

MORE SOLUTIONS: CHAPTER 4, CLASS OF JULY 20

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Theoretical, 4. Notice that N is integer valued. Consequently, $P(N \geq i) = \sum_{j=i}^{\infty} P(N = j)$. According to the hint:

$$\sum_{i=1}^{\infty} P(N \geq i) = \sum_{i=1}^{\infty} \sum_{j=i}^{\infty} P(N = j) = \sum_{j=1}^{\infty} \sum_{i=1}^j P(N = j) .$$

However, $\sum_{i=1}^j P(N = j) = jP(N = j)$. Therefore, the sum becomes

$$\sum_{j=1}^{\infty} jP(N = j) = \mathbb{E}(N) ,$$

with the last equality follows from the definition of the expectation.