Extra solution: Chapter 2, Self-Test Problems, Class of July 9

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Problem 7

(a) The A point in the sample space corresponds to a specific selection of 8 numbers out of the 40 given numbers. The event of getting all numbers right is an event that contains only one point. Hence, the probability of this event is $1/\binom{40}{8} = 1/76904685$.

(b) There are $\binom{8}{7}\binom{32}{1} = 8 \cdot 32 = 256$ points in the event where exactly 7 numbers are guessed correctly. The probability of that event is $\approx 3.329 \times 10^{-06}$.

(6) There are $\binom{8}{6}\binom{32}{2} = 13888$ points in the event where exactly 6 numbers are guessed correctly. The probability of guessing at least 6 numbers contains $13888+256+1 = 14145$ points and the probability of the event is $\approx 1.839 \times 10^{-04}$.

Problem 15

Observe that the probability of A_i^c is equal to zero, for all i, since the probability of A_i is one. The union $\cup_{i=1}^{\infty} A_i^c$ is an event in the sigma algebra. It was shown in class that the probability of the union is bounded by the sum of the probabilities. However, since the sum of a countable number of zeros is zero one obtains that the probability of the union of the complimentary events is bounded by zero. Since all events have non-negative probabilities, it follows that the probability of the union is zero. The intersection of the original is equal to the complimentary of the union of the complimentary events. Hence, the probability of the intersection $\cap_{i=1}^{\infty} A_i$ is one.

Problem 20

Associate points in the sample space with paths of a random walk. Let a move to the right correspond to selecting a blue ball and a move up to the selection of a red ball. All paths terminate at the point $(10, 20)$. The event under consideration corresponds to reaching (hitting) the line $y = 20$ before reaching the line $x = 10$. This event can be divided into the disjoint sub-events of first hitting the horizontal line after j steps, for $j = 20, 21, 29$. (In the case $j = 30$ the vertical line is reached first.) It follows that the probability we seek is:

$$\sum_{j=20}^{29} \frac{\binom{j-1}{19}}{\binom{30}{20}}$$