

Computational Game Theory - Spring 2008/2009

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Problem set 2

1. Consider a simple network, where two disjoint edges connect a source vertex s to a destination vertex t . Each edge is labeled with a cost function $c()$, which describes the cost (e.g., travel time) incurred by users of the edge, as a function of the amount of traffic routed on the edge. The upper edge has the constant cost function $c(x) = 1$, and thus represents a route that is relatively long but immune to congestion. The cost of the lower edge, which is governed by the function $c(x) = x$, increases as the edge gets more congested. In particular, the lower edge is cheaper than the upper edge if and only if less than one unit of traffic uses it.

Suppose that there is one unit of traffic, representing a very large population of players, and that each player chooses independently between the two routes from s to t . Assume that each player aims to minimize its cost.

- (a) Is there a (strongly / weakly) dominant strategy in the game described above?
 - (b) Describe all the Nash equilibria of the game.
 - (c) Suppose we have a “social objective” – we wish to minimize the average cost incurred by players. What is the action profile that minimizes the social objective? What is the social cost of this optimum? What is the social cost of the Nash equilibrium?
 - (d) Discuss the implications of rational behavior in the above game.
2. **(bonus question)** Consider a two-player game given in matrix form where each player has n strategies. Assume that the payoffs for each player are in the range $[0, 1]$ and are selected independently and uniformly at random. Show that the probability that this random game has a pure (deterministic) Nash equilibrium approaches $1 - \frac{1}{e}$ as n goes to infinity. You may use the fact that $\lim(1 - \frac{1}{n})^n = \frac{1}{e}$ as n goes to infinity.
 3. Characterize all pure Nash equilibria of the following games:

- (a) *War of Attrition - 2 players:* Each player has a private value for an item, and the following auction is run: every period the price for the item is raised until one of the participants “quits”. Each player should decide on the time step in which he quits, and the players also pay for the wasted time. The payoff of player 1 is as follows (and analogously for player 2):

$$p_1(a_1, a_2, v) = \begin{cases} -a_1 & a_1 < a_2 \\ v - a_2 & a_1 > a_2 \\ 0.5v - a_1 & a_1 = a_2 \end{cases}$$

- (b) *Beauty Contest - 2 players:* Each player chooses a value $a \in [0, 1]$. The winner is the player closest to 70% of the average value.
- (c) *Platform choice - 2 players:* Two parties want to choose their platform on a specific issue. The public opinion is uniformly distributed on the line $[0, 1]$. Each party chooses a platform $a \in [0, 1]$ under the assumption that each voter votes for the party closest to his opinion (in case of a tie the voter chooses randomly). The aim of each party is to attract as many voters as possible.

(d) *Platform choice - three parties*: Same as above but with three parties.

4. Computation of equilibria.

- (a) Find all the correlated equilibria and Nash equilibria (mixed and pure) in the prisoner's dilemma game.
- (b) Same for the battle of the sexes game.
- (c) **(bonus question)** Find the complexity class of the following problem: "Given a game G and a number k , does G have a correlated NE with social welfare at least k "?

5. Answer the following:

- (a) Consider transforming all payoffs in a game by a positive affine transformation ($y(x)$ is a positive affine transformation of x if there exist $a > 0, b$ such that $y(x) = ax + b$).
 - i. How will the set of Nash equilibria (pure and mixed) be affected? Prove your answer.
 - ii. Suppose that we transformed only player 1's payoffs with a positive affine transformation, and left player 2's payoffs unchanged. How will this affect the set of Nash equilibria in the game? Prove your answer.
- (b) Explain why proving the existence of a Nash equilibrium (not necessarily pure) in the game depicted below proves the existence in any 2x2 game where no payoffs are exactly the same.

	L	R
U	0,0	a,b
D	1,c	d,e