Gaussian Mixture Copula Function

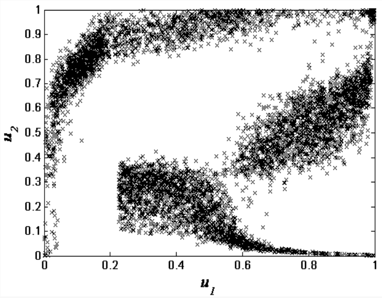
Objective: Design a Copula function to capture dependencies in multimodal distributions.

Motivation: Known Copula families are not designed to capture dependencies in multimodal distributions (*absence of location parameter* !!!)

Example: *Clayton* Copula

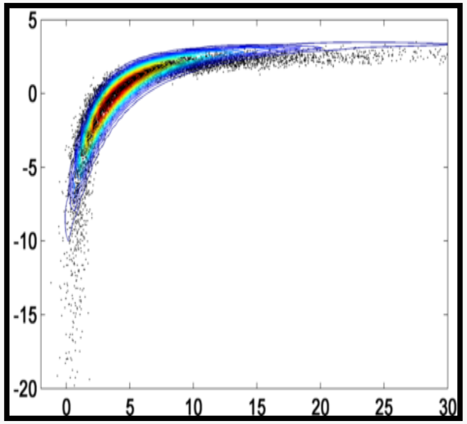
$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$$

Not suitable to model a multimodal distribution

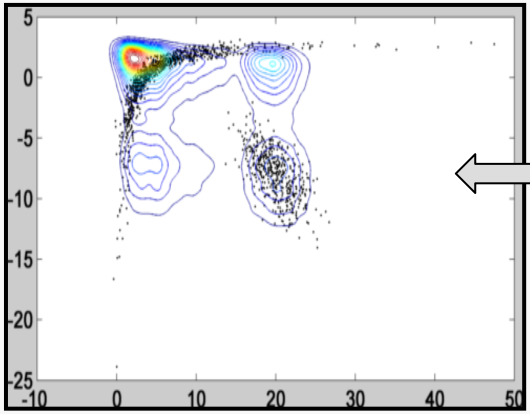


***Gaussian* Copula Based Joint Density Fit on:**

Unimodal Dataset



Bimodal Dataset



Gaussian copula could not capture the dependence in a bimodal dataset. !!!

Problem formulation and preliminary results.

Copula function derived from an arbitrary density

$$c(u_1, u_1 \cdots u_n) = \frac{f\left(F_1^{-1}(u_1), \cdots \cdots F_n^{-1}(u_n)\right)}{f_1\left(F_1^{-1}(u_1)\right) \times f_2\left(F_2^{-1}(u_1)\right) \cdots f_n\left(F_n^{-1}(u_n)\right)}$$

Substitute

Sum of Gaussians

$$f = \psi = \sum_k \alpha^{(k)} \phi^{(k)}\left(x_1, x_2 \dots x_n; \theta^{(k)}\right)$$

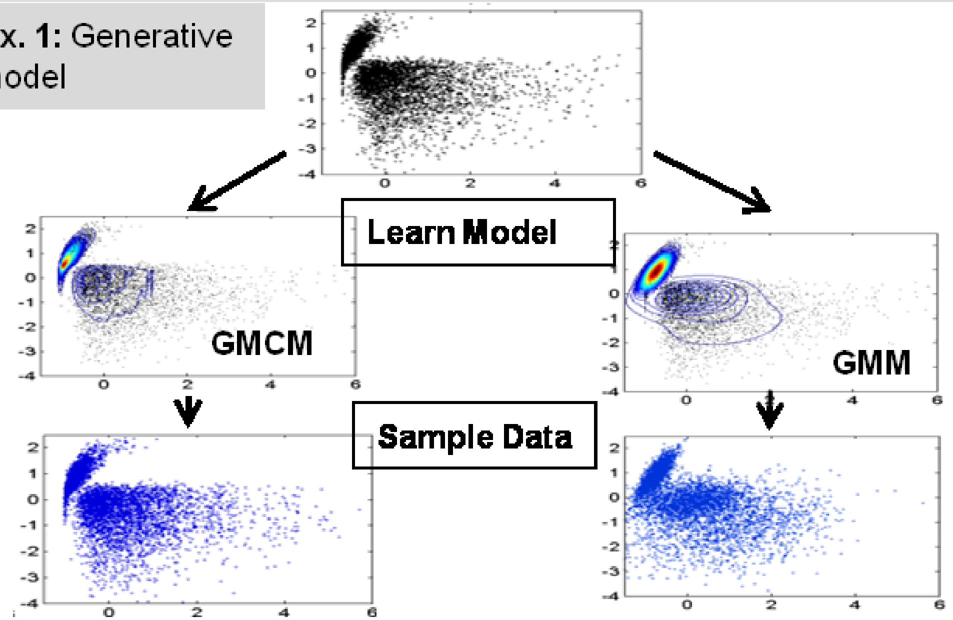
$$f_j = \psi_j$$ ← Marginal Density GMM

$$F_j^{-1} = \Psi_j^{-1}$$ ← Inverse function of GMM marginal CDF

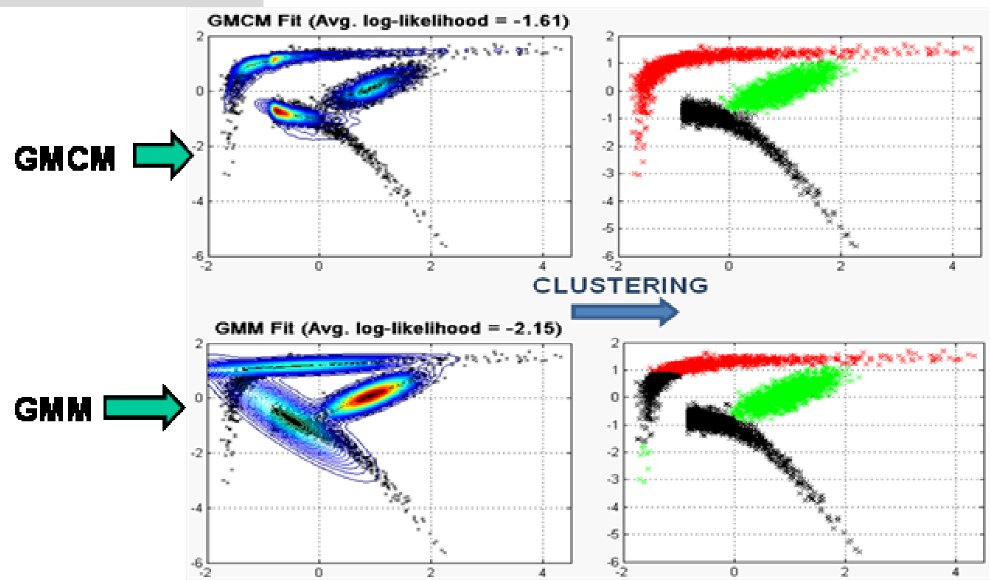
Gaussian Mixture Copula Function

$$c_{gmc}(u_1, u_2 \dots u_n; \Theta) = \frac{\psi\left(\Psi_1^{-1}(u_1), \Psi_2^{-1}(u_2) \cdots \Psi_n^{-1}(u_n); \Theta\right)}{\psi_1\left(\Psi_1^{-1}(u_1)\right) \times \psi_2\left(\Psi_2^{-1}(u_2)\right) \cdots \psi_n\left(\Psi_n^{-1}(u_n)\right)}$$

Ex. 1: Generative model



Ex. 2: Clustering



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