Significant Occurrence in Even Musical Texture in Bach's Preludes

A Study Using Mathematical Tools

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Israel

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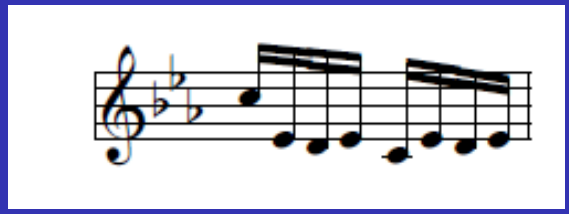
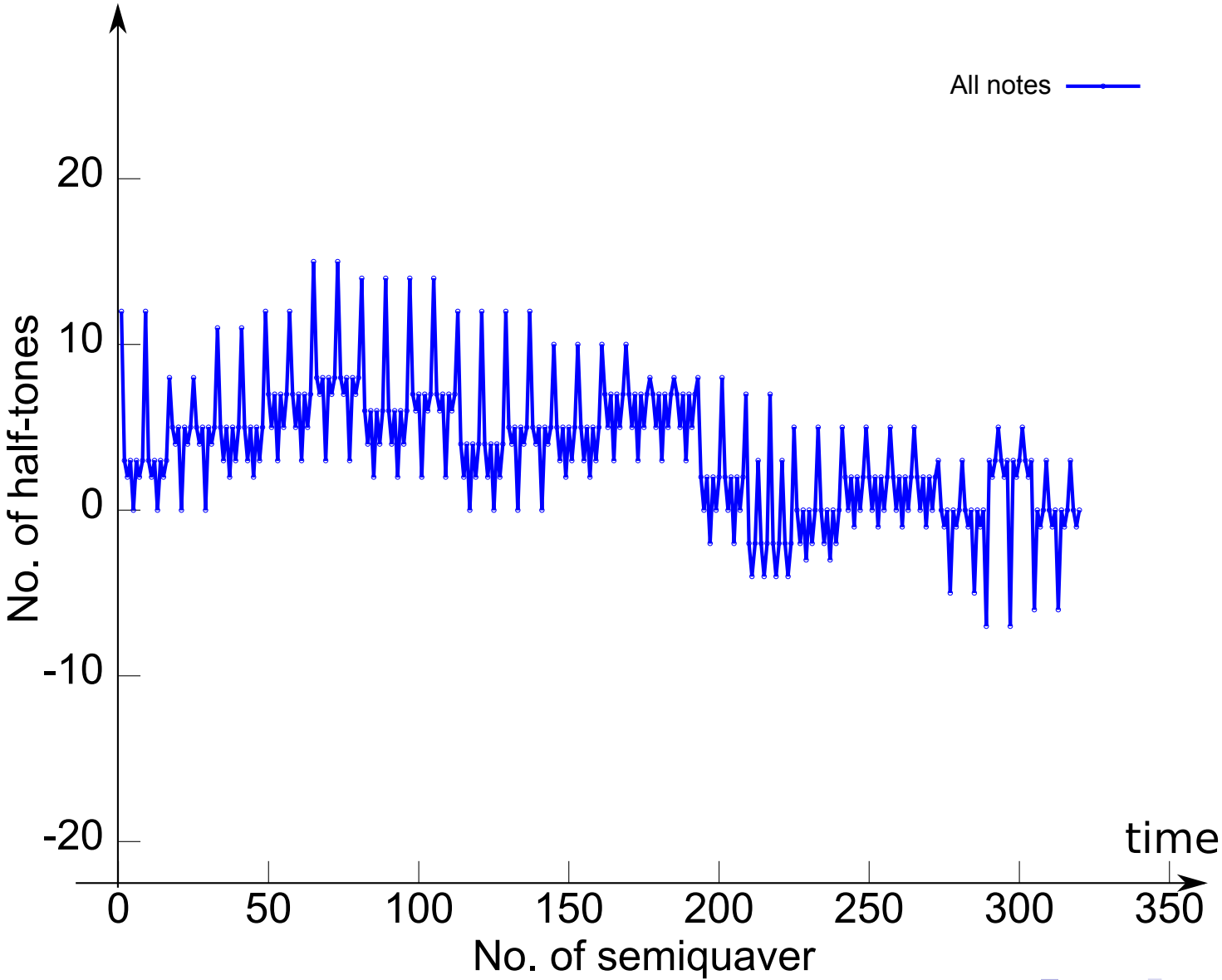
Curve of all notes of Do minor





Basic Definitions

1. **"Learned" schemata**
   - Collection of notes
   - Intervals
   - Harmonies
   - Tonal organization

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   - Intervals
   - Harmonies
   - Tonal organization
2. **"Natural" schemata**
   - Range of occurrence related to normative range
   - Types of curves of change (in our study - only of **pitch**)
   - Cognitive **operations**: augmentation/diminishing, contrast, shift, splitting and gathering, equivalence
   - Concurrence and non-concurrence, rarity

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   - includes small, hidden, meaningful deviations of all the natural schemata
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2. "**Evenness**" - a constant parameter or characteristic
   - Duration
   - Pitch curve of the **pattern** (up to small deviations)

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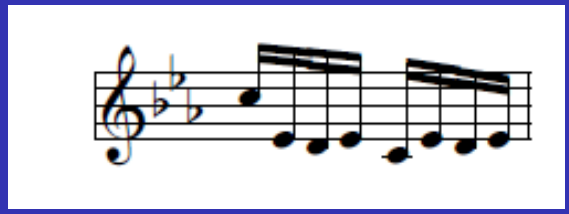

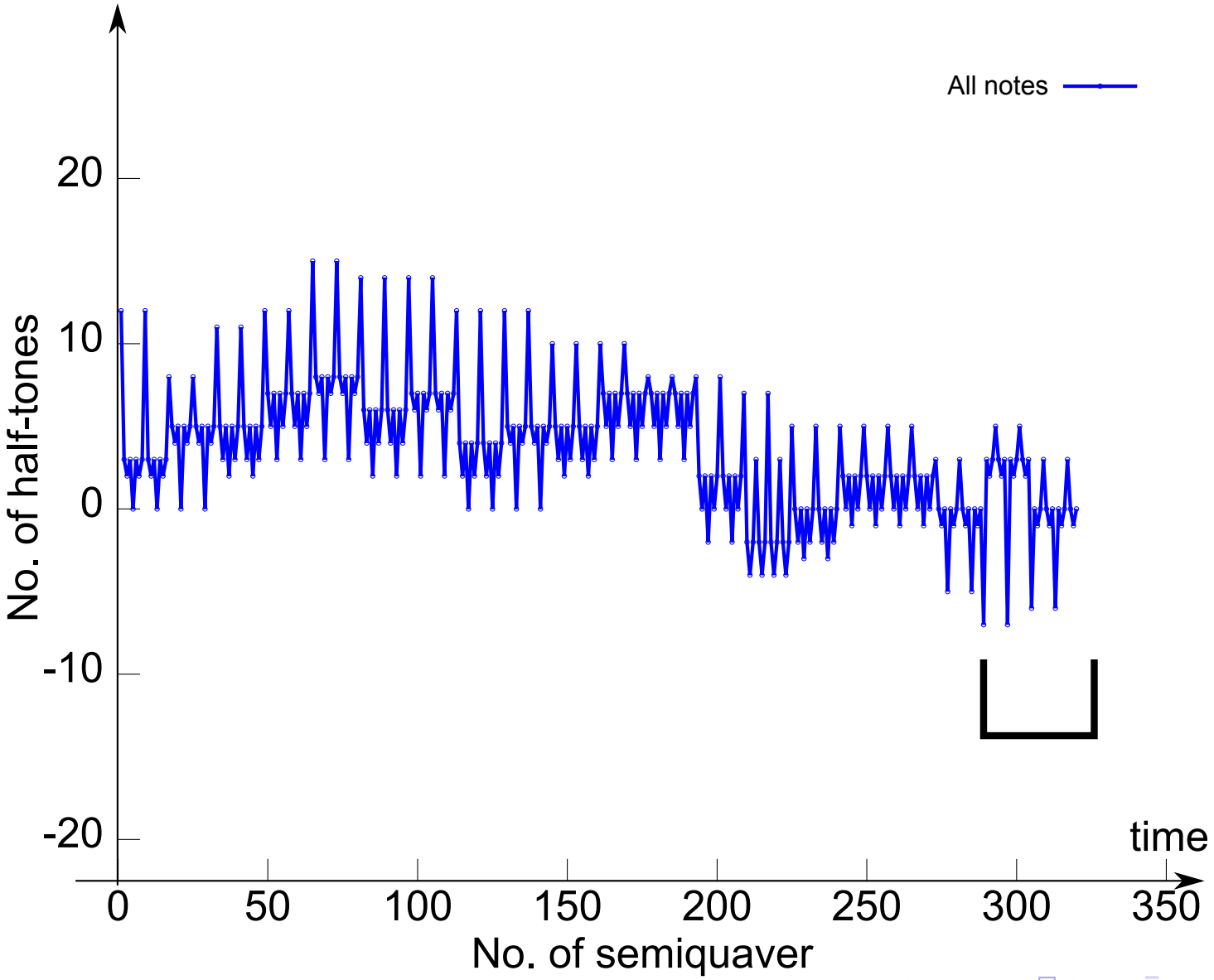

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Selecting Preludes from the **WTC-I by Bach**
with common characteristics

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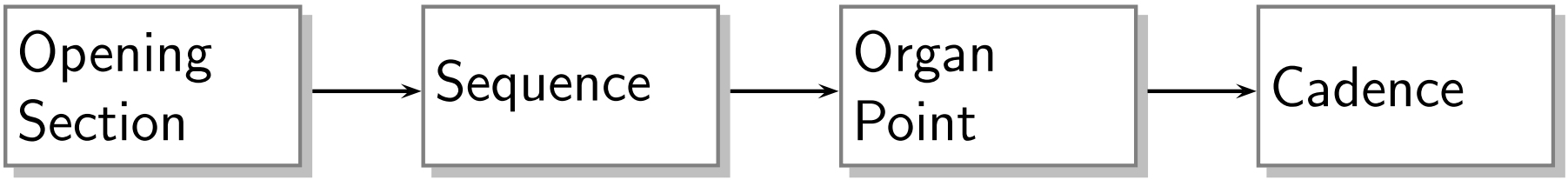
- **similar structure**

The Generic Model of an Even Prelude by Bach



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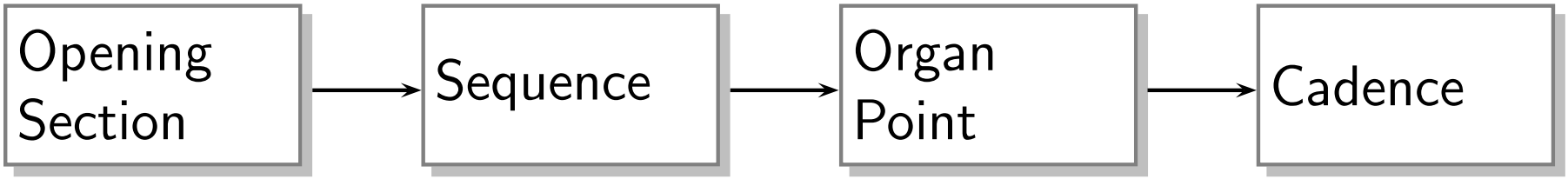

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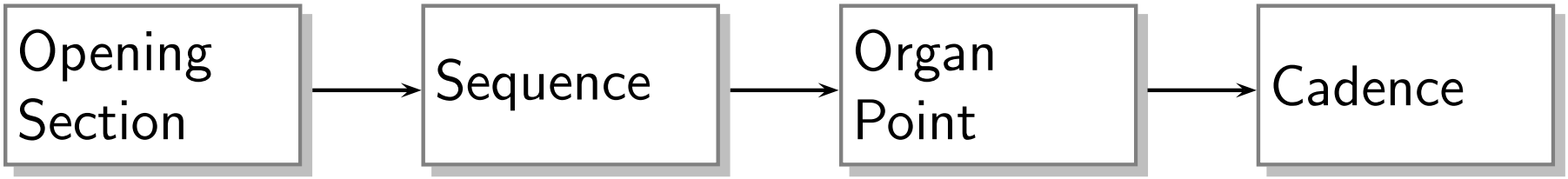

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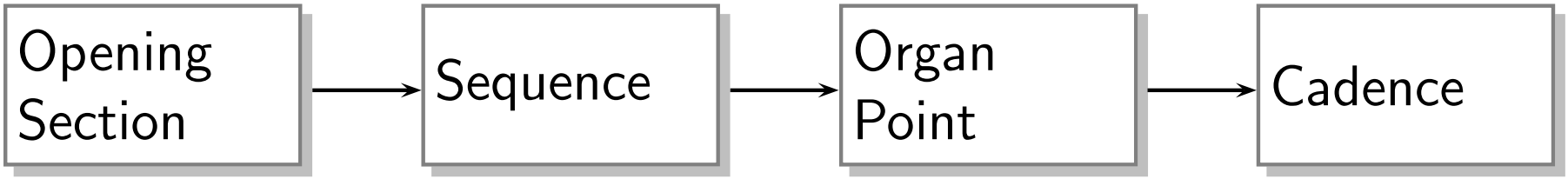

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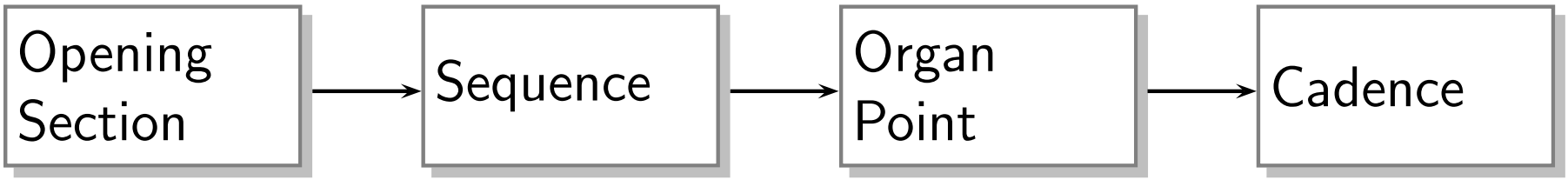
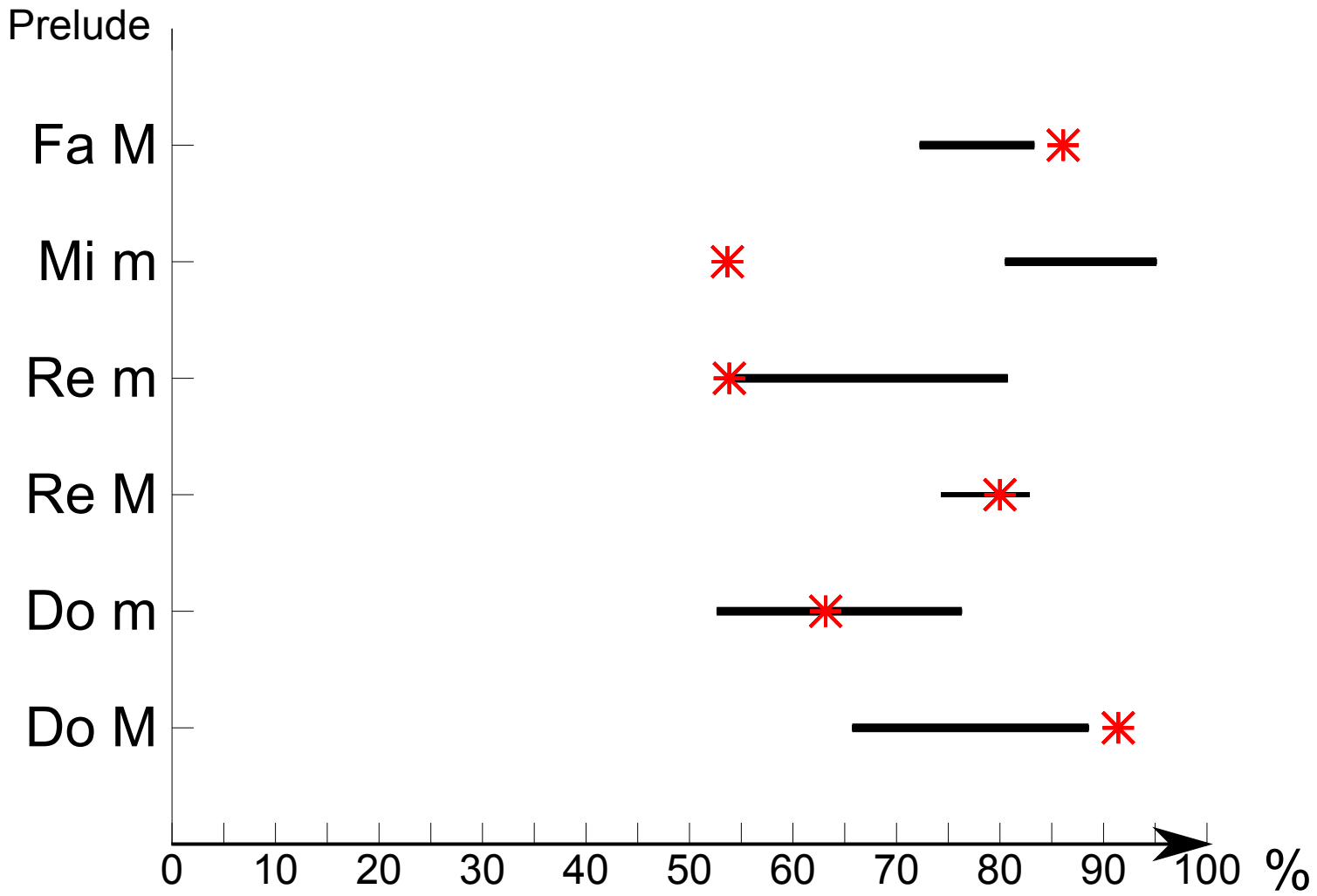


1. I,II,V,I; or I, IV, VII, I;
2. Bach is using one of his famous **Figured Bass Formulas** e.g. (second, sext), 2,6,2,6,...
3. The Organ Point - appears usually in the second half of the prelude
4. The Cadence - '**Bach the Believer**'

The Location of the Organ points



Basic Musical Units

1. Nuclei

Basic Musical Units

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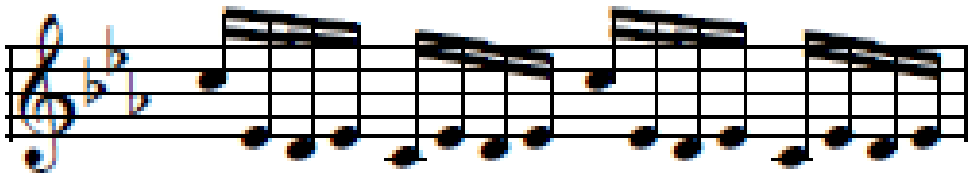
2. Patterns

Basic Musical Units

1. Nuclei
2. Patterns
3. Groups of patterns (higher levels)

Convex and Concave curves of Nuclei and Patterns

1. First pattern (repeated), of Do minor prelude (no. 2)



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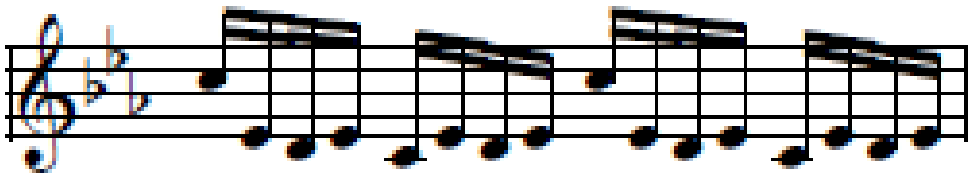
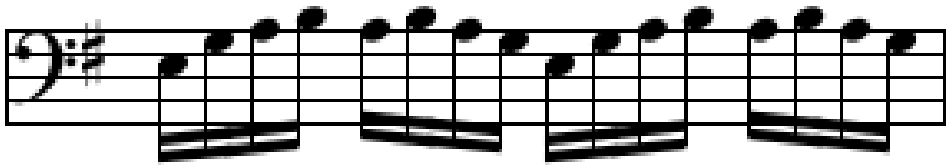


2. First pattern (repeated), of Mi minor prelude (no. 10)



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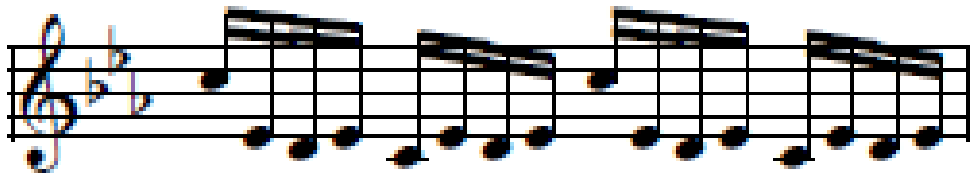

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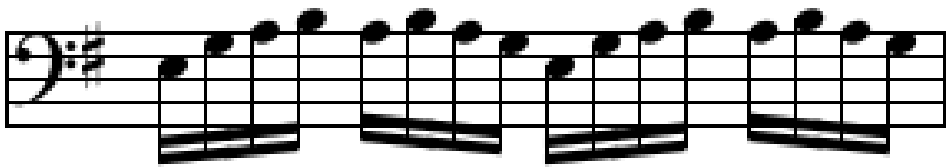
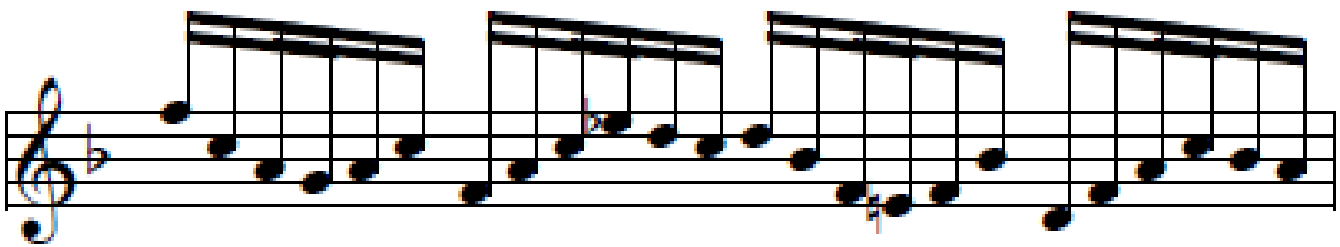
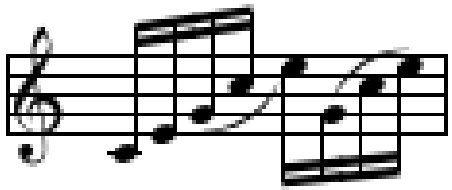


3. First and second patterns of Fa M prelude (no. 11)



Straight-Lines Curves in Different Directions

1. First pattern of Do M prelude (no. 1)



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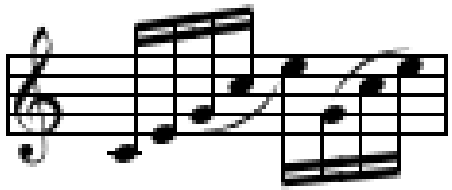
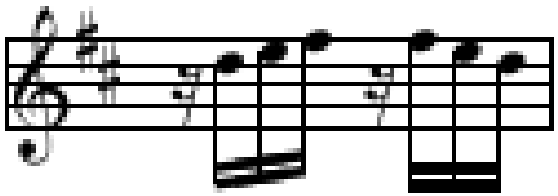


2. First pattern of Re M prelude (no. 5)



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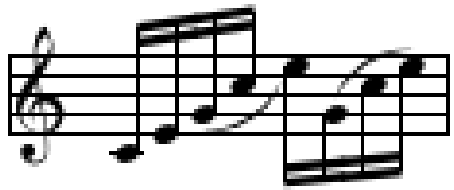

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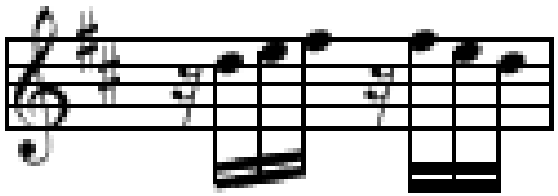
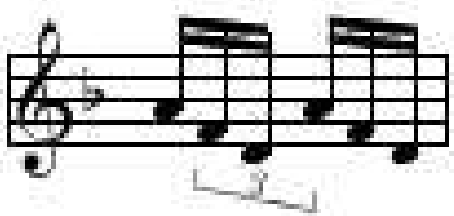
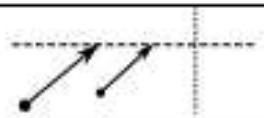
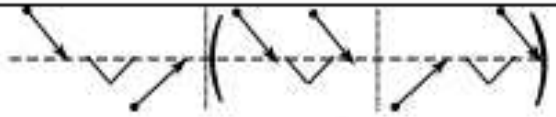
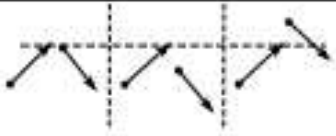
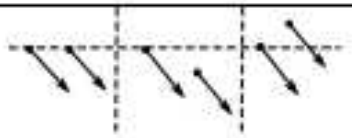
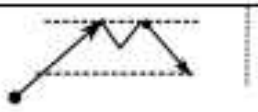
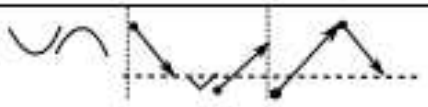


3. First pattern of Re minor prelude (no. 6)

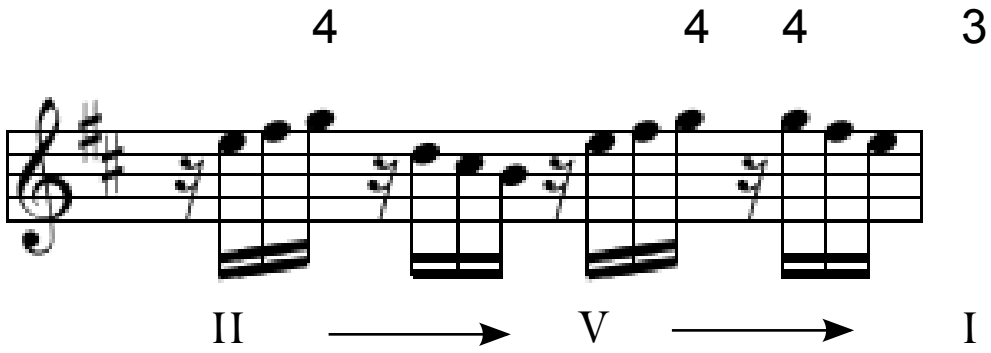


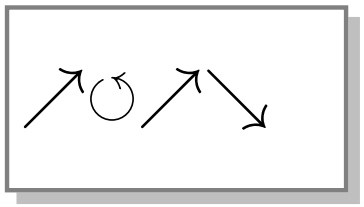
Basic Graphical Presentation of Nuclei

WTC-I Prelude	→ All Nuclei Directions	→ Basic Direc.	→ 1st Nucleus	→ 2nd Nucleus
Do M		↗↗	↑△	↑△
Do m		↘↗	↓≒, ∨	↑≒, ∨
Re M		↗↘	↑≮, ≒	↓≮, ≒
Re m		↘↘	↓△	↓△
Mi m^*		↗↘	↑≒, ≮	∨, ↓≮
Fa M^{**}		↘↗; ↗↘	↓△, ∨, ↑≒	↑△, ↓≮

Special Grouping of Nuclei in Re M

Second Bar of Re M Prelude (no. 5)

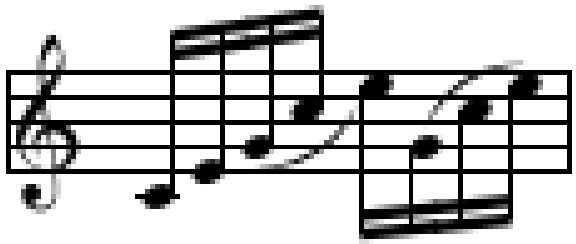




- bar 2 $\rightarrow$ I; bar 5 $\rightarrow$ V
- bar 10 = $\mathrm{III}\rightarrow\mathrm{II}$; bar 12)=$\mathrm{II}\rightarrow\mathrm{I}$
- bar 18 = VI; bar 19 $\rightarrow$ V:IV
- bar 21 $\rightarrow$$\mathrm{IV}$; bar 24 $\rightarrow$ I

Second step

1. Expressing curves of pitch by numbers, always related to the first pattern - the reference pattern.
   [0, 4, 7, 12, 16], is the first nucleus of Do M.



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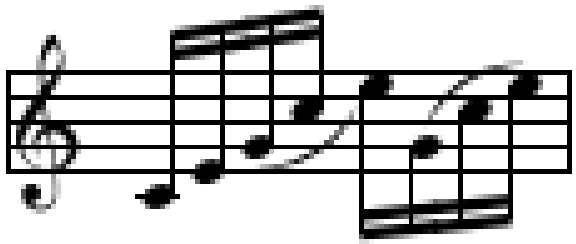

2. Analyzing them by using **Musical and Mathematical** tools

Mathematical Tools

1. **Peak notes and low notes** (of the upper voice)

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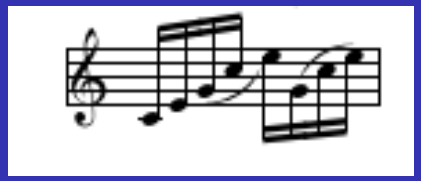
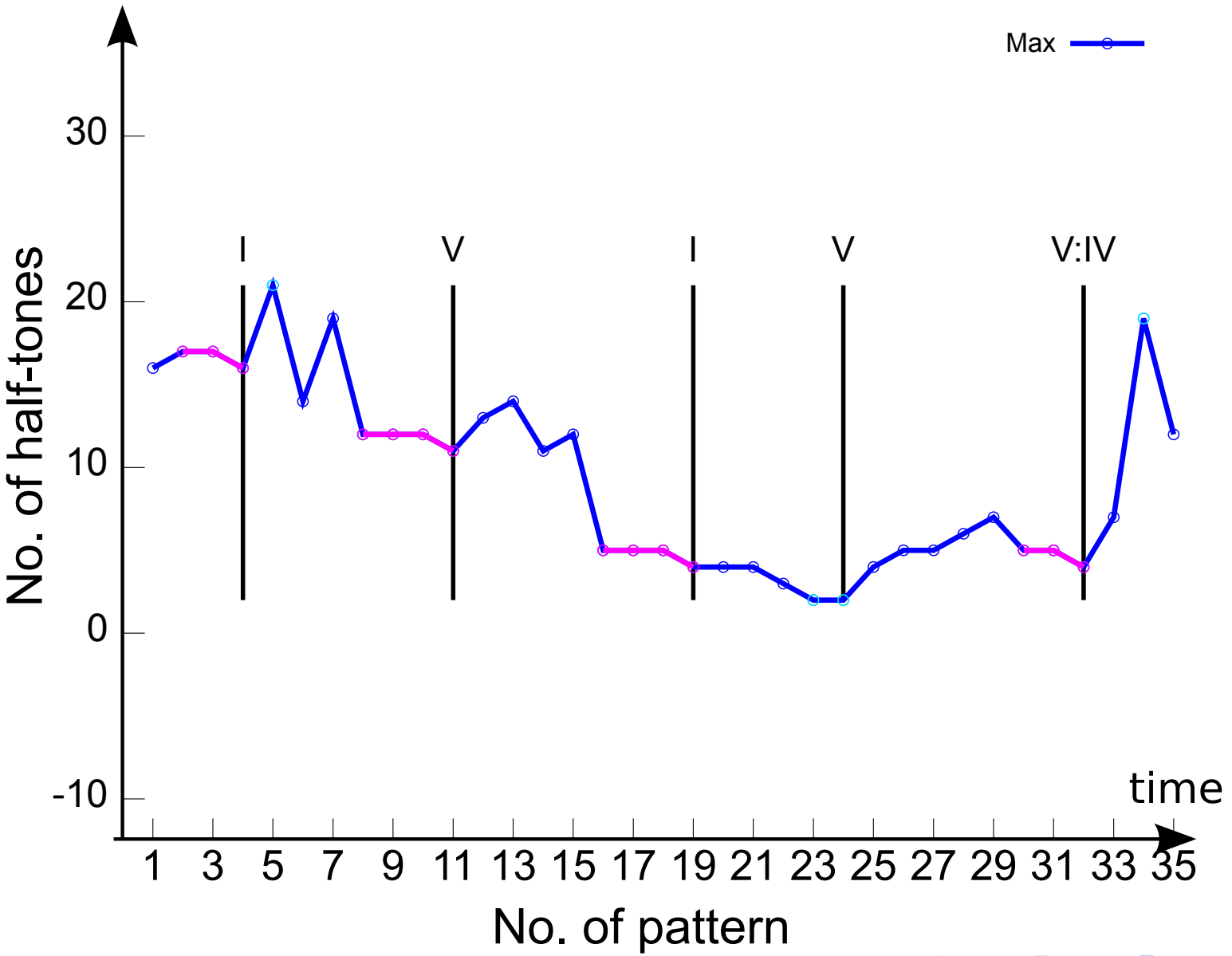
Curve of all Peak-notes of Do M





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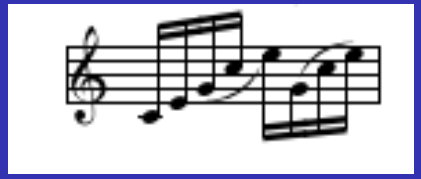
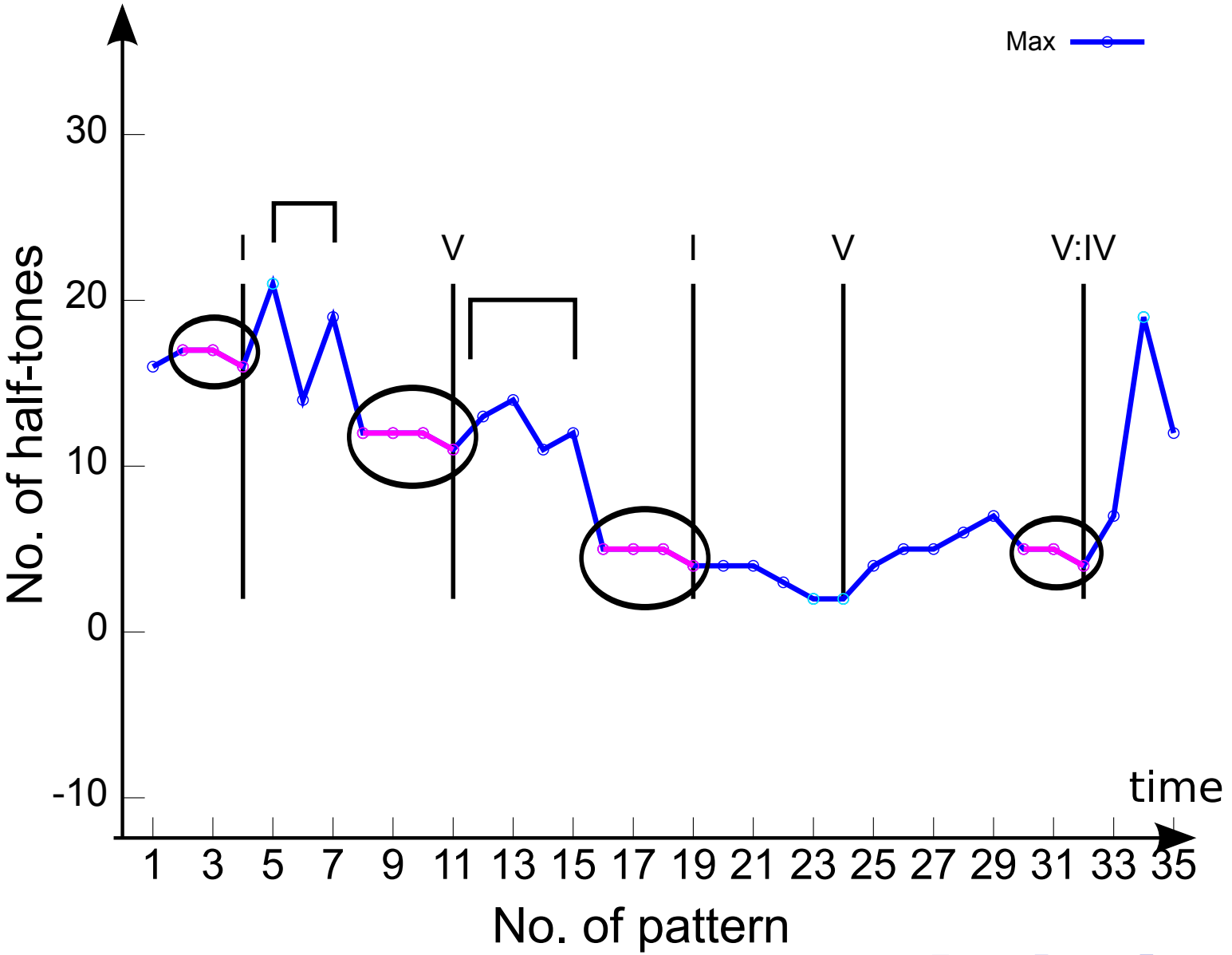
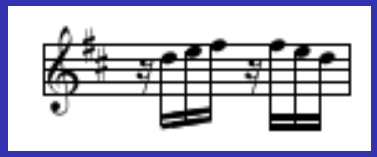
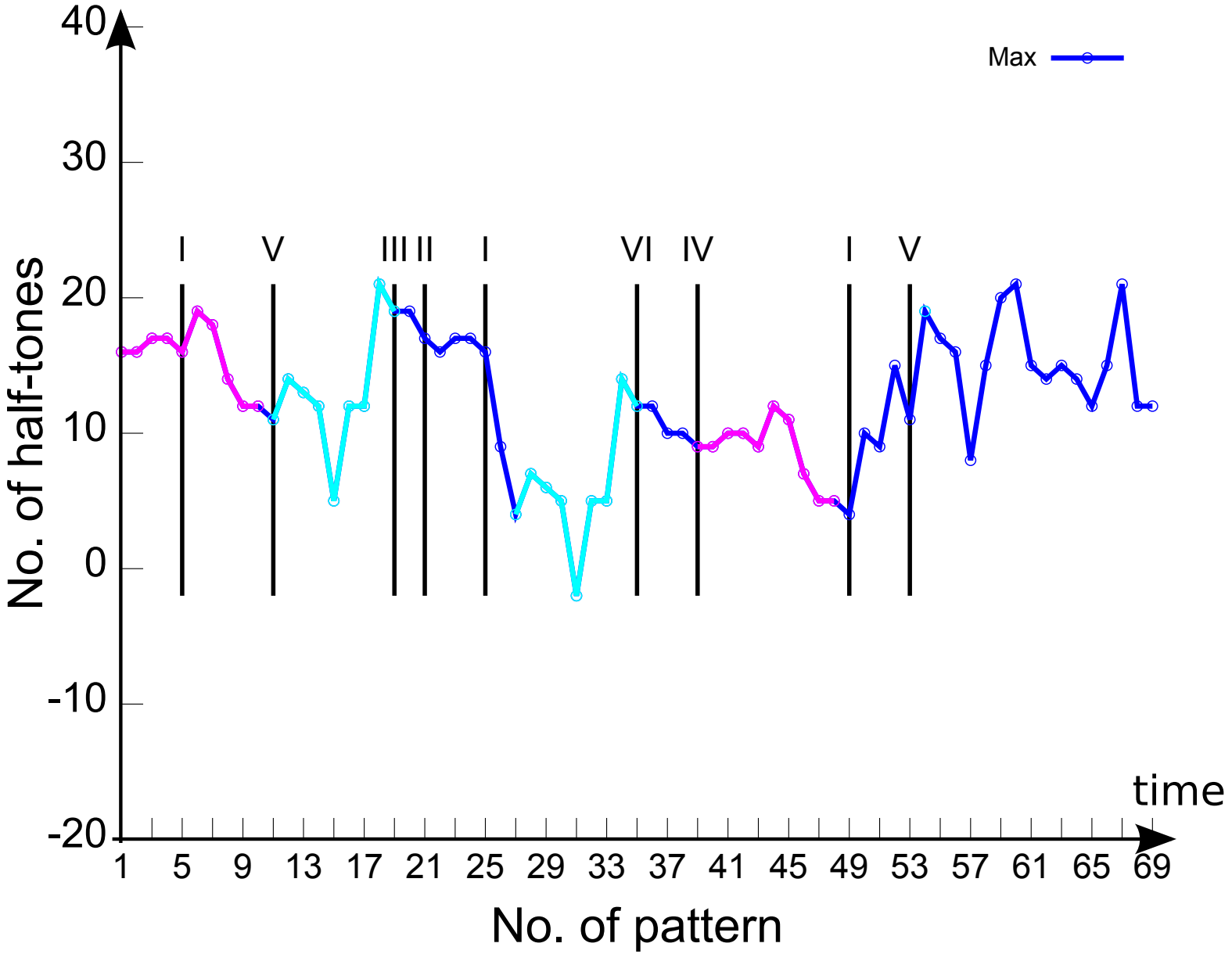




Curve of all Peak-notes of Re M





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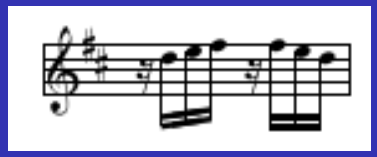
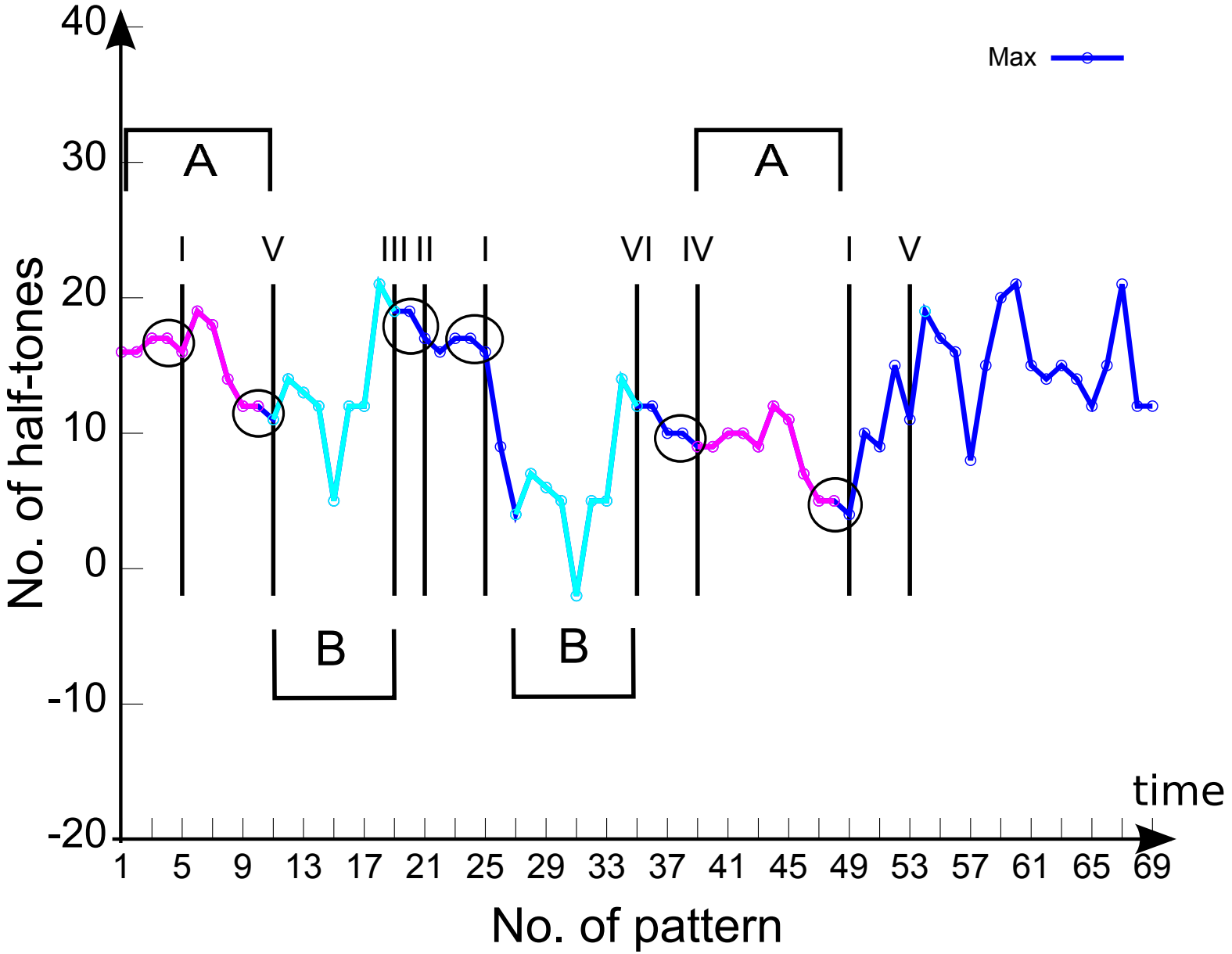




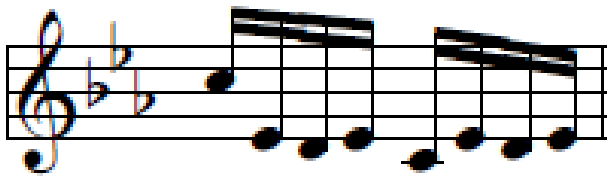
Mathematical Tools - cont.

Statistical Averages

Median - 'center of gravity' or an 'inner organ point'

Median - a sort of mean of one pattern

First pattern of Do minor prelude (no. 2)



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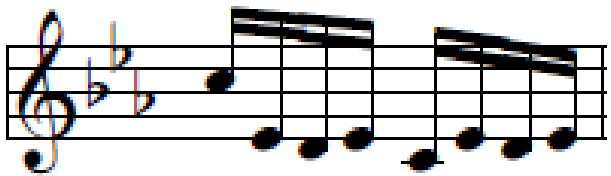
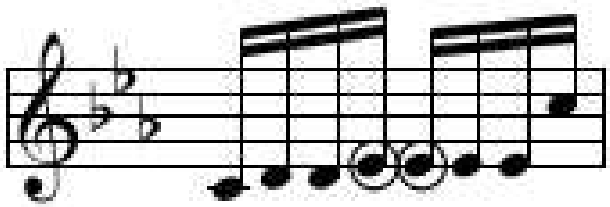
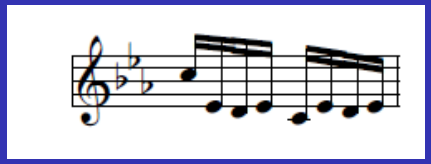


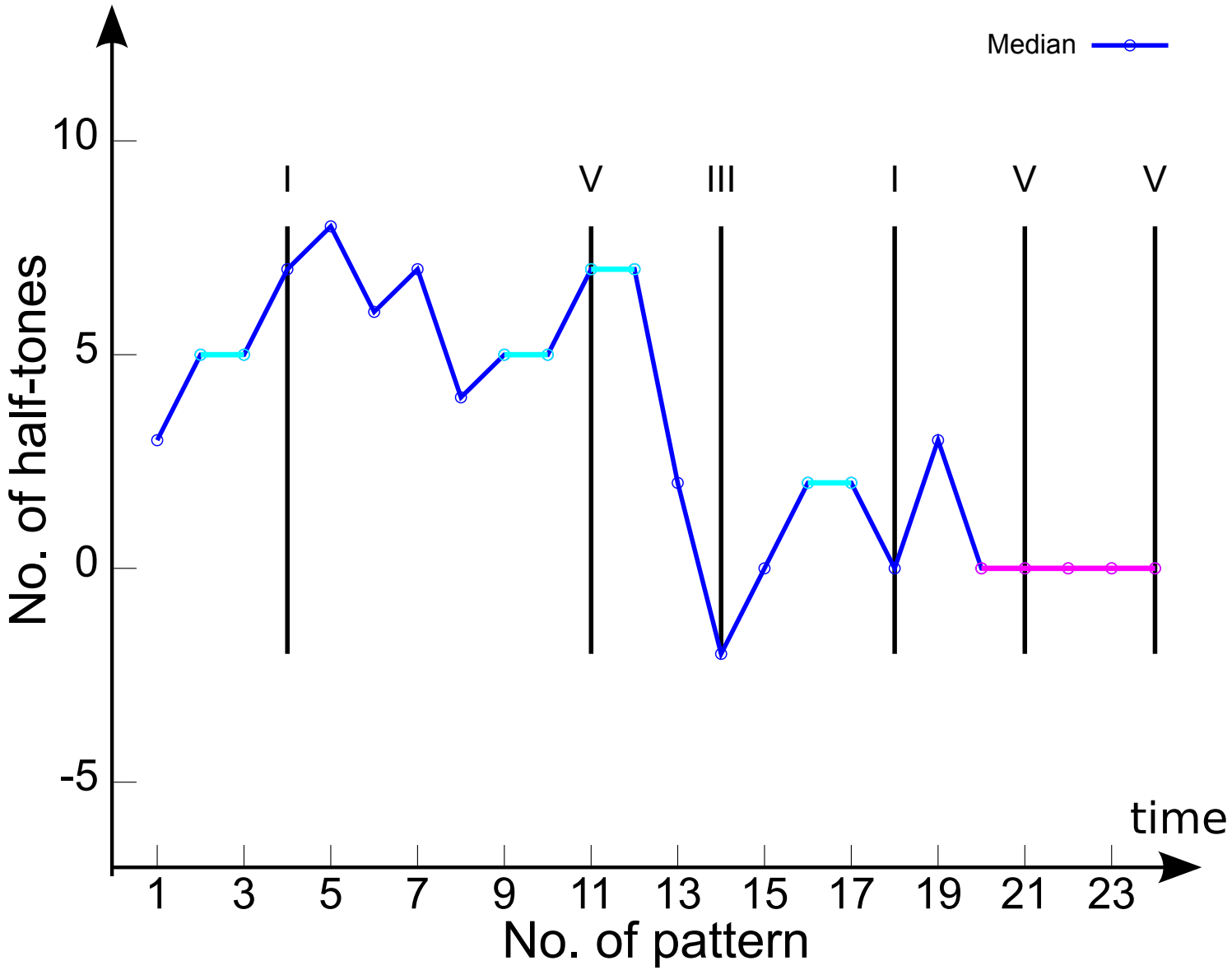
In order to obtain the **Median**, we arrange all pitches according to increasing pitch.



The pitch that is located in the middle of the row, is the median - **Mi** ♭.

Curve of all Medians of Do minor





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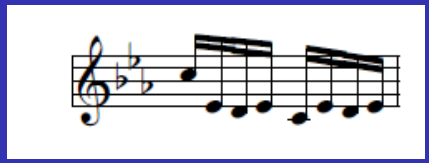
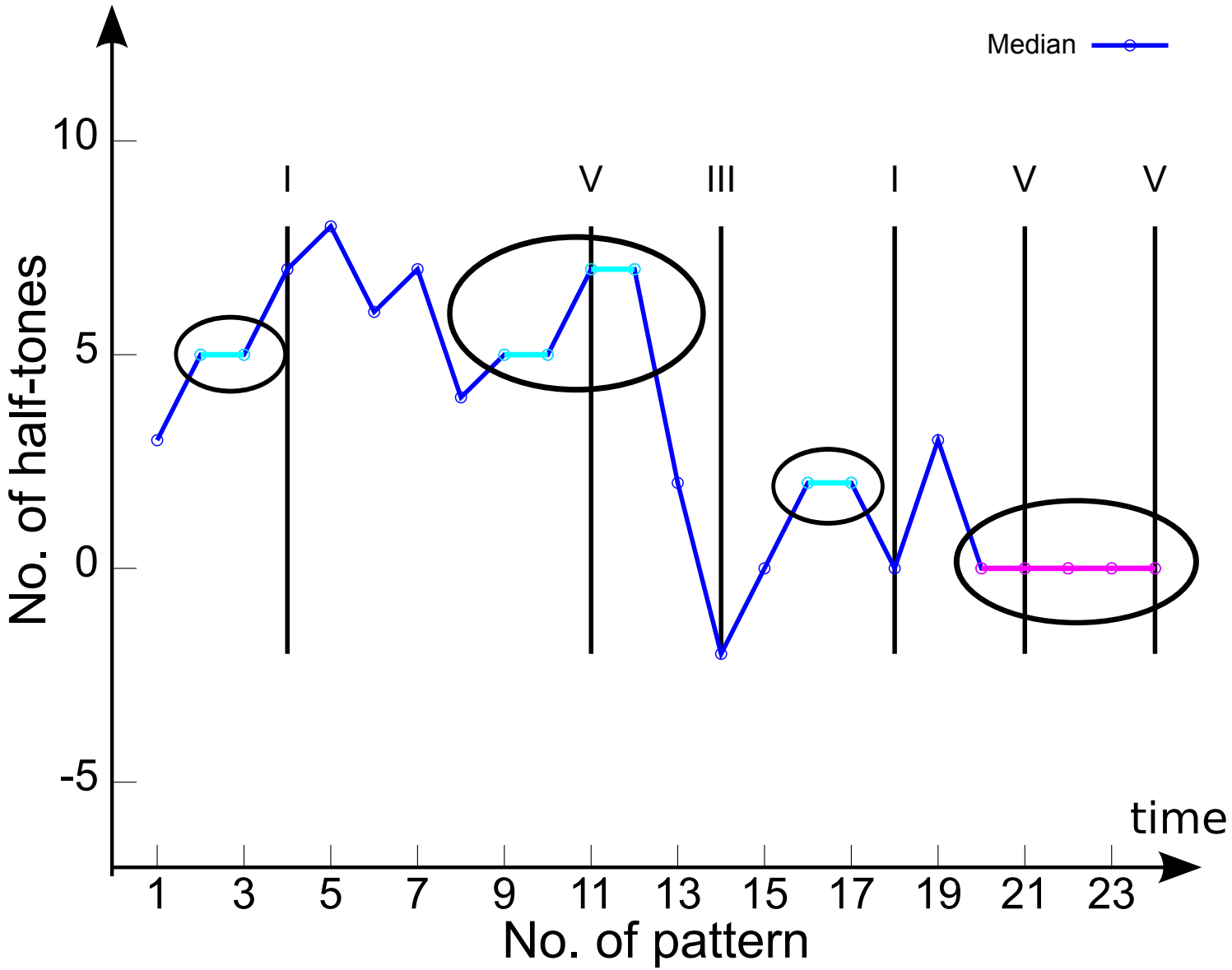




Mathematical Tools - cont.

Geometric Statistics

- **Linear Regression**
- **Parabolic Regression**

Regression line - a comparison between two patterns

$$y = a \cdot x + b$$

$$\text{Find} \quad \min_{a,\, b} S(a, b), \quad \text{where :}$$

$$S(a, b) = \sum_{i=1}^{n} \left(y_i - (a \cdot i + b)\right)^2$$

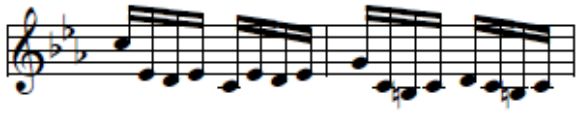
$$\frac{\partial S}{\partial a} = 0$$

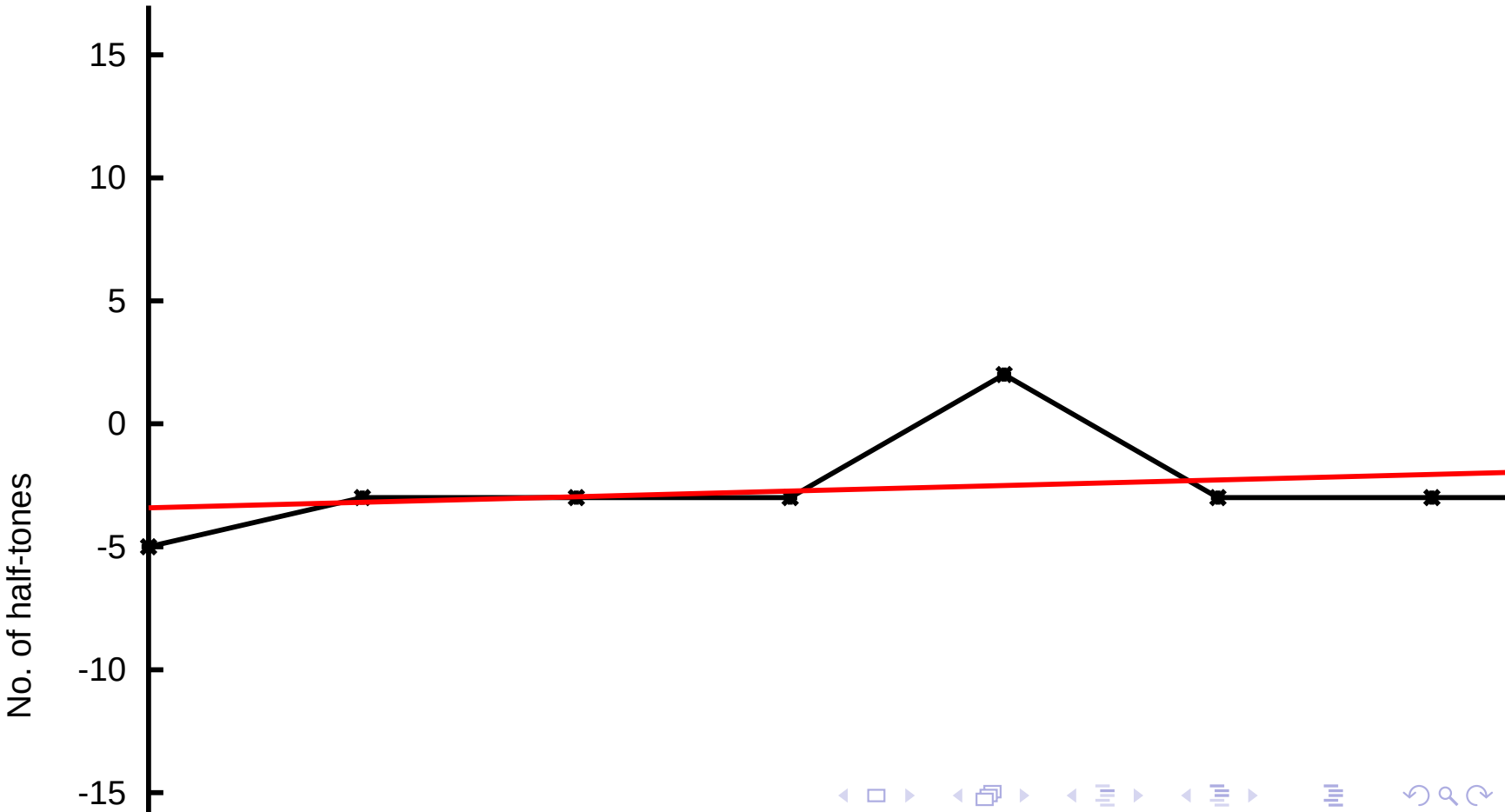
$$\frac{\partial S}{\partial b} = 0$$

$$\Rightarrow a, b = \ldots$$

Creating a Regression Line

Patterns $1, 23$**, in Do** m.





Parabolic Regression - a comparison between two patterns

$$y = a \cdot x^2 + bx + c$$

$$\text{Find} \quad \min_{a,\, b,\, c} S(a, b, c), \quad \text{where :}$$

$$S(a, b, c) = \sum_{i=1}^{n} \left(y_i - (a \cdot i^2 + b \cdot i + c)\right)^2$$

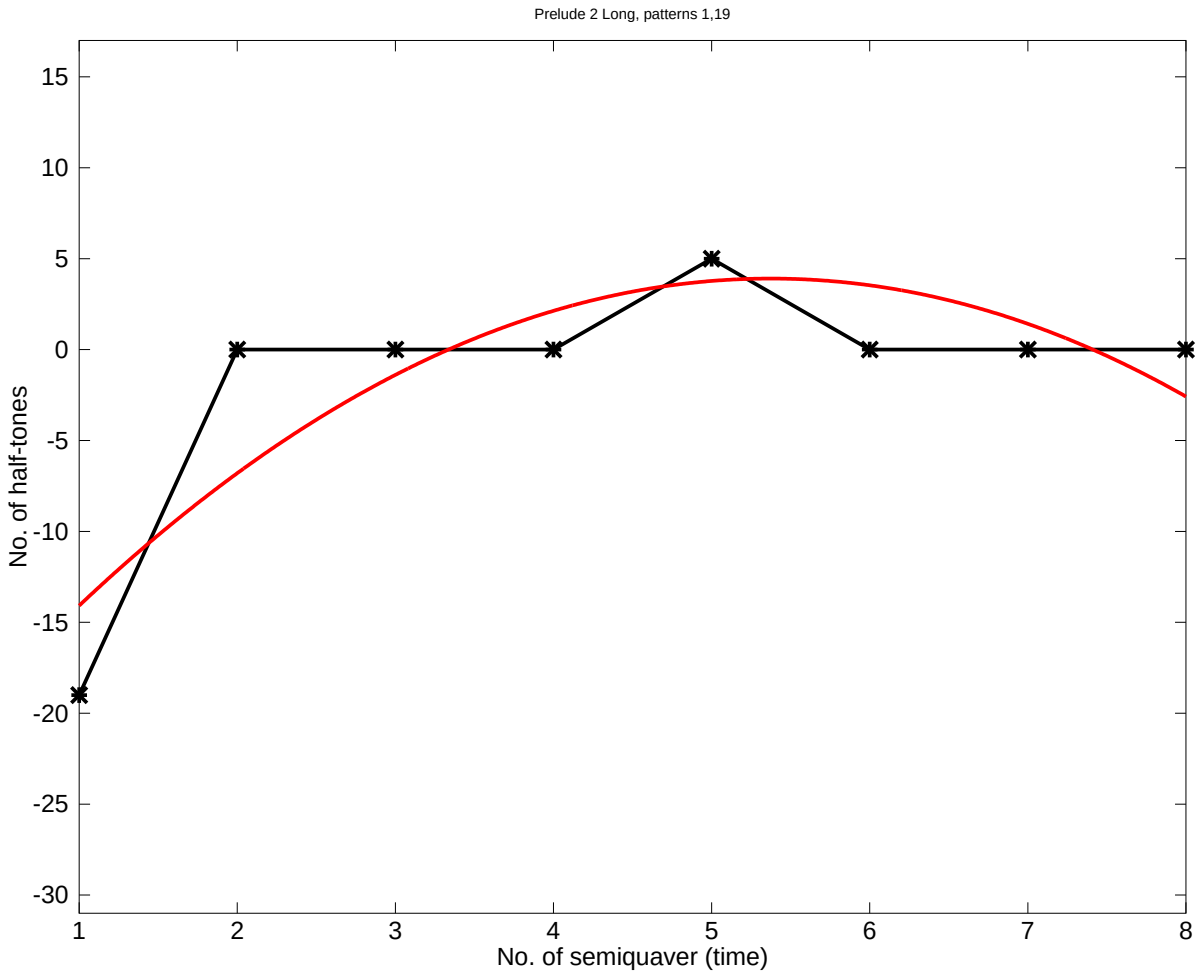
$$\frac{\partial S}{\partial a} = 0$$

$$\frac{\partial S}{\partial b} = 0$$

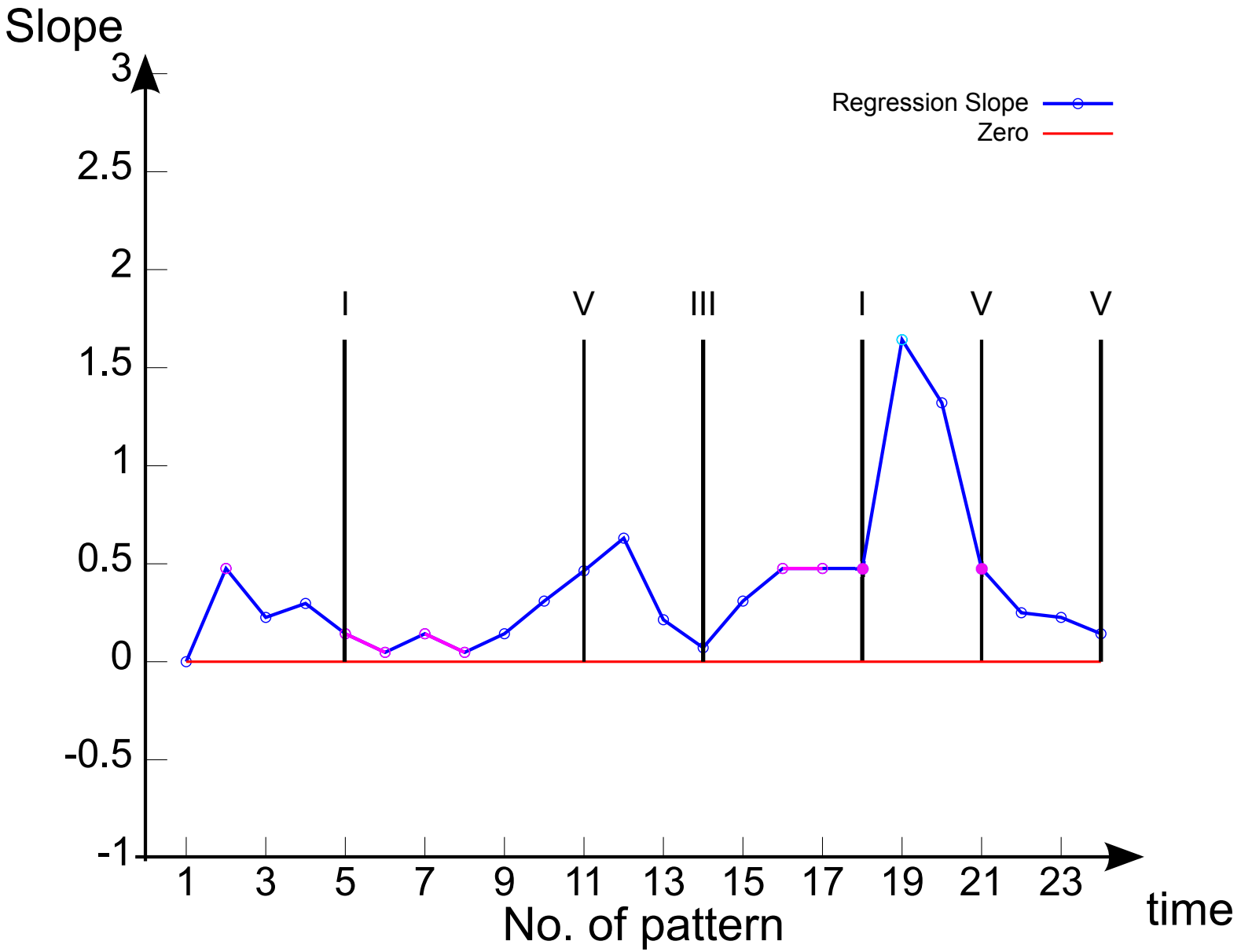
$$\frac{\partial S}{\partial c} = 0$$

$$\Rightarrow a, b, c = \ldots$$

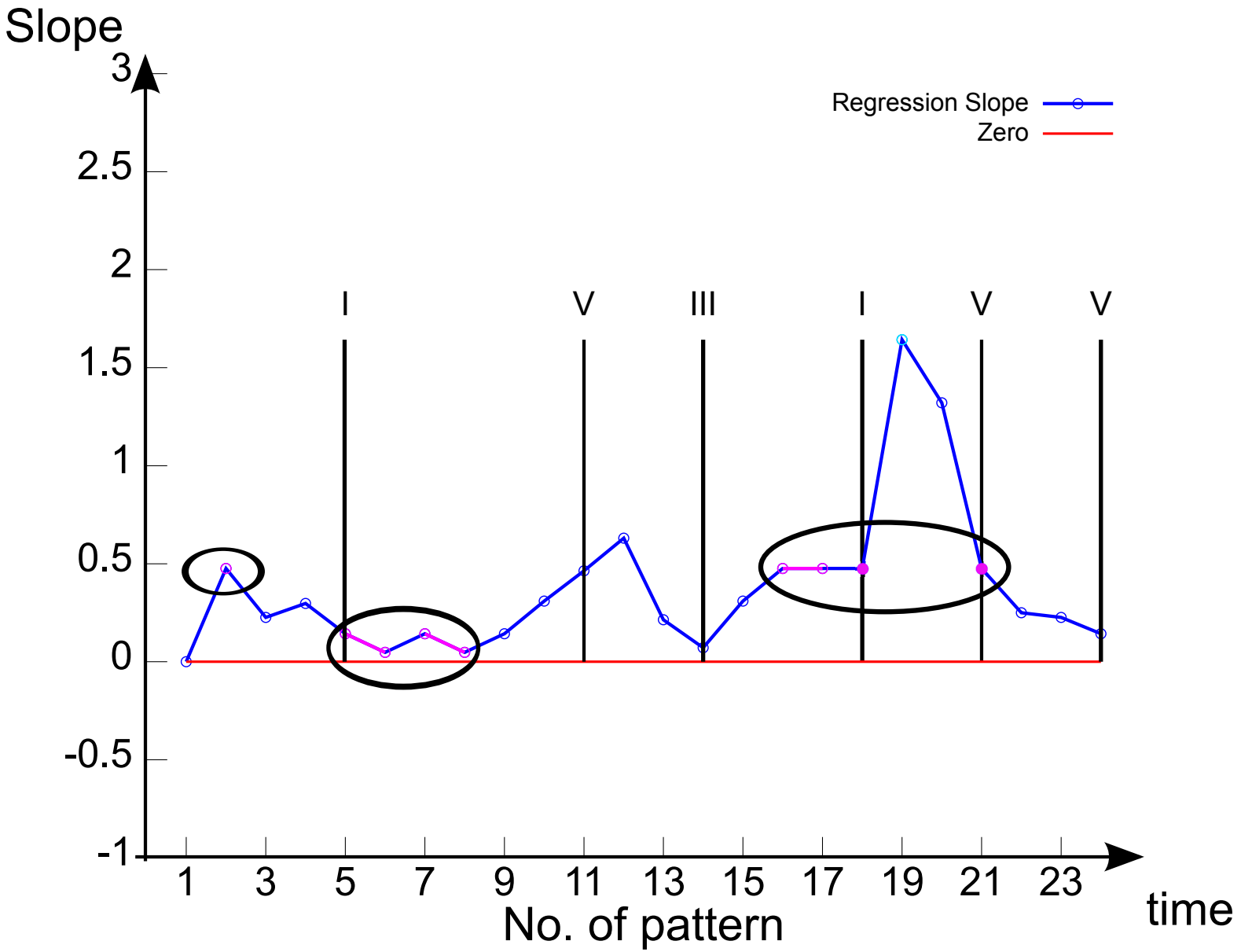
Creating a Parabolic Regression Curve - Do minor



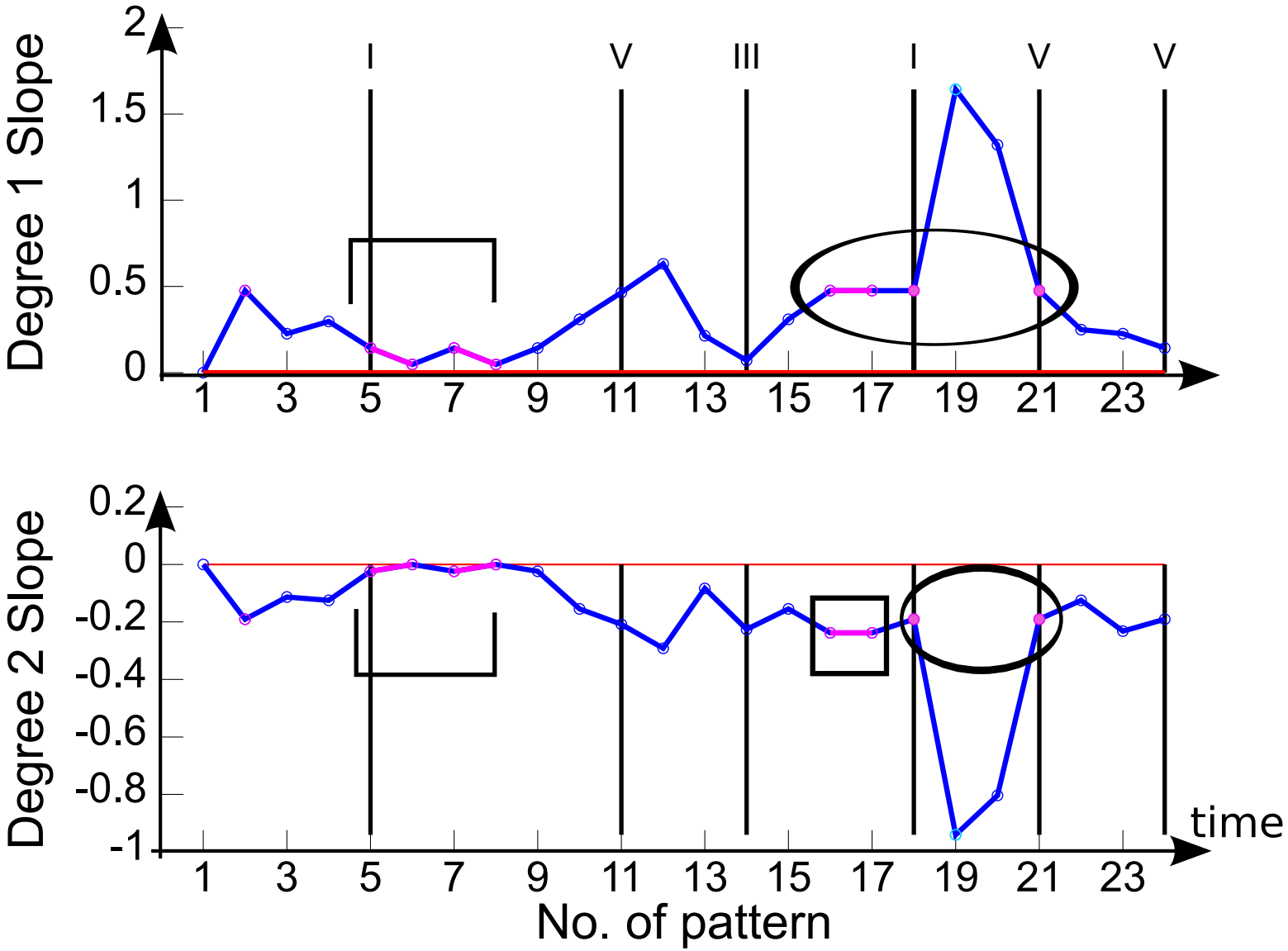
Linear Reg. of Do minor



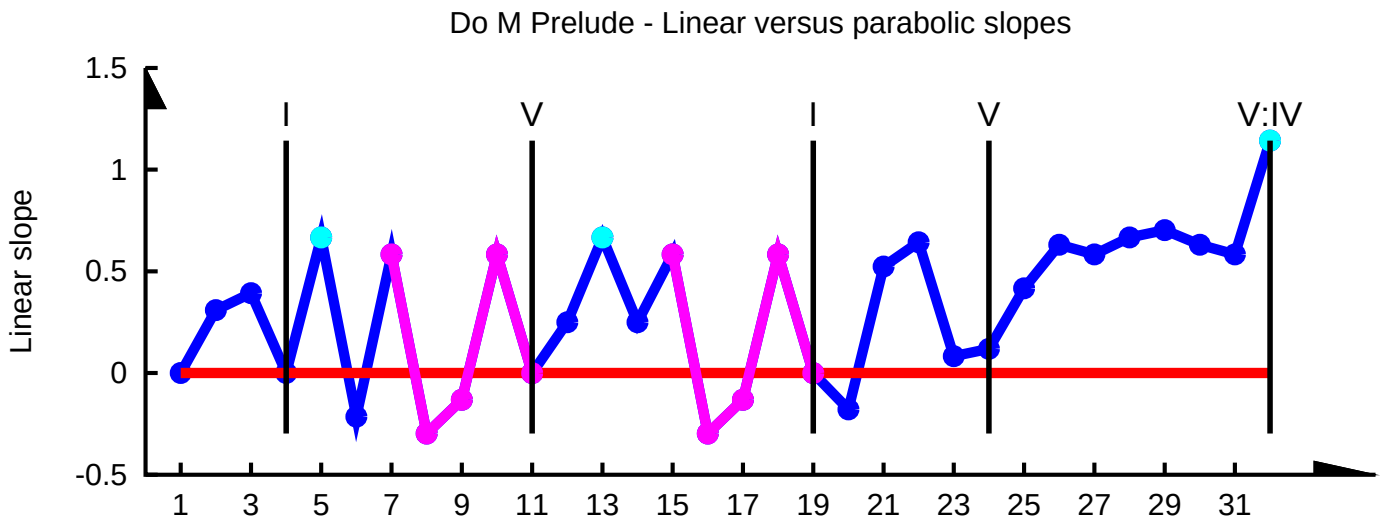
Linear Reg. of Do minor



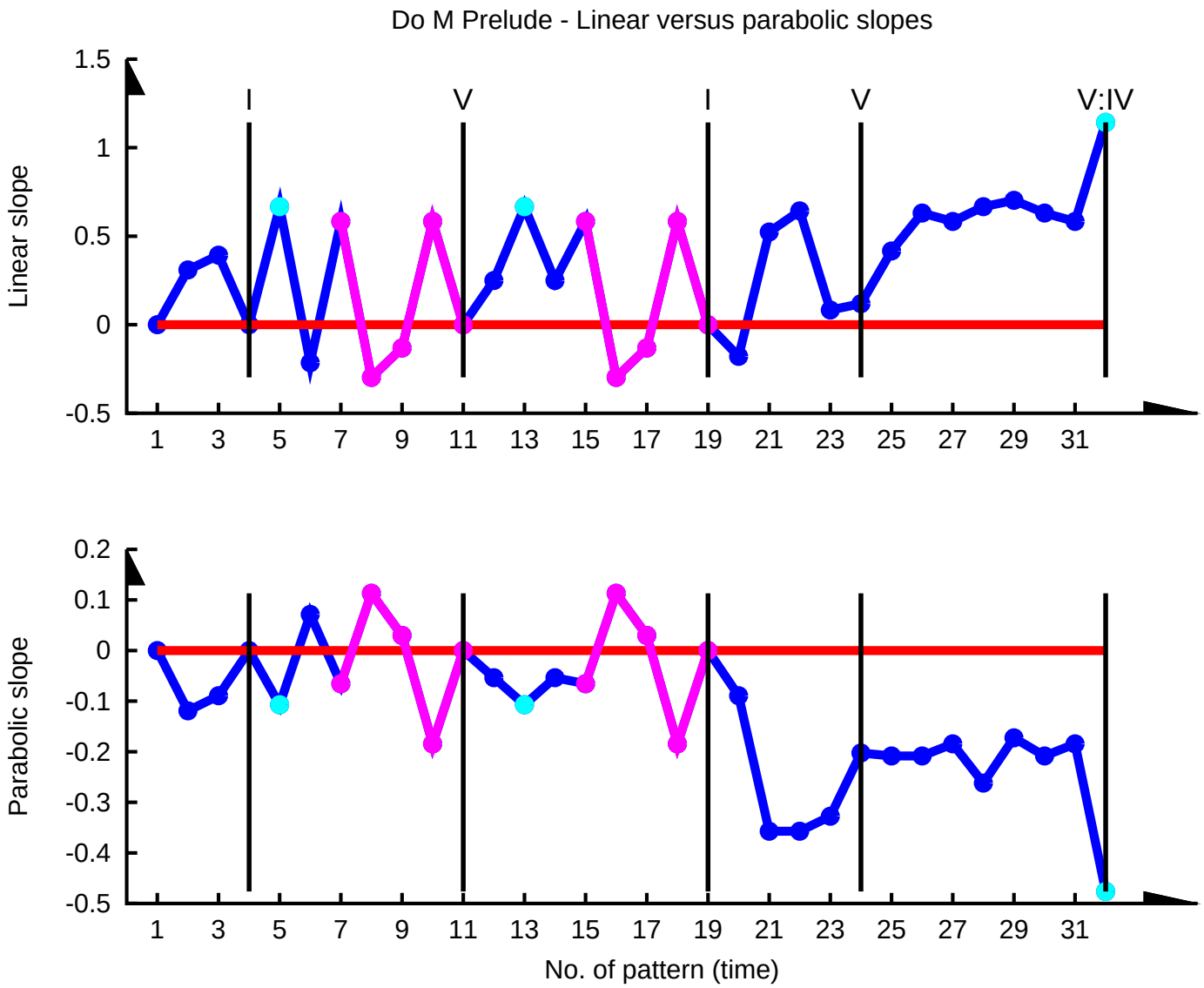
Intersecting Linear and Parabolic Reg. of Do minor



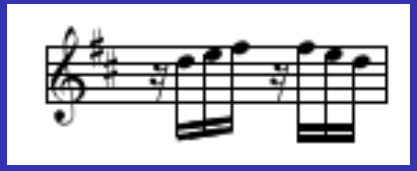
Linear Reg. of Do M

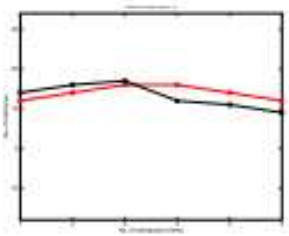
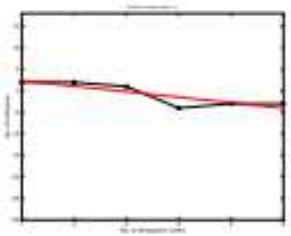
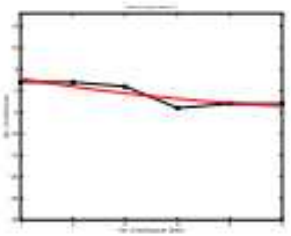
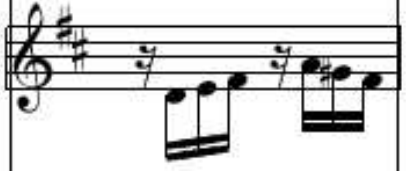
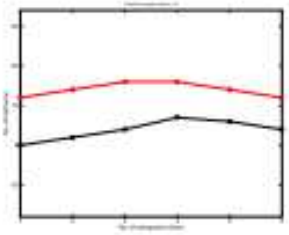
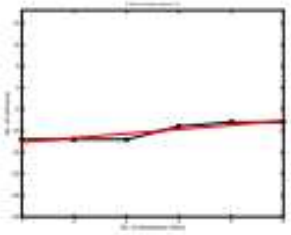
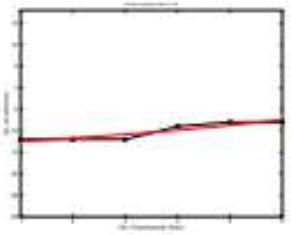


Intersecting Linear and Parabolic Reg. of Do M

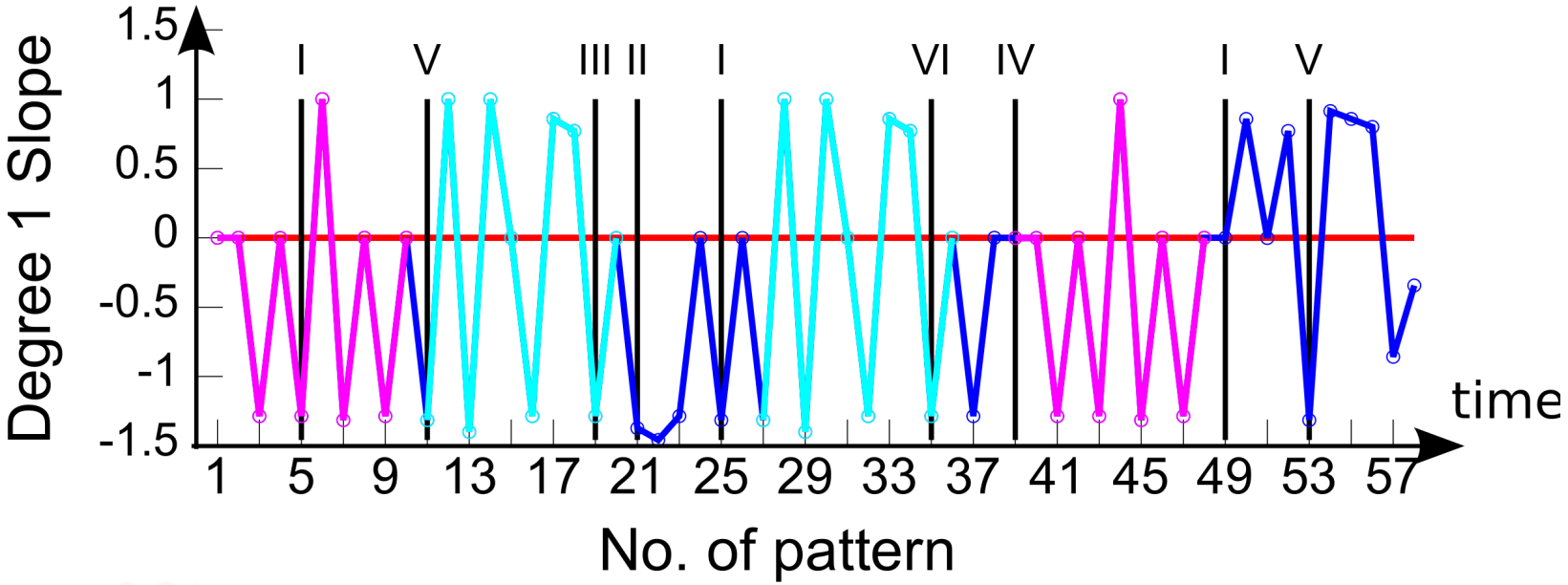


Slopes of Re M

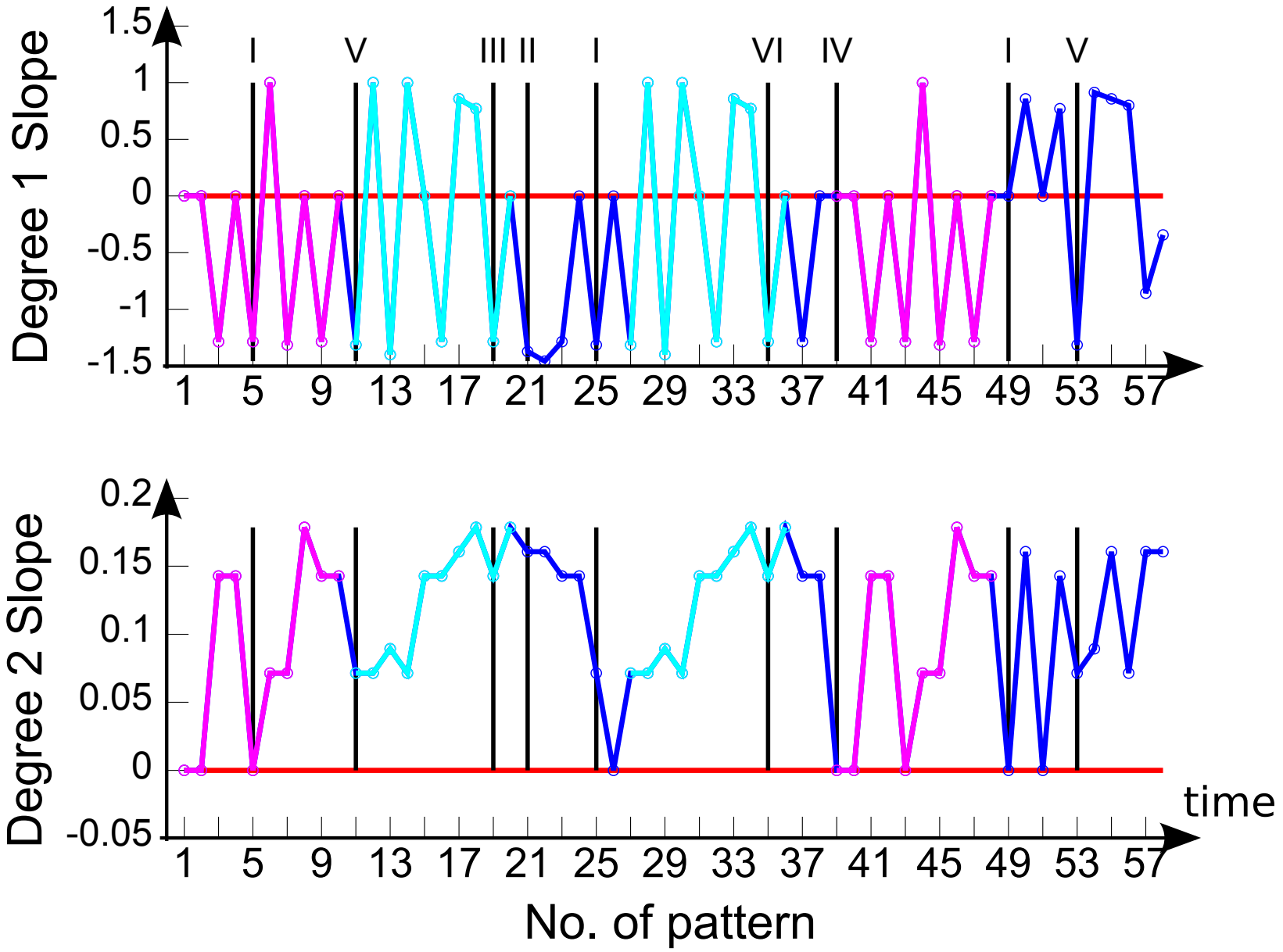


pattern	pat.versus pat.	Line. Slope	value	Parab. Slope	value
3 			−1.29		0.14
28 			1.00		0.07

Linear Reg. of Re M



Intersecting Linear and Parabolic Reg. of Re M



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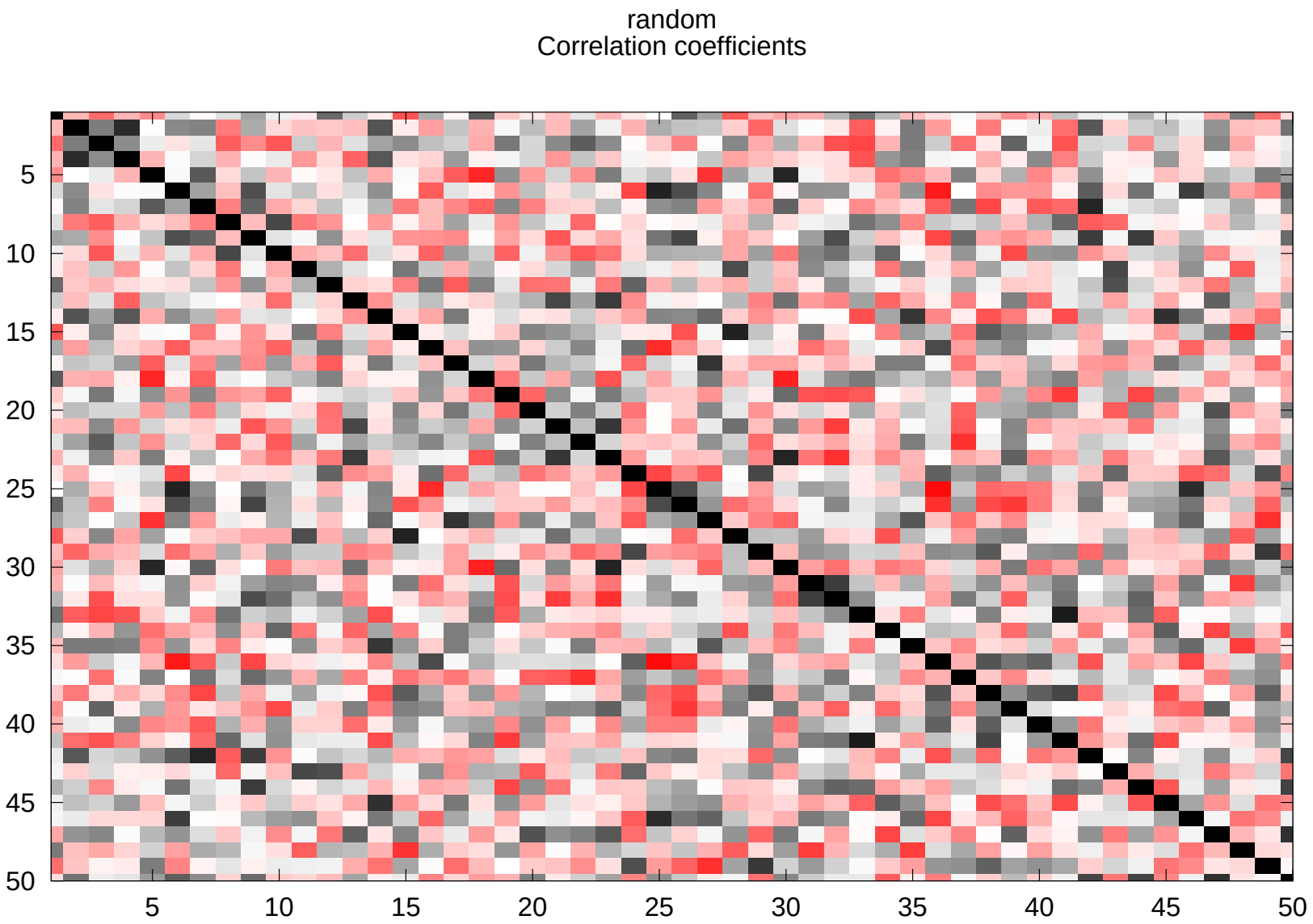
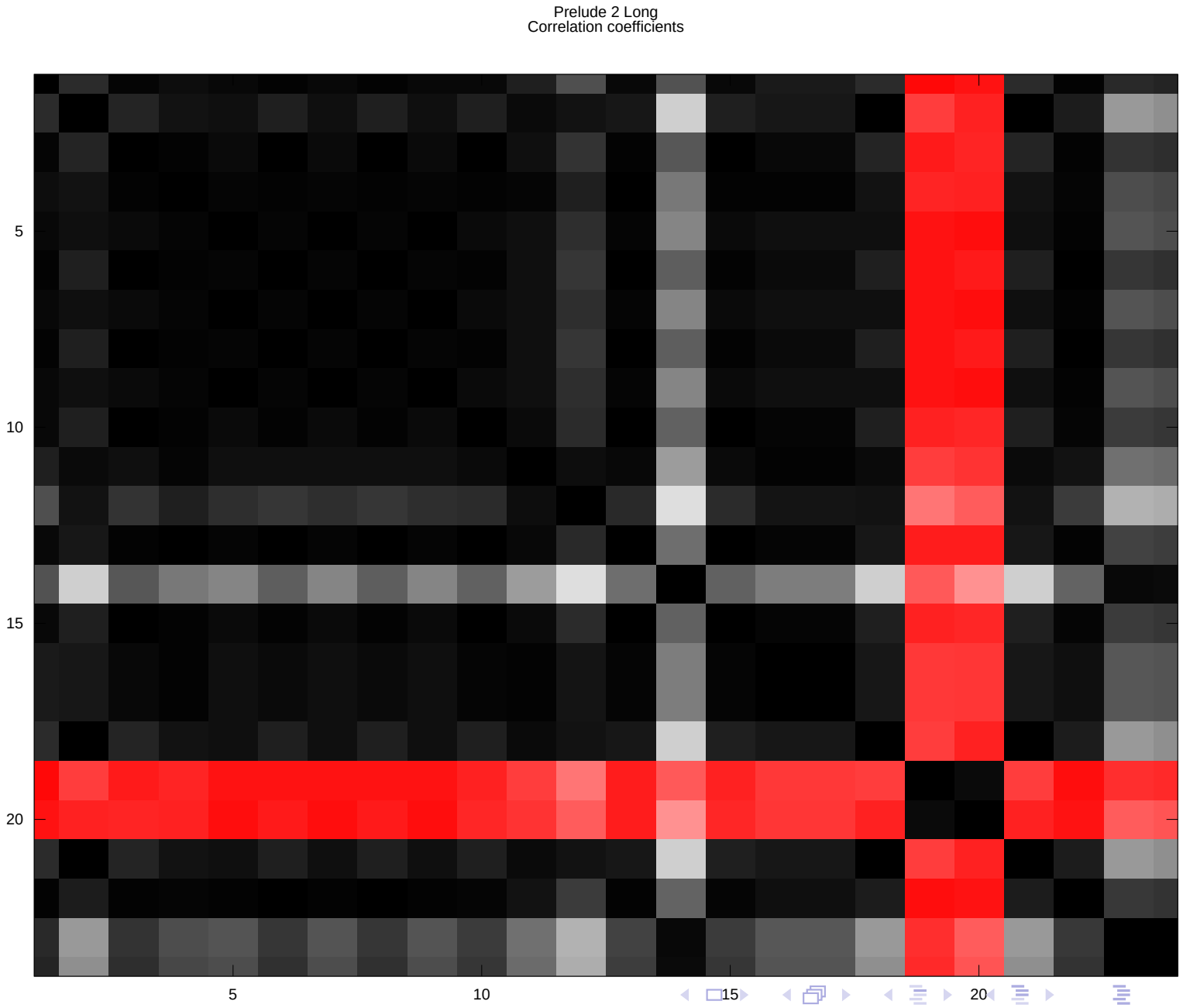
Random Matrix



Do m Matrix of Correlation Coefficients Slopes



Do m Matrix of Parabolic Reg. Coefficients