Computational Game Theory - Spring 2008/2009

Lecturer: Michal Feldman *TA: Ilan Nehama*

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1. Model the following games. I.e. define - The set of players
   - The action set for each of the players
   - The payoff function for each of the players

   (a) Beauty Contest:
   Each player chooses a value $a \in [0, 1]$
   The winner is the player closest to 70% of the average value.

   (b) Platform choice - two parties:
   Assume two parties want to choose their platform on a specific issue.
   The opinions on this issue can be viewed as the continuous line $[0, 100]$. It is known that the public opinion is uniformly distributed on the line $[0, 100]$.
   Each party chooses a platform $a \in [0, 100]$. Each voter votes for the party closest to his opinion. (In case of a tie he chooses randomly). The aim of each party is to attract as many voters as possible.

   (c) Platform choice - three parties:
   Same as above but with three parties.

   (d) Voting:
   Assume we have a group of 5 individuals who want to choose an alternative among 4 possible alternatives (For instance choosing a prime minister).
   Each individual has a **private** valuation describing the value he gains from each of the alternatives.
   The voting protocol: Each voter announces his top two alternatives and the alternative that receives the maximal number of votes is chosen. In the case of a tie, all the alternatives receiving the maximal number of votes win and hence the utility of each voter is the average of the chosen alternatives according to his valuation.

2. Specify the Best Response (BR) function for the following games (defined in Question 1).

   (a) Beauty Contest

   (b) Platform choice - two parties

   (c) Platform choice - three parties

   (d) Voting (In this case the best response is also a function of the private valuation)

3. Model the following games. I.e. define - The set of players
   - The action set for each of the players
   - The payoff function for each of the players

   (a) Single Item Sealed-Bid Auction - First Price:
   A single item is sold on an auction.
   Every bidder out of the n bidders has a private valuation (in Shekels) of the item, where v_i denotes the valuation of bidder i.
   The auction procedure is: Every bidder submits a sealed envelope containing his bid. The

highest bidder gets the item and pays his bid. (In case of a tie the first bidder according to a lexicographical order gets the object and pays his bid)

(b) Single Item Sealed-Bid Auction - Second Price:
A single item is sold on an auction.
Every bidder out of the n bidders has a private valuation (in Shekels) of the item - v_i.
The auction procedure is: Every bidder submits a sealed envelope containing his bid. The highest bidder gets the item and pays the second highest bid. (In case of a tie the first bidder according to a lexicographical order gets the object and pays the second highest bid, which equals to his bid in this case).

4. An auction mechanism is called **incentive compatible** if no bidder can benefit from bidding any value different from his true value for any bidding profile of the other bidders (regardless of whether they bid truthfully).

- For the following types of auctions (defined in question 3): prove that the mechanism is incentive compatible or demonstrate (by an example) why it is not incentive compatible.
  - Single Item Sealed-Bid Auction - First Price
  - Single Item Sealed-Bid Auction - Second Price
- Discuss the relation between the notion of incentive compatibility and the notion of dominance between actions.

Note: Be careful not to mix between the two notions of bids and values.

5. Assume that only two firms produce cars in the car market. All cars are sold with the same price, which is determined by the total number of cars produced. Let q_1 and q_2 denote the integer quantities produced by firms 1 and 2, respectively. Let $P(Q) = a - Q$ be the market-clearing price when the aggregate quantity of the market is $Q = q_1 + q_2$ (More precisely, $P(Q) = a - Q$ for $Q < a$, and $P(Q) = 0$ for $Q \geq a$).

Assume that the total cost to firm i of producing quantity q_i is

$$C_i(q_i) = cq_i$$

That is, there are no fixed costs and the marginal cost is constant at c (where we assume $0 < c < a$). The profit of a firm is the difference between its revenue and cost.

(a) Suppose the firms choose their quantities simultaneously.
  i. Translate this setting into a normal-form representation of a game (players, strategies, payoffs).
  ii. Describe a pure Nash equilibrium of the game.

(b) Suppose the firms choose their quantities sequentially (First the first firm and second the second firm after observing q_1)
  i. Translate this setting into a normal-form representation of a game (players, strategies, payoffs).
  ii. Describe a pure Nash equilibrium of the game.

remark: assume that output is continuously divisible and that negative outputs are not feasible.

6. Let G be a two-player zero-sum game, and suppose (τ_1, τ_2) and (σ_1, σ_2) are two Nash equilibria of G. Let π be the payoff value for player 1. i.e., $\pi(x, y)$ is the payoff of player 1 when his strategy is x and player 2's strategy is y.

   Prove:

   (a) (σ_1, τ_2) and (τ_1, σ_2) are also Nash equilibria of G.

   (b) $\pi(\tau_1, \tau_2) = \pi(\sigma_1, \tau_2) = \pi(\tau_1, \sigma_2) = \pi(\sigma_1, \sigma_2)$.

   Note: Notice that the players might have more than two available actions.