x\to a}$ & $\varnothing = [\![NP]\!]^w \cap [\![F_a]\!]^{w,x\to a}$ $\leftrightarrow$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w$ & $\exists\langle T_a, F_a\rangle \forall b{\neq}a[\, b \in [\![NP]\!]^w \rightarrow (b \notin [\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w$ & $\forall b{\neq}a[\, b \in [\![NP]\!]^w \rightarrow ([\![p(b)]\!]^{w,\, x\to b}=0$ or $[\![S(x)_{p(x)}]\!]^{w,\, x\to b}=0]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards
$a \in [\![NP]\!]^w$ & $\neg\exists b{\neq}a{:}b \in [\![NP]\!]^w$ & $[\![p]\!]^{w,\, x\to b}=1$ & $[\![S(x)_{p(x)}]\!]^{w,\, x\to b} = 1$

Hence *Some(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

C. Understanding the consequences of r-incrementalization

To get the Heim/Schlenker Generalization, we will strengthen the assertability condition by weakening our global notion of relevance to what we call potential-relevance (Rel_p). It will be easy to see that what we said in section 11 is correct: the incrementalization of Rel_p will be equivalent to the r-incrementalization of our earlier notion Rel.

(52) Let φ(S(x)$_{p(x)}$) be a sentence that dominates S(x)$_{p(x)}$ where x is a variable of type α, the single to-be-bound variable in S(x)$_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if

$\forall w \in C\ \forall a \in D_\alpha\ (Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \rightarrow [\![p(x)]\!]^{w,x\rightarrow a} = 1)$

(53) $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists T_a, F_a$
a. $[\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![F_a]\!]^{w,x\rightarrow\alpha})$ and
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

Equivalently:
(53)' $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists\psi\exists T_a\exists F_a$
a. $T_a=[x=a \vee \psi]$ and $F_a=[x\neq a \wedge \psi]$
b. $[\![\varphi(T_a)]\!]^{w} \neq [\![\varphi(F_a)]\!]^{w}$

C.1. Binding by an expression of type e

(54) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
$S(x)_{p(x)}$ (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

For every w:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a=John.$

Proof (trivial):
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![F_a]\!]^{w,x\rightarrow\alpha})\ \&$
$[\![\text{John } \lambda x T_a]\!]^{w} \neq [\![\text{John } \lambda x F_a]\!]$ $\leftrightarrow$ by lambda conversion

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![F_a]\!]^{w,x\rightarrow\alpha})\ \&$
$[\![T_a]\!]^{w,x\rightarrow John} \neq [\![F_a]\!]^{w,x\rightarrow John}$ $\Leftrightarrow$

$a = John$

Hence (54) presupposes that John has a unique mother.

C.2. Quantification

φ: Every(NP)(x (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \Leftrightarrow a \in [\![NP]\!]^{w}$

Proof:
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a}=1\ \&\ [\![F_a]\!]^{w,x\to a}=0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![\text{every NP } \lambda x T_a]\!]^{w} \neq [\![\text{every NP } \lambda x F_a]\!]^{w}$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a}=1\ \&\ [\![F_a]\!]^{w,x\to a}=0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![NP]\!]^{w}\subseteq[\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^{w}\subseteq[\![F_a]\!]^{w,x\to a})$ $\leftrightarrow$

$a \in [\![NP]\!]^{w}$

Hence *Every(NP)(λx(S(x)*$_{p(x)}$*)* presupposes that p holds of every member of the denotation of NP

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w\in C \forall a\in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a \in [\![NP]\!]^{w}$

Proof:
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a}=1\ \&\ [\![F_a]\!]^{w,x\to a}=0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![\text{some NP } \lambda x T_a]\!]^{w} \neq [\![\text{some NP } \lambda x F_a]\!]^{w}$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a}=1\ \&\ [\![F_a]\!]^{w,x\to a}=0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![NP]\!]^{w}\cap[\![T_a]\!]^{w,x\to a} \neq \varnothing$ and $[\![NP]\!]^{w}\cap[\![F_a]\!]^{w,x\to a} = \varnothing$ $\leftrightarrow$

$a \in [\![NP]\!]^{w}$

Hence *Some(NP)(λx(S(x)*$_{p(x)}$*)* presupposes that p holds of every member of the NP domain.

D. Problem from Infinite Domains

(55) An infinite number of boys drove their car to school.
$[_{\varphi}$ An infinite number of boys $[_{S(x)}$ x drove x's car to school$]_{\text{x has a unique car}}]$

$\forall w\in C\ \neg\exists a\in D_e\ (Rel(S(x), \varphi, w, a))$.
Hence the sentence should presuppose nothing.

Revision:

(56) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$

φ is assertable in C only if
$\forall w\in C\ \forall A\subseteq D_{\alpha}\ S(x)_{p(x)}\ Rel_{SUB\text{-}SET}(S(x), \varphi, w, A) \rightarrow \exists A'\subseteq A(\forall a\in A'[\![p(x)]\!]^{w,x\rightarrow a}=1)$[5]

Equivalently: φ is assertable in C only if
$\forall w\in C\ \forall A\subseteq D_{\alpha}\ S(x)_{p(x)}\ Rel_{SUB\text{-}SET}(S(x), \varphi, w, A) \rightarrow \exists a\in A([\![p(x)]\!]^{w,x\rightarrow a}=1)$

(57) $Rel_{SUB\text{-}SET}(S(x), \varphi, w, A) \Leftrightarrow_{def}$
$\exists T_A, F_A$
a. $\langle T_A,F_A\rangle$ is an A-differing-extension of $S(x)_{p(x)}$ (an A-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_A)]\!]^{w,g} \neq [\![\varphi(F_A)]\!]^{w,g}$

(58) $\langle T_A,F_A\rangle$ is an A-DE of $S(x)_{p(x)}$ if
$\forall w\ \forall a\in A[\![T_A]\!]^{w,x\rightarrow a}=1\ \&\ [\![F_a]\!]^{w,x\rightarrow a}=0\ \&$
$\forall\alpha\notin A\ [([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![F_a]\!]^{w,x\rightarrow\alpha})\ \&$
$[([\![p(x)]\!]^{w,x\rightarrow\alpha}=1)\rightarrow([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![S]\!]^{w,x\rightarrow\alpha})]]$

Note: this assertability condition is stronger than what we had previously since:

a. $\forall S,\varphi,w,a[Rel(S(x), \varphi, w, a) \rightarrow Rel_{SUB\text{-}SET}(S(x), \varphi, w, \{a\})]$
b. If $|A|=\infty\ \exists S,\varphi,w\ [Rel_{SUB\text{-}SET}(S(x), \varphi, w, A)\ \&\forall a\in A\neg Rel(S(x), \varphi, w, a)]$

E. More General Statement (for a language with variables)

Hopefully some other time

[5] '$Rel_{SUB\text{-}SET}(S(x), \varphi(S(x)), w, A)$' should be read as the value of S(x) is relevant for the value of φ in w for some subset of A.