

The following homework assignment should be submitted in writing no later than Tuesday, 30/6/10. No delays are allowed, unless they are both justified and pre-approved by me. You may submit the project in pairs.

A system is observed in 4 time points: $0 < t_1 < t_2 < t_3 < t_4$. At each point the incremental movement of a particle since the previous observation is assessed. Let X_i be the assessment made at time t_i . Assume that the expectation of X_i is the actual incremental movement, the variance is σ^2 (known) and the correlation between observations X_i and X_j is $\rho^{|i-j|}$, for some known ρ . It is known that the actual increment movements are non-negative and that particle moves a total distance of one between time 0 and time t_4 . It is hypothesized that the total distance that the particle moved until time t_2 is at least half the overall distance.

1. Formulate the problem of estimating the incremental movements based on the observations in terms of a minimization problem with constraints. Give a justification for this formulation under the assumption that the observations have a joint Gaussian distribution.
2. Select increments that satisfy the assumptions of the system and select values for σ^2 and ρ . Generate observations X_1, \dots, X_4 under the Gaussian assumption and according to the given parameters. Use the simulated observations in order to estimate the actual increments. Present the R code you used. (You may use the function `mvrnorm` from the library `MASS` in order to generate multi-normal observations.)