Speed of Light Measurement

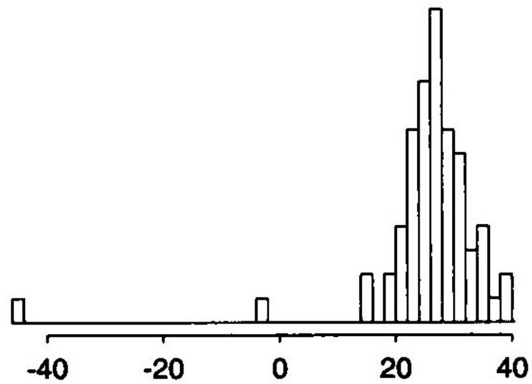
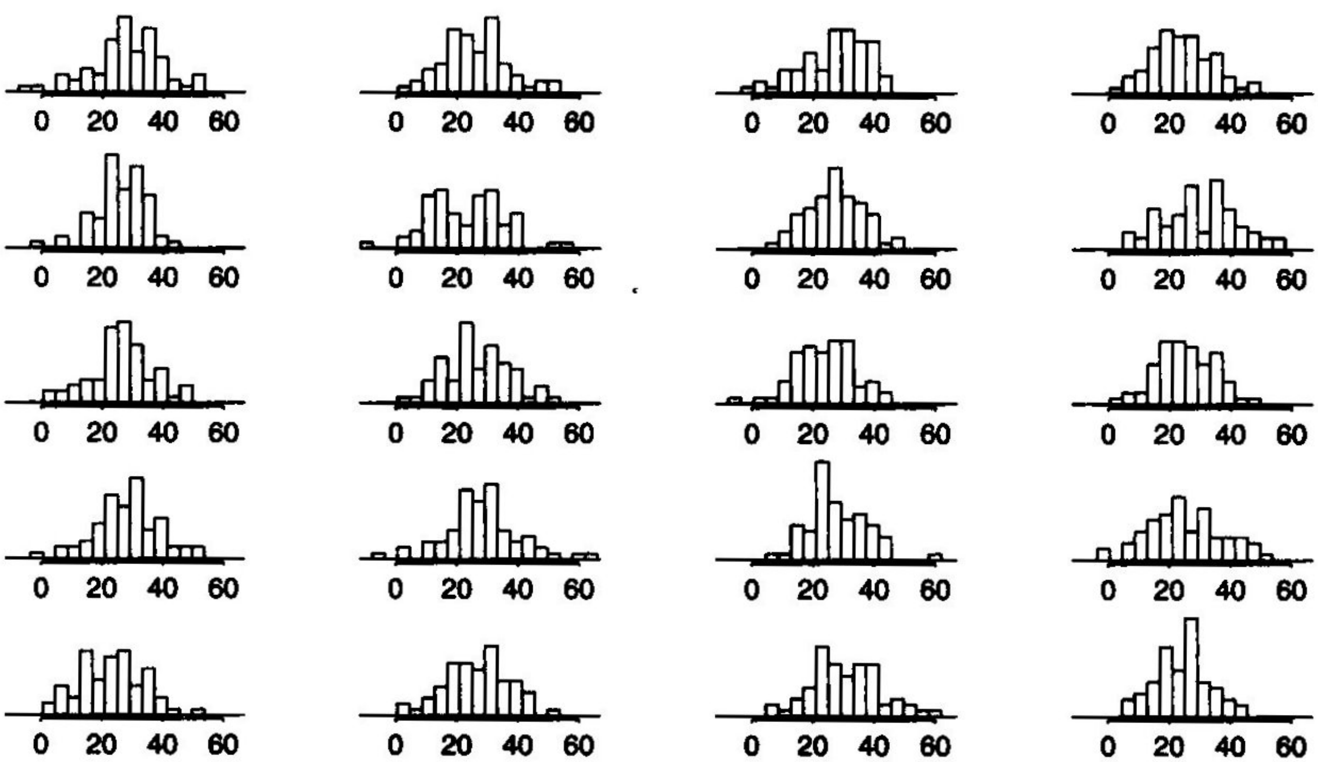


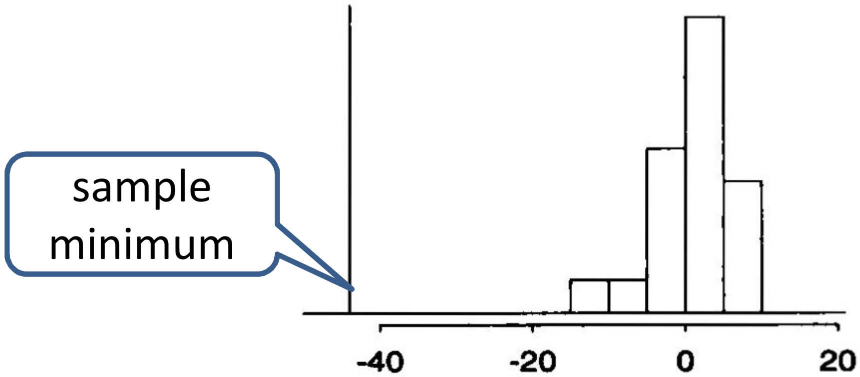
Figure 3.1 *Histogram of Simon Newcomb's measurements for estimating the speed of light, from Stigler (1977). The data are recorded as deviations from* 24,800 *nanoseconds.*

20 Replications from predictive posterior



Speed of Light Measurement

Posterior distribution of smallest observation



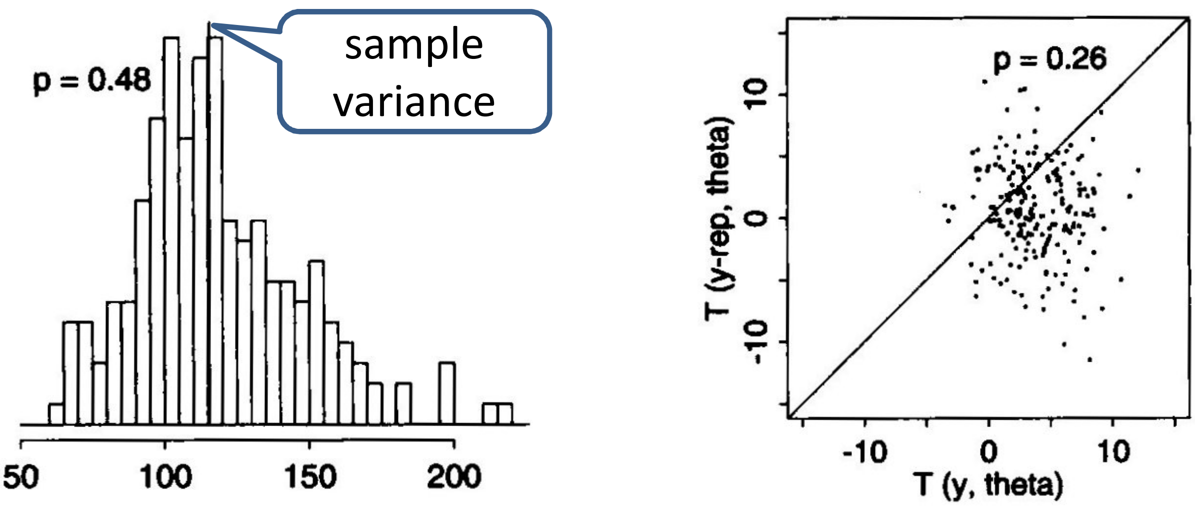


Figure 6.5 *Realized vs. posterior predictive distributions for two more test quantities in the speed of light example: (a) Sample variance (vertical line at 115.5), compared to 200 simulations from the posterior predictive distribution of the sample variance. (b) Scatterplot showing prior and posterior simulations of a test quantity:* $T(y,\theta) = |y_{(61)} - \theta| - |y_{(6)} - \theta|$ *(horizontal axis) vs.* $T(y^{\text{rep}},\theta) = |y^{\text{rep}}_{(61)} - \theta| - |y^{\text{rep}}_{(6)} - \theta|$ *(vertical axis) based on 200 simulations from the posterior distribution of* (θ, y^{rep})*. The p-value is computed as the proportion of points in the upper-left half of the plot.*

Patients and Symptoms

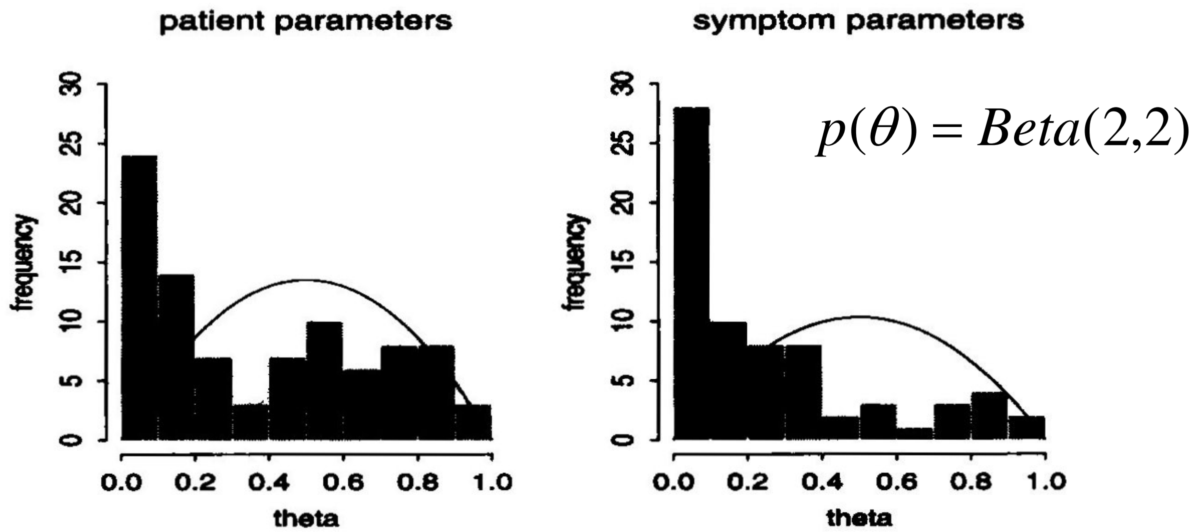


Figure 6.8 *Histograms of (a) 90 patient parameters and (b) 69 symptom parameters, from a single draw from the posterior distribution of a psychometric model. These histograms of posterior estimates contradict the assumed Beta*$(2, 2)$ *prior densities (overlain on the histograms) for each batch of parameters, and motivated us to switch to mixture prior distributions. This implicit comparison to the values under the prior distribution can be viewed as a posterior predictive check in which a new set of patients and a new set of symptoms are simulated.*

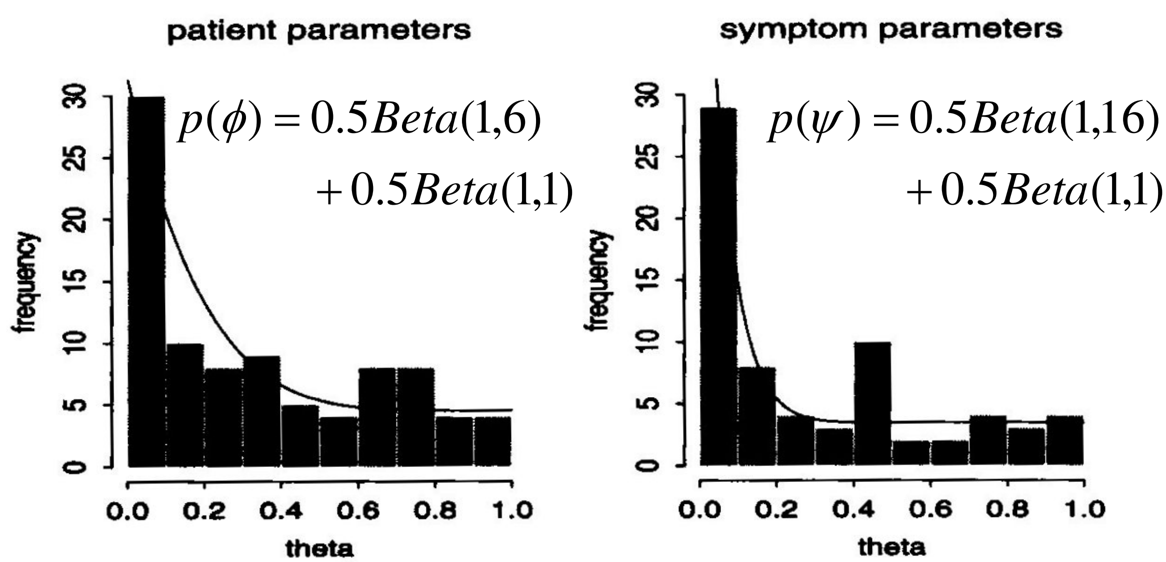
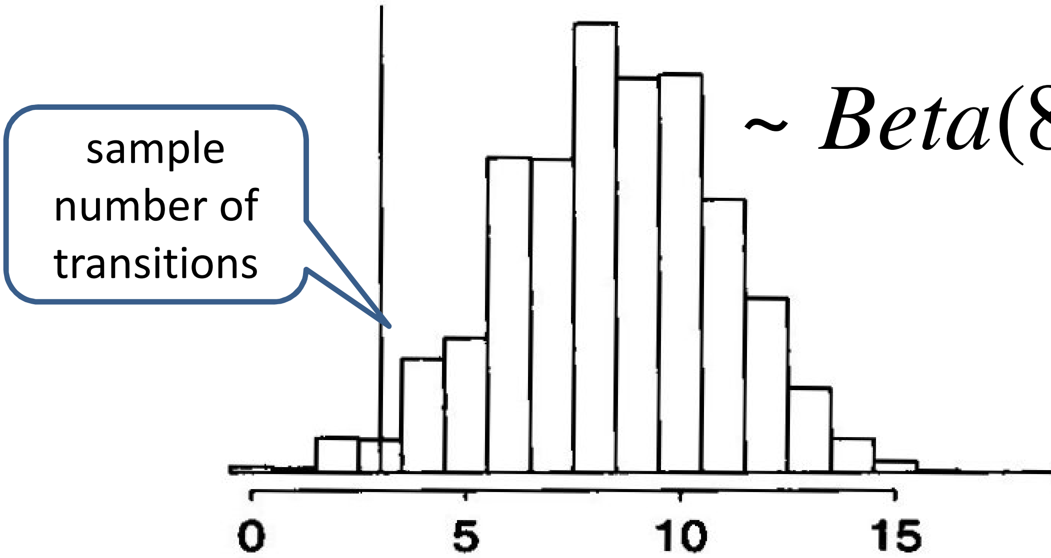


Figure 6.9 *Histograms of (a) 90 patient parameters and (b) 69 symptom parameters, as estimated from an expanded psychometric model. The mixture prior densities (overlain on the histograms) are not perfect, but they approximate the corresponding histograms much better than the Beta*$(2, 2)$ *densities in Figure 6.8.*

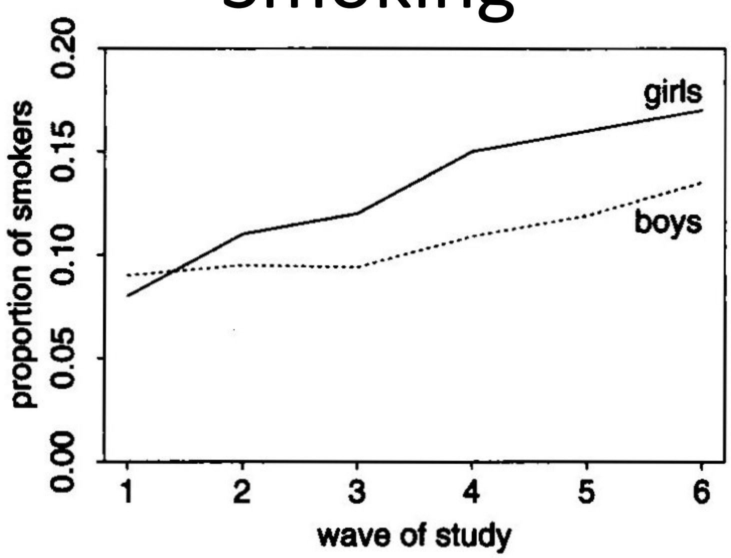
Binomial Trials

y=1,1,0,0,0,0,0,1,1,1,1,1,0,0,0,0,0,0,0



$\sim Beta(8,14)$

Smoking



		Model 1		Model 2	
Test variable	$T(y)$	95% int. for $T(y^{\text{rep}})$	p-value	95% int. for $T(y^{\text{rep}})$	p-value
% never-smokers	77.3	[75.5, 78.2]	0.27	[74.8, 79.9]	0.53
% always-smokers	5.1	[5.0, 6.5]	0.95	[3.8, 6.3]	0.44
% incident smokers	8.4	[5.3, 7.9]	0.005	[4.9, 7.8]	0.004

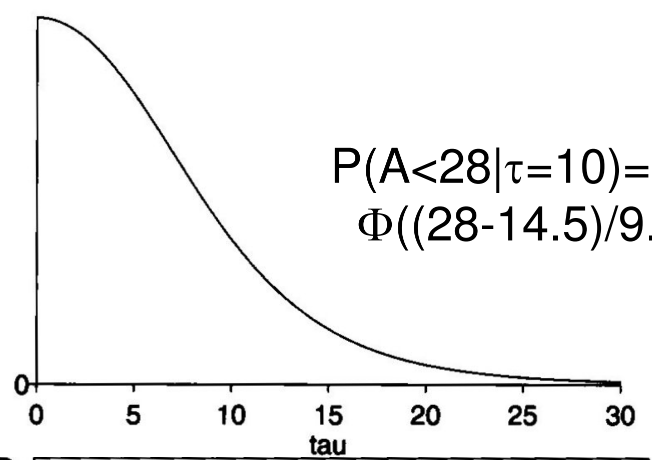
SAT Preparation Courses

Goal: estimate the effectiveness of short-term preparation at imroving SAT-Verbal scores in 8 schoold

School	Estimated treatment effect, y_j	Standard error of effect estimate, σ_j
A	28	15
B	8	10
C	−3	16
D	7	11
E	−1	9
F	1	11
G	18	10
H	12	18

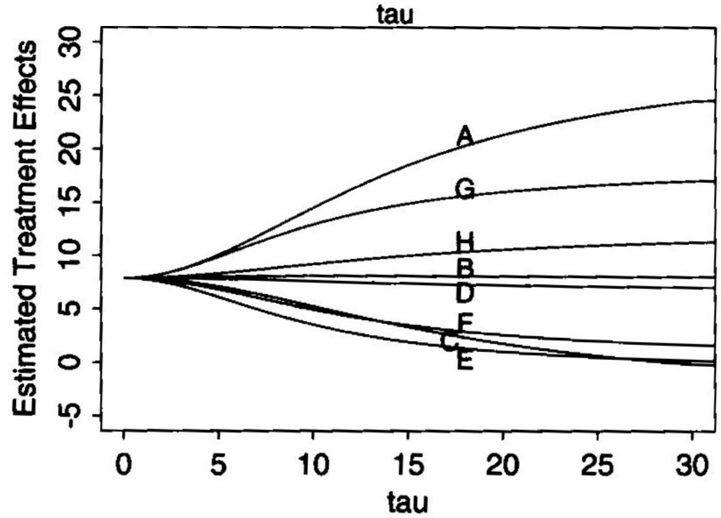
Naïve pooled estimate results in mean 7.9 with standard deviation 4.2

$P(\tau|y)$



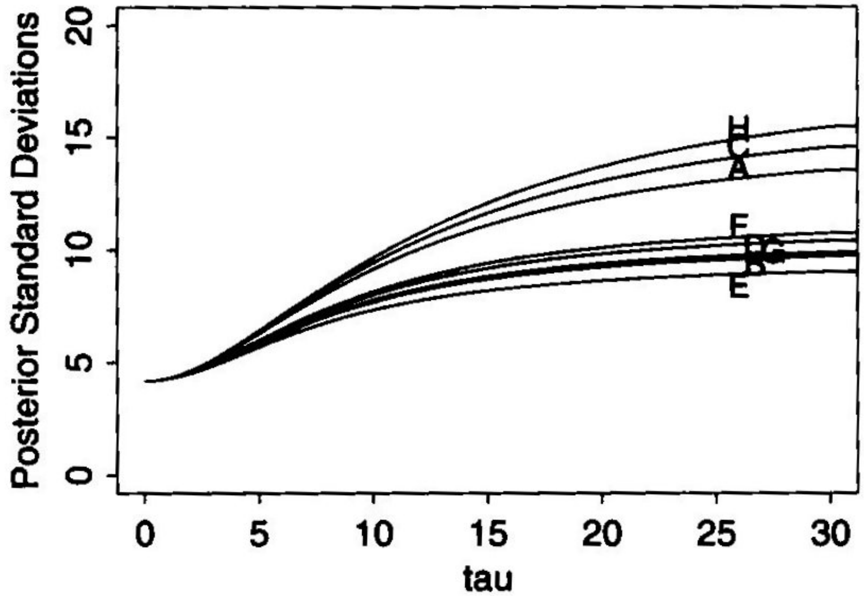
$P(A<28|\tau=10)=$
$\Phi((28-14.5)/9.1)=93\%$

$E(\theta_j|\tau,y)$



Bayesian Analysis

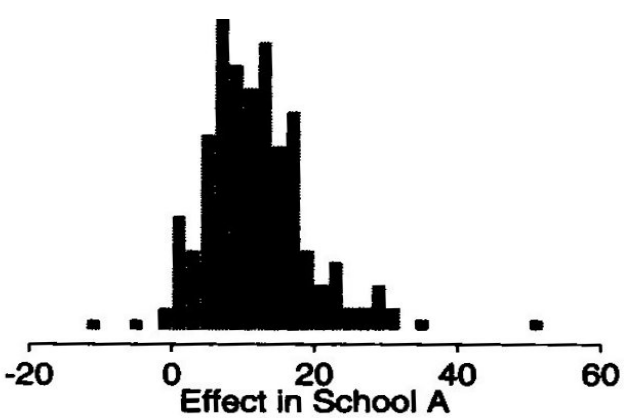
std($\theta_j|\tau$,y)

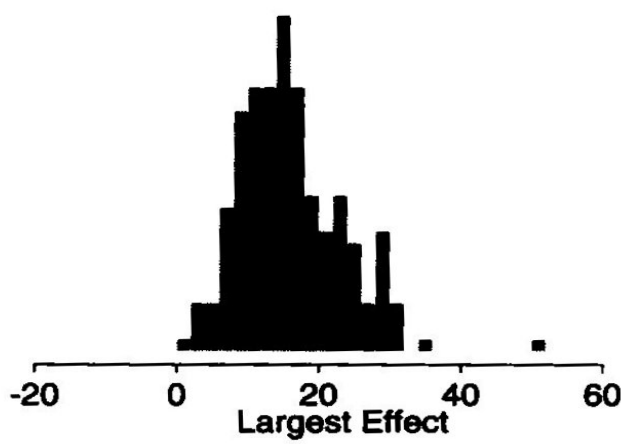


Summary of 200 posterior samples:

School	Posterior quantiles 2.5%	25%	median	75%	97.5%
A	−2	7	10	16	31
B	−5	3	8	12	23
C	−11	2	7	11	19
D	−7	4	8	11	21
E	−9	1	5	10	18
F	−7	2	6	10	28
G	−1	7	10	15	26
H	−6	3	8	13	33

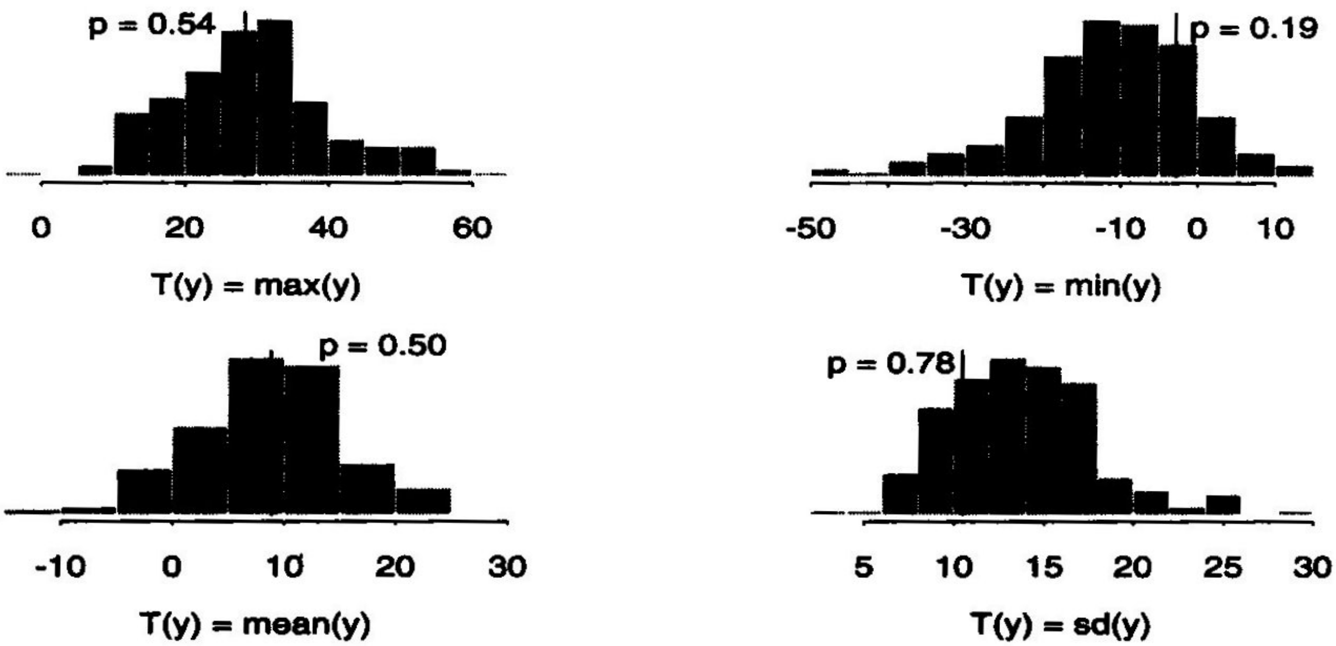
P(maxθ>28.4)=0.22
P(θ_A> θ_C)=0.705





Model	$D_{\hat{\theta}}$	$\widehat{D}_{\text{avg}}$	p_D	DIC
no pooling ($\tau = \infty$)	54.9	62.6	7.7	70.3
complete pooling ($\tau = 0$)	59.5	60.5	1.0	61.5
hierarchical (τ unknown)	57.8	60.6	2.8	63.4

Posterior test statistics



School	Posterior quantiles 2.5%	25%	median	75%	97.5%
A	−2	6	11	16	34
B	−5	4	8	12	21
C	−14	2	7	11	21
D	−6	4	8	12	21
E	−9	1	6	9	17
F	−9	3	7	10	19
G	−1	6	10	15	26
H	−8	4	8	13	26

Table 17.1 *Summary of 2500 simulations of the treatment effects in the eight schools, using the t_4 population distribution in place of the normal. Results are similar to those obtained under the normal model and displayed in Table 5.3.*

Model Checking

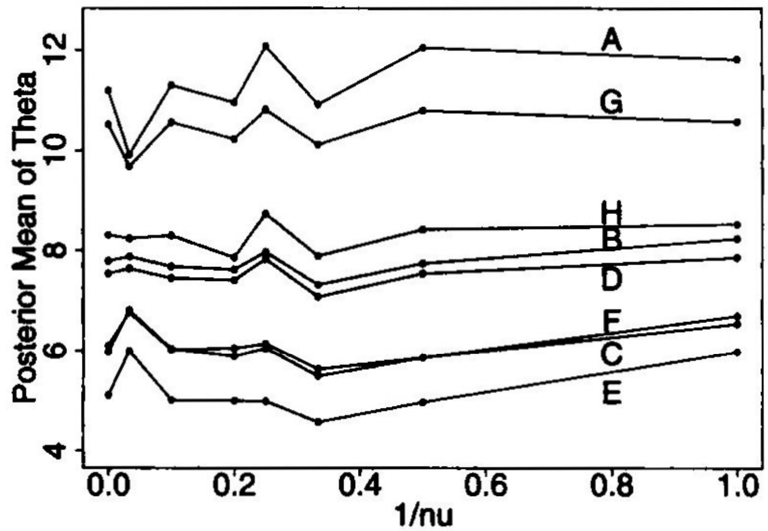


Figure 17.1 *Posterior means of treatment effects as functions of ν, on the scale of $1/\nu$, for the sensitivity analysis of the educational testing example. The values at $1/\nu=0$ come from the simulations under the normal distribution in Section 5.5. Much of the scatter in the graphs is due to simulation variability.*

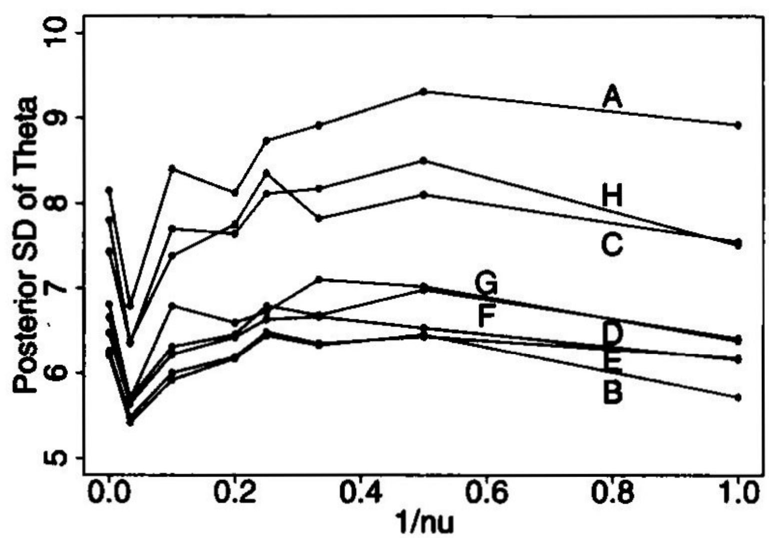


Figure 17.2 *Posterior standard deviations of treatment effects as functions of ν, on the scale of $1/\nu$, for the sensitivity analysis of the educational testing example. The values at $1/\nu=0$ come from the simulations under the normal distribution in Section 5.5. Much of the scatter in the graphs is due to simulation variability.*