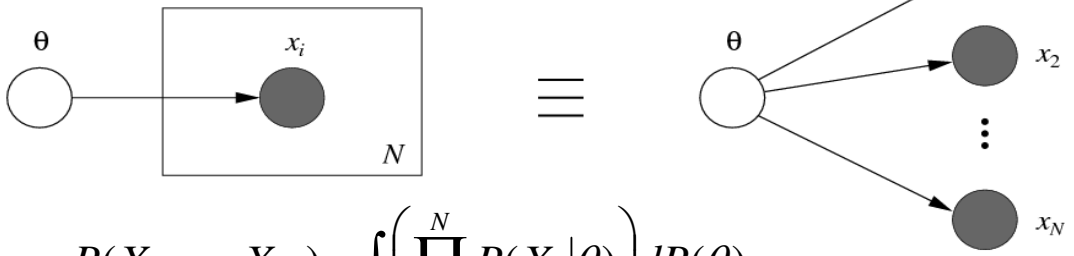
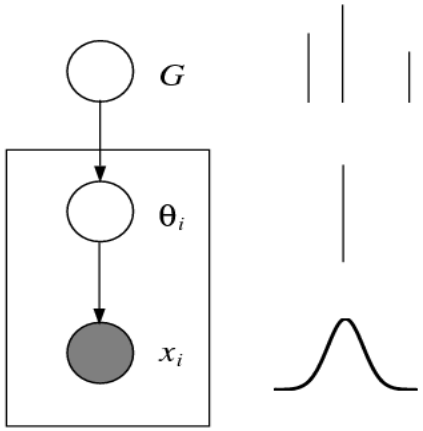
# Mixtures Models

**Plate notation**




$$P(X_1,\ldots,X_N) = \int \left( \prod_{i=1}^{N} P(X_i|\theta) \right) dP(\theta)$$

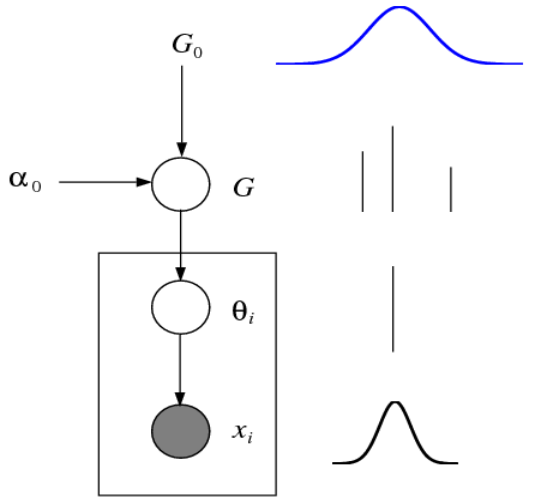
**Finite Mixture Model**




$$\begin{aligned} G &= \sum_{k=1}^{K} \pi_k \, \delta_{\phi_k} \\ \theta_i &\sim G \\ x_i &\sim p(\cdot \mid \theta_i) \end{aligned}$$

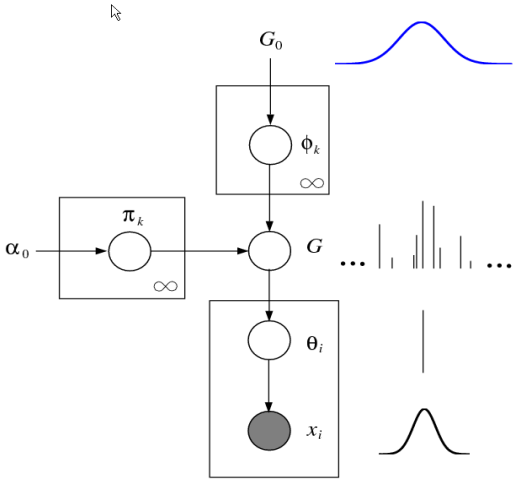
**Bayesian Finite Mixture Model**

$$\begin{aligned} \phi_k &\sim G_0 \\ \pi_k &\sim \mathrm{Dir}(\alpha_0/K, \ldots, \alpha_0/K) \\ G &= \sum_{k=1}^{K} \pi_k \, \delta_{\phi_k} \\ \theta_i &\sim G \\ x_i &\sim p(\cdot \mid \theta_i) \end{aligned}$$

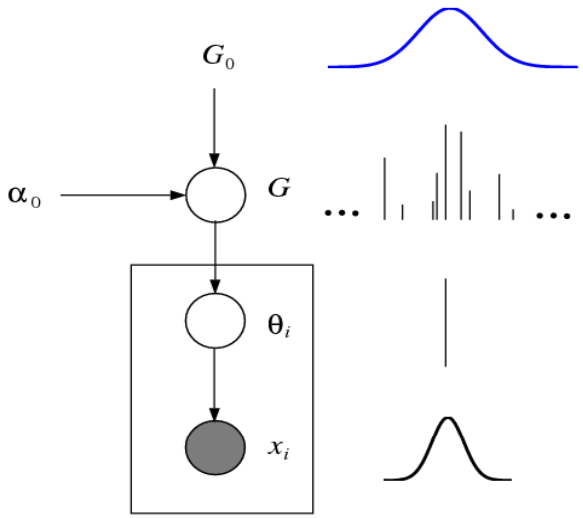


# Dirichlet Processes and Extensions

## Stick Breaking Construction

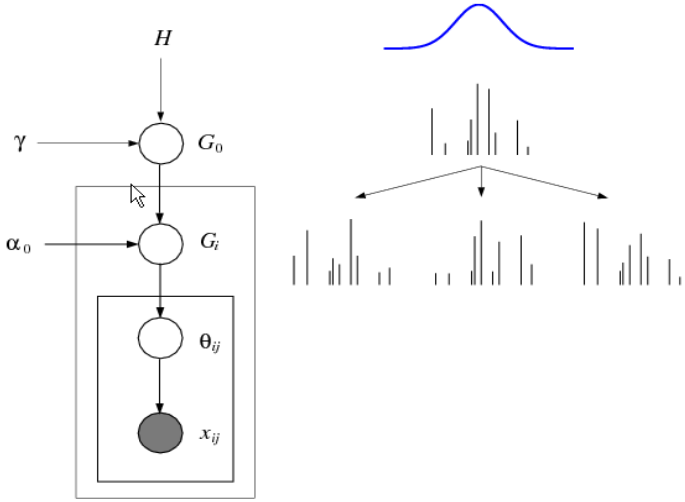



## Dirichlet Process Mixture



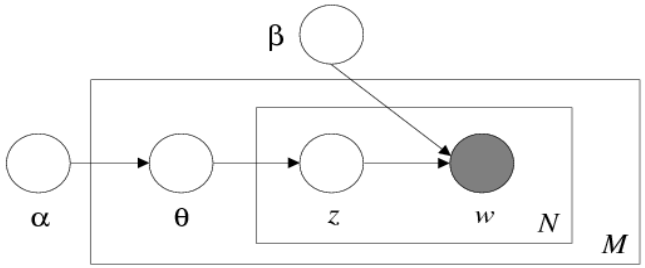

$$
\begin{aligned}
G &\sim \mathrm{DP}(\alpha_0 G_0) \\
\theta_i \,|\, G &\sim G \qquad i \in 1, \ldots, n \\
x_i \,|\, \theta_i &\sim F(x_i \,|\, \theta_i) \qquad i \in 1, \ldots, n
\end{aligned}
$$

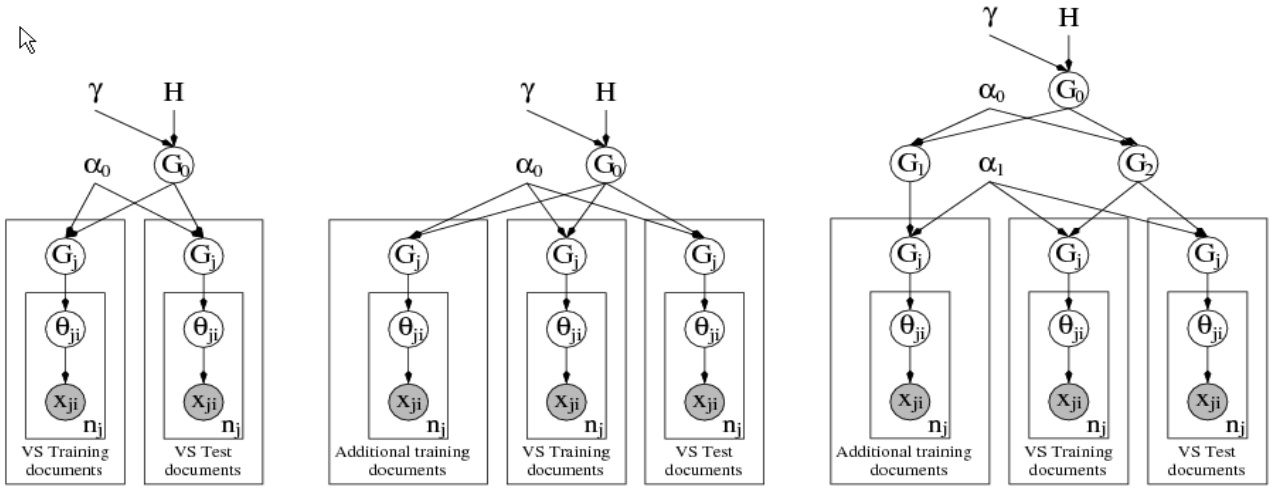
## Hierarchical Dirichlet Process Mixture

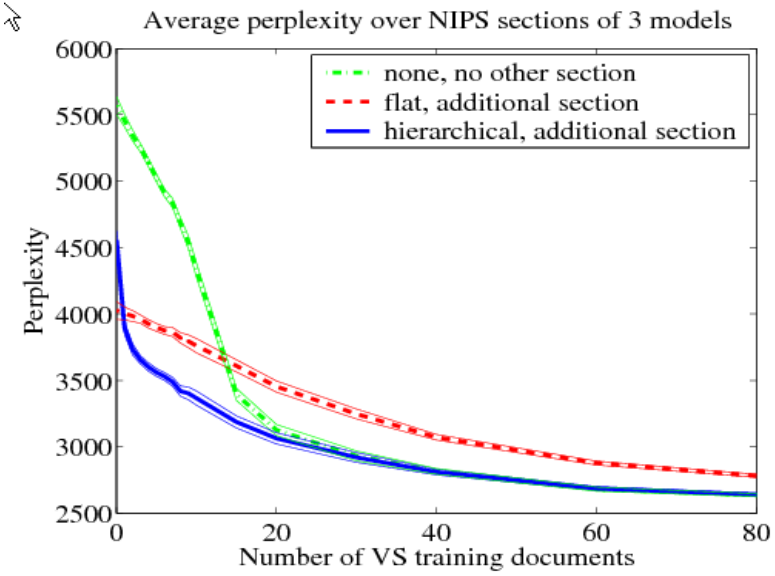



$$
\begin{aligned}
G_0 \,|\, \gamma, H &\sim \mathrm{DP}(\gamma H) \\
G_i \,|\, \alpha, G_0 &\sim \mathrm{DP}(\alpha_0 G_0) \\
\theta_{ij} \,|\, G_i &\sim G_i \\
x_{ij} \,|\, \theta_{ij} &\sim F(x_{ij}, \theta_{ij})
\end{aligned}
$$

# Latent Dirichlet Allocation




| CS | NS | LT | AA | IM | SP | AP | CN |
|---|---|---|---|---|---|---|---|
| task | cells | signal | algorithms | processing | visual | approach | tree |
| representation | cell | layer | test | pattern | images | based | pomdp |
| pattern | activity | gaussian | approach | approach | video | trained | observable |
| processing | response | cells | methods | architecture | language | test layer | strategy |
| trained | neuron | figure | based | single | image | features | class |
| representations | visual | nonlinear | point | shows | pixel | table | stochastic |
| three | patterns | rate | problems | simple | acoustic | classification | history |
| process unit | pattern | equation | large | based | delta | rate | strategies |
| patterns | single | cell | paper | large | lowpass | paper | density |
| examples | visual | large | distance | motion | signals | image | policy |
| concept | cells | examples | tangent | visual | separation | images | optimal |
| similarity | cortical | form | image | velocity | signal | face | reinforcement |
| bayesian | orientation | point | images | flow | sources | similarity | control |
| hypotheses | receptive | see | transformation | target | source | pixel | action |
| generalization | contrast | parameter | transformations | chip | matrix | visual | states |
| numbers | spatial | consider | pattern | eye | blind | database | actions |
| positive | cortex | random | vectors | smooth | mixing | matching | step |
| classes | stimulus | small | convolution | direction | gradient | facial | problems |
| hypothesis | tuning | optimal | simard | optical | eq | examples | goal |