

SOLUTIONS FOR THE “MIDTERM”

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Problem 1.

- (1) $10^8 \times 26 \times 25 \times 24 \times 23 \times 22$.
- (2) $10^8 \times 26^5 \times \binom{8+3}{3}$.
- (3) $10^8 \times 26^5 \times \binom{8+5-4}{5}$.

Problem 2.

- (1) $\frac{(n-6) \times 6 + 2 \times 5 + 2 \times 4 + 2 \times 3}{n!}$.
- (2) $\frac{n \times 6}{n!}$.

Problem 3.

(1)

$$P(E) = \left[\frac{8}{13} \frac{10}{15} \right] \times \left[\frac{1}{2} \frac{9}{14} + \frac{1}{2} \frac{9}{14} \right] + \left[\frac{5}{13} \frac{5}{15} \right] \times \left[\frac{1}{2} \frac{8}{14} + \frac{1}{2} \frac{10}{14} \right] \\ + \left[\frac{8}{13} \frac{5}{15} \right] \times \left[\frac{1}{2} \frac{7}{12} + \frac{1}{2} \frac{11}{16} \right] + \left[\frac{5}{13} \frac{10}{15} \right] \times \left[\frac{1}{2} \frac{8}{12} + \frac{1}{2} \frac{10}{16} \right].$$

- (2) $\left\{ \left[\frac{8}{13} \frac{5}{15} \right] \times \left[\frac{1}{2} \frac{7}{12} + \frac{1}{2} \frac{11}{16} \right] + \left[\frac{5}{13} \frac{10}{15} \right] \times \left[\frac{1}{2} \frac{8}{12} + \frac{1}{2} \frac{10}{16} \right] \right\} / P(E)$.

Problem 4.

- (1) $3/5$.
- (2) $(1 - 5/28)^{20}$.
- (3) $2/5$.

Problem 5.

- (1) $E(X) = 4$, $\text{Var}(X) = 12 + 4 - 4^2 = 0$. Hence, $E(Y) = 1.5 \times 4 - 3 = 3$, $\text{Var}(Y) = 0$.
- (2) Any selection of a and b such that $b = -4a$ will produce zero mean but no selection can produce unit variance.