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Spatial Vector Autoregressions

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ABSTRACT *A spatial vector autoregressive model (SpVAR) is defined as a VAR which includes spatial as well as temporal lags among a vector of stationary state variables. SpVARs may contain disturbances that are spatially as well as temporally correlated. Although the structural parameters are not fully identified in SpVARs, contemporaneous spatial lag coefficients may be identified by weakly exogenous state variables. Dynamic spatial panel data econometrics is used to estimate SpVARs. The incidental parameter problem is handled by bias correction rather than more popular alternatives such as generalized methods of moments (GMM). The interaction between temporal and spatial stationarity is discussed. The impulse responses for SpVARs are derived, which naturally depend upon the temporal and spatial dynamics of the model. We provide an empirical illustration using annual spatial panel data for Israel. The estimated SpVAR is used to calculate impulse responses between variables, over time, and across space. Finally, weakly exogenous instrumental variables are used to identify contemporaneous spatial lag coefficients.*

Autoregressions du vecteur spatial

RÉSUMÉ *Un 'spatial vector autoregressive model' (modèle autorégressif de vecteur spatial—SpVAR) se définit comme VAR, qui inclut des décalages spatiaux et temporaires parmi un vecteur de variables d'état stationnaire. Les SpVAR peuvent contenir des perturbations qui sont en corrélation au niveau spatial et temporel. Bien que les paramètres structurels ne soient pas entièrement identifiés dans les SpVAR, des coefficients contemporains de variable aléatoire décalée peuvent être identifiés par des variables d'état faiblement exogènes. L'économétrie de données dynamiques spatiales recueillies au moyen d'un panel est utilisée pour faire une estimation des SpVAR. Le problème du paramètre annexe est traité par correction erreur systématique plutôt que par des alternatives plus populaires, telles que le GMM. La relation entre la stationnarité temporelle et spatiale est discutée. Les réponses impulsionnelles pour les SpVAR en sont dérivées, ce qui dépend naturellement de la dynamique temporelle et spatiale du modèle. Nous fournissons une illustration empirique à l'aide des données annuelles spatiales du panel pour Israël. Le SpVAR estimé est utilisé pour le calcul des réponses impulsionnelles entre les variables, sur une période de temps et à travers l'espace. Enfin, des variables instrumentales faiblement exogènes sont utilisées pour identifier des coefficients contemporains de variables aléatoires décalées.*

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Autoregresiones Vectoriales Espaciales

RESUMEN *Un modelo autoregresivo vectorial espacial (SpVAR) se define como un VAR que incluye retardos espaciales además de temporales entre un vector de variables de estado estacionario. Los SpVARs pueden contener disturbios correlacionadas espacialmente además de temporalmente. Aunque los parámetros espaciales no están identificados del todo en los SpVARs, se pueden identificar coeficientes de retardo espacial mediante variables de estado débilmente exógenos. Se utilizan económetras de datos de panel dinámicos espaciales para estimar SpVARs. El problema del parámetro incidental se manipula con una corrección de la tendencia en vez de con las alternativas más populares, tales como GMM. Se habla de la interacción entre la inmovilidad temporal y la espacial. Se derivan las respuestas de impulso de los SpVARs, que naturalmente dependen de las dinámicas espaciales y temporales del modelo. Proporcionamos una ilustración empírica utilizando datos anuales de panel de datos espaciales para Israel. Los SpVAR estimadas se utilizan para calcular respuestas de impulso entre variables, en el tiempo, y por el espacio. Finalmente, se utilizan variables instrumentales débilmente exógenos para identificar coeficientes de retardo espacial contemporáneos.*

KEYWORDS: *Spatial econometrics; spatial autocorrelation; vector autoregressions; spatial panel data*

JEL CLASSIFICATION: *C21; C22; C23; C53*

1. Introduction

Regional scientists have shown that spatial dependence in economic data may alter, and even reverse, the results of standard time series models. For example, Rey & Montouri (1999) have shown that beta convergence tests depend upon spatial spillovers in the USA. A similar finding is reported by Badinger *et al*. (2004) for the EU. These and other studies establish the importance of integrating spatial and temporal lags in the econometric analysis of regional data. However, the literatures on temporal and spatial dynamics have more or less progressed along separate tracks. In this paper we try to bring the two literatures together.

We begin by recalling some key results in the econometric analysis of time series data. In particular we discuss the relationship between structural models and vector autoregressions (VARs) and the identification problem that arises in structural vector autoregressions (SVARs). In VAR models the dynamics are naturally temporal. Next, we recall key results in the spatial econometric analysis of cross-section data, especially regarding the identification of spatial lags and spatial autocorrelation. In cross-section data the dynamics are naturally spatial. Spatial and temporal dynamics meet in spatial panel data models, since they incorporate cross-section and time series dimensions. A very large body of literature (Hsiao, 2003; Baltagi, 2005) deals with the econometric analysis of temporal dynamics in panel data. A considerably smaller body of literature (Elhorst, 2003, 2004; Lee, 2004) is concerned with the econometric analysis of spatial dynamics in temporally static panel data models. We recall the main results of these quite separate literatures on panel data. There is, however, virtually no literature at all on spatial panel data models which embody both spatial and temporal dynamics.[1]

Our purpose in this paper is therefore to consider how spatial panel data may be used to estimate models which jointly specify temporal dynamics as in VARs and spatial dynamics. We refer to such models as spatial vector autoregressions[2] (SpVARs). SpVARs differ from VARs in that they incorporate spatial as well as temporal dynamics, and they differ from spatial models because they incorporate temporal dynamics. SpVARs contain two types of spatial dynamics. Variables at time t may depend upon contemporaneous spatial lags as in spatial models for cross-section data. In addition, variables at time t may depend upon spatial lags at time $t-\tau$ ($\tau > 0$). We refer to the latter as 'lagged spatial lags'. In the absence of spatial lags, SpVARs are identical to VARs, and in the absence of temporal lags SpVARs are identical to spatial panel models.

We ask whether SpVARs identify all the structural parameters to be estimated. These parameters include the model's underlying parameters in addition to its spatial and temporal lag coefficients. Since it is well known that structural VAR models (SVARs) generally fail to identify all the structural parameters, it is not surprising that the same applies to SpVARs. We show that the eigenvalues in SpVARs depend upon spatial and temporal dynamics, therefore stationarity depends upon both types of dynamics, as noted by Mur & Trivez (2003).[3]

We distinguish between SpVARs with and without spatial autocorrelation (SAC) in the residuals. We compare SpVARs in which there are no spatial lags but the residuals are spatially correlated, with SpVARs in which there are spatial lags but the residuals are spatially uncorrelated. The former is nested in the latter and a common factor test may be used to distinguish empirically between them. We also show that the impulse responses of SpVARs with spatial autocorrelation are a simple transformation of the impulse responses in which regional shocks are assumed to be uncorrelated.

We illustrate the methodology with an application to regional data in Israel. The estimated SpVAR contains four variables in nine regions over 18 years. The SpVAR is estimated as a homogeneous stationary panel in which regions are specified to have specific effects, and within-variable shocks are assumed to be spatially correlated. Panel unit root tests are used to determine the order of differencing in the SpVAR. Finally, the impulse responses of the estimated SpVAR are calculated under the assumption that regional shocks are independent, and under the assumption that they are spatially correlated.

2. Econometric Theory

In what follows regions are labelled as $n = 1, 2, \ldots, N$; time periods are labelled as $t = 1, 2, \ldots, T$; endogenous variables are labelled as Y_k, where $k = 1, 2, \ldots, K$; exogenous variables are labelled as X_p, where $p = 1, 2, \ldots, P$; and lag orders are labelled as $j = 1, 2, \ldots, q$.

2.1. SVARs and VARs

The main purpose of this section is to recall that structural parameters are under-identified in VARs.[4] Let Y_t denote a $K \times 1$ vector of variables measured at time t with elements Y_{kt}. We write the structural VAR model for Y as:

$$Y_t = AY_t + \sum_{j=1}^{q} B_j Y_{t-j} + u_t, \tag{1}$$

where A is a $K \times K$ matrix of α coefficients with zeros along the leading diagonal, B_j is a $K \times K$ matrix of coefficients and u_t is a vector of autocorrelated disturbances:

$$u_t = \sum_{j=1}^{q} R_j u_{t-j} + \varepsilon_t, \tag{2}$$

where R_j is a $K \times K$ matrix of jth order serial correlation coefficients, and ε_t is a vector of white noise residuals with variance–covariance matrix Ω. In the iid case $\Omega = \sigma^2 I_K$. Equation (1) constitutes a linear structural model in which α_{km} is the contemporaneous causal effect of Y_m on Y_k.

In equation (1) the $K(K-1)$ unknown elements of A are not identified because Y_t and u_t are not independent. Nor are the K^2 elements of B_j identified because Y_{t-j} is not weakly exogenous unless $R_j = 0$. The VAR model implied by equation (1) is obtained by solving equation (1) for Y_t:

$$Y_t = \sum_{j=1}^{q} \Pi_j Y_{t-j} + \nu_t, \tag{3}$$

where

$$\Pi_j = MB_j = (I_K - A)^{-1} B_j$$

$$\nu_t = Mu_t.$$

Provided $R_j = 0$, Π is identified in equation (3) and $\Sigma_\nu = M\Omega M'$ is the symmetric variance–covariance matrix of the VAR residuals. When, for example, $q = 1$, equation (3) contains K^2 parameters from Π and $\frac{1}{2}K(1+K)$ parameters from Σ_ν, making a total of $1\frac{1}{2}K^2 + \frac{1}{2}K$ altogether. The number of unknown structural parameters include $K(K-1)$ elements of A, K^2 elements of B and $\frac{1}{2}K(1+K)$ elements of Ω, making a total of $2\frac{1}{2}K^2 - \frac{1}{2}K$. Therefore, the structural parameters are under-identified by a factor of $K(K-1)$. If, however, $\Omega = \sigma^2 I_K$ then the number of unknown structural parameters is reduced to $2K^2$, and the identification deficit reduces to $\frac{1}{2}K(K-1)$. The identification deficit is always positive and increases non-linearly with the number of variables participating in the SVAR. In the degenerate case when $K = 2$ and ε is iid there are eight structural parameters to be identified but the VAR contains seven restrictions, in which case the identification deficit is 1.

Another point we wish to make is that an SVAR with $R = 0$ nests within it a static model in which $R \neq 0$. In the latter, $B = 0$, and, after substituting equation (2) into (1) equation (1), becomes:

$$Y_t = AY_t + R(I_K - A)Y_{t-1} + \varepsilon_t. \tag{4}$$

In the former case equation (1) is:

$$Y_t = AY_t + BY_{t-1} + \varepsilon_t. \tag{5}$$

Whereas in equation (4) the coefficient matrix of Y_{t-1} is constrained to be related to the coefficient matrix of Y_t, in equation (5) the two matrices are unconstrained. Therefore, equation (4) is a restricted version of equation (5); it contains a common factor restriction of $(I_K - RL)$, where L denotes the lag operator. This common factor restriction may be tested using a Wald test (Hendry, 1995, chap. 7.7).

2.2. Spatial Econometrics: Cross-section Data

Whereas in Subsection 2.1 there was only one region and there were no exogenous variables ($N = 1$ and $P = 0$), in this subsection there are many regions ($N > 1$) but since the data are cross-sections there is only one time period ($T = 1$). Y_k is an $N \times 1$ vector of observations on variable k and $Y_k^* = WY_k$ denotes the value of variable k in the 'neighbourhood' of region n. W is an $N \times N$ connectivity matrix with known elements w_{ni} with $w_{nn} = 0$.

The structural model for variable k in region n is represented by:

$$Y_{kn} = \sum_{p=1}^{P} \gamma_{kp} X_{pn} + \sum_{i=1}^{K} \theta_{ki} Y_{in}^* + e_{kn}, \tag{6}$$

where X_p are exogenous variables. In equation (6) θ_{kk} are spatial lag coefficients, θ_{ki} are cross-spatial lag coefficients, $e_k = \Phi We_k + \varepsilon_k$, where Φ is a $K \times N$ matrix of spatial autocorrelation coefficients, and ε_k is an $N \times 1$ vector of iid residuals. Writing equation (6) in matrix notation yields:

$$Y = \Gamma^* X + \Theta^* W^* Y + e \tag{7}$$

$$e = \Phi^* W^* e + \varepsilon, \tag{8}$$

where Y is an $NK \times 1$ vector of endogenous variables (stacked by n), X is an $NP \times 1$ vector of exogenous variables, $\Gamma^* = I_N \otimes \Gamma$ is an $NK \times NP$ block diagonal matrix with Γ along the diagonal, where Γ is a $K \times \mathrm{P}$ matrix of γ coefficients, $\Theta^* = I_N \otimes \Theta$ is an $NK \times \mathrm{NK}$ block diagonal matrix with Θ along the diagonal, where Θ is a $K \times \mathrm{K}$ symmetric matrix of *?* coefficients, $W^* = I_K \otimes W$ is an $NK \times NK$ block diagonal matrix with W along the diagonal, and $\Phi^* = I_N \otimes \Phi$ is a $KN \times KN$ block diagonal matrix of spatial autocorrelation coefficients with Φ along the diagonal.

Identification of the structural parameters requires that the X variables are not perfectly spatially collinear (Manski, 1993) and $w_{ni} < 1$. If there are no X variables, the spatial lag coefficients are not identified. Therefore, just as the SVAR coefficients are under-identified in the absence of exogenous variables, so the spatial lag coefficients are under-identified in the absence of exogenous variables.

In equation (7) the spatially lagged dependent variables are not independent of e, hence OLS estimates of equation (6) are biased and inconsistent. The solution to this problem is to rewrite equation (7) as:

$$(I_{NK} - \Theta^* W^*) Y = \Gamma^* X + e \tag{9}$$

whose parameters may be estimated non-linearly by maximum likelihood (Anselin, 1988), provided, of course, that Γ and Θ are identified.

Equation (9) solves for the spatial impulse response profile, which shows the effect of X_p in region i on Y_k in region n (h_{pkni}):

$$Y = H(X + e) \tag{10}$$

$$H = (I_{NK} - \Theta^* W^*)^{-1}.$$

In the absence of spatial lags $h_{pkni} = 0$ when $i \neq n$. In this case shocks to X_k in region j do not propagate beyond region j.

In Subsection 2.1 we saw that temporal static models with serially correlated errors are restricted versions of dynamic models with serially independent errors. The same applies to spatial models. Spatially static models with spatially autocorrelated errors are restricted versions of spatially dynamic models with spatially independent errors. We demonstrate this by setting $K = 1$ for simplicity. The spatially static model is:

$$Y = \Gamma X + e$$

$$e = \Phi We + \varepsilon,$$

which may be written as:

$$(I_N - \Phi W)Y = \Gamma(I_N - \Phi W)X + \varepsilon \tag{11}$$

The spatially dynamic model is:

$$(I_N - \Theta W)Y = \Gamma X + \varepsilon. \tag{12}$$

Equation (11) contains the common factor restriction $I - \Phi W$, whereas equation (12) does not. Therefore equation (12) is a restricted version of equation (11). This common factor restriction may be tested, as in Subsection 2.1, using a common factor test (Anselin, 1988, pp. 226–229).

2.3. Spatial Panel Data

Spatial panel data have been discussed by Anselin (1988), chap. 10), Elhorst (2003) and Lee (2004) in a temporally static context, i.e. models in which there are spatial dynamics but no temporal dynamics.[5] Introducing spatial dynamics into temporally static panel data models is a complication that does not, however, substantively alter the theory of panel data econometrics. In terms of equation (6), time subscripts are appended to the variables in the model so that the dependent variable becomes Y_{knt}, the independent variables become X_{pnt}, the spatial lagged dependent variables become Y^*_{knt}, and the residual error becomes ε_{knt}. Estimators that are consistent when the data are independent are also consistent when they are spatially dependent; however, the asymptotics are typically slowed down (Lee, 2004).[6]

By way of introduction we consider the following univariate model ($K = 1$) with first-order temporal lags ($q = 1$):

$$Y_{nt} = \mu_n + \theta Y^*_{nt} + \beta Y_{nt-1} + \lambda Y^*_{nt-1} + u_{nt} \tag{13}$$

$$u_{nt} = \rho u_{nt-1} + \delta u^*_{nt} + \gamma u^*_{nt-1} + \varepsilon_{nt} \tag{14}$$

$$\sigma_{ni} = \mathrm{cov}(\varepsilon_n \varepsilon_i),$$

where μ_n denotes a regional-specific effect. The spatial lag coefficient is θ and the temporal lag coefficient is β. Equation (13) also contains a 'lagged spatial lag' coefficient (λ) since there may be a temporal lag in the spatial lag. Equation (14) specifies the structure of the error term. The SAC coefficient is denoted by δ, the temporal autocorrelation coefficient (TAC) by ρ, and the lagged SAC coefficient by γ. Finally, there will be spatial correlation (SC) between the εs if $\sigma_{ni} \neq 0$, i.e. the residual covariance matrix Ω is non-diagonal. Whereas SAC arises if shocks are clustered by space, SC arises if shocks happen to be correlated without any clear spatial pattern, as in seemingly unrelated regression (SUR). We may distinguish between four spatially correlated cases:

(i) All the correlation between the residuals is due to SAC. In this case $\Omega = \sigma^2 I_N$ (is diagonal) and $d \neq 0$.
(ii) All the correlation between the residuals is due to SUR (SC). In this case $\delta = 0$ and $\Omega \neq \sigma^2 I_N$.
(iii) The correlation between the residuals is due to both SAC and SUR. In this case $d \neq 0$ and $\Omega \neq \sigma^2 I_N$.
(iv) There is no correlation between the residuals. In this case $d = 0$ and $\Omega = \sigma^2 I_N$.

The identification of the parameters in equation (13), including the spatial lag coefficient θ, requires that Y_{nt-1} and Y^*_{nt-1} be weakly exogenous. If they are not, these variables are not independent of u_{nt}. It is easy to show that these variables are weakly exogenous when $\rho = \gamma = 0$. If $\rho \neq 0$, u_{nt-1} affects both u_{nt} and Y_{nt-1}, in which case Y_{nt-1} and u_{nt} are not independent. If $\gamma \neq 0$, u^*_{nt-1} affects both u_{nt} and Y^*_{nt-1}, in which case Y^*_{nt-1} and u_{nt} are not independent. In short, temporal autocorrelation and/or lagged SAC mean that the parameters of the model cannot be identified.

The structural multivariate counterpart of equation (13) is:

$$Y_{knt} = \mu_{kn} + \sum_{i=1}^{K} (\alpha_{ki} Y_{\text{int}} + \beta_{kj} Y_{\text{int}-1} + \theta_{ki} Y^*_{\text{int}} + \lambda_{ki} Y^*_{\text{int}-1}) + \varepsilon_{knt} \tag{15}$$

where the μs are region-specific effects, the αs are within-region contemporaneous causal effects between the Ys with $\alpha_{kk} = 0$, the θs are spatial lag coefficients, the βs are temporal lag coefficients, and the λs are lagged spatial lag coefficients. For simplicity, we assume that the residuals are neither spatially nor temporally autocorrelated and are homoscedastic. When $\lambda = \theta = 0$, equation (15) reverts to an SVAR. When $\beta = \lambda = 0$ it reverts to a spatial panel model. When both spatial and temporal dynamics are present, equation (15) is a structural spatial VAR (SpSVAR).

We denote by A, B, Θ and Λ the $K \times K$ coefficient matrices in equation (15) for the αs, βs, θs and λs, respectively. We may express equation (15) in terms of matrices as follows:

$$Y_t = \mu + A^* Y_t + B^* Y_{t-1} + \Theta^* Y^*_t + \Lambda^* Y^*_{t-1} + \varepsilon_t, \tag{16}$$

where Y is an $NK \times 1$ vector of observations stacked by n, μ is an $NK \times 1$ vector of regional-specific effects, $A^* = I_N \otimes A$, $B^* = I_N \otimes B$, $\Theta^* = I_N \otimes \Theta$, and $\Lambda^* = I_N \otimes \Lambda$ are $NK \times NK$ block diagonal matrices. In equation (16) Y_{t-1} and Y^*_{t-1} are weakly exogenous because ε is not temporally autocorrelated, but Y_t and Y^*_t are not independent of ε_t.

In Subsection 2.1 we saw that VARs under-identify the structural parameters. What happens in SpVARs? To answer this question we solve equation (16) for Y_t:

$$Y_t = \Pi_0 + \Pi_1 Y_{t-1} + \Pi_2 Y^*_t + \Pi_3 Y^*_{t-1} + v_t \tag{16a}$$

$$M = (I_{NK} - A^*)^{-1}$$

$$\Pi_0 = M\mu \qquad \Pi_1 = MB^* \qquad \Pi_2 = M\Theta^* \qquad \Pi_3 = M\Lambda^* \qquad \nu = M\varepsilon.$$

There are $K(K-1)$ unknown A coefficients, K^2 unknown coefficients for each of B, Θ, and Λ, and there are NK unknown variances for Σ_ε, making a total of $K(4K-1)+K=4K^2$ unknown structural parameters in equation (16). In equation (16a) there are $3K^2$ data restrictions from the Πs and Σ_ν provides ½$K(K+1)$ further restrictions. Therefore, the SpVAR under-identifies the structural parameters and the identification deficit is ½$K(K-1)$. Nevertheless, equation (16a) reveals some information. If Π_2 is statistically significant then this points to the presence of spatial dynamics. If Π_1 is statistically significant then this points to the presence of temporal dynamics. Finally, if Π_3 is statistically significant then this points to the presence of temporal-spatial dynamics.

2.4. The Incidental Parameter Problem

Badinger *et al*. (2004) have suggested that dynamic panel data econometrics developed for spatially uncorrelated data may be applied to spatially correlated data if the data are first spatially filtered. This two-stage procedure assumes that spatial dependencies in the data are nuisance parameters, which are entirely independent of the underlying 'spaceless' model to be estimated. If this is not the case, their two-stage procedure may filter away important components in the underlying model.[7] Just as it is inadvisable to use seasonally filtered data in dynamic time series models (Hendry, 1995), we think it is inadvisable to use spatially filtered data in dynamic spatial panel data models. Instead, we take the view that spatial lags and spatial autocorrelation should be estimated jointly with temporal lags and temporal autocorrelation in dynamic panel data models.

If spatial-specific effects are specified in equation (16a) the 'incidental parameter problem' arises in temporal dynamic panels, giving rise to bias in the estimates of Π_1. The econometric implications of estimating fixed effects in dynamic panels, such as in equation (16), have attracted much attention in the literature. The basic problem is that least squares dummy variable (LSDV) estimates of the Bs are biased downwards when T is finite, with the bias being $O(T^{-1})$. Hsiao (1986) shows that when $q=1$ the asymptotic bias is equal to:

$$b = -\frac{\dfrac{1+\beta}{T-1}\left(1-\dfrac{1-\beta^T}{T(1-\beta)}\right)}{\left(1-\dfrac{2\beta}{(1-\beta)(T-1)}\left(1-\dfrac{1-\beta^T}{T(1-\beta)}\right)\right)}, \tag{17}$$

where $p\lim(\hat{\beta})=\beta+b$. The bias tends to zero as T tends to infinity. If the panel is short (T is small) this bias may not be negligible. In Figure 1 we use equation (17) to plot the relationship between $p\lim(\hat{\beta})$ and β for various values of T. As expected, the plotted schedules approach the 45 degree line from below as T increases. In our empirical example $T=16$. Equation (17) implies in this case that the asymptotic bias is -0.0991 when $\beta=0.5$, in which case $p\lim(\hat{\beta})=0.4009$. If $\beta=0$ the asymptotic bias is -0.06, in which case $p\lim(\hat{\beta})=-0.06$. Also, the bias varies directly with β. For example, when $\beta=-0.5$ the bias is -0.031, which is a third of its counterpart when $\beta=0.5$.

The most popular solution to the incidental parameter problem (Arellano & Bond, 1991; Blundell & Bond, 1998) is based on instrumental variable estimation

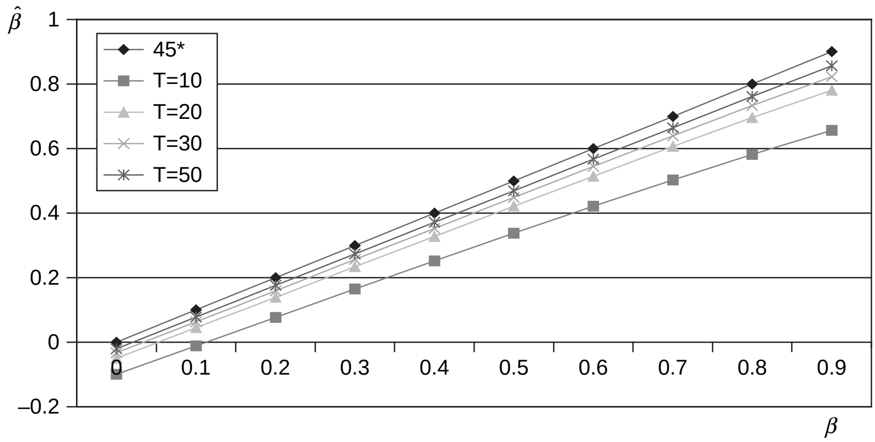

Figure 1. The inconsistency of the panel autoregressive coefficient.

by GMM, where sufficiently lagged values of Δy_j and y_j are used to instrument y_{jt-i} in equation (15) (see, for example, Badinger *et al*., 2004). Apart from the 'weak instrument' problem, there is a further problem since q in equation (15) is unknown. If $q = 1$, as is typically assumed, matters are easier. But q is unknown, as is the autocorrelation structure of the residuals in equation (15). We are therefore sceptical of the wisdom of applying GMM to the estimation of equation (15).

Kiviet (1995) and Hahn & Kuersteiner (2002) have suggested bias correction as an alternative to maximum likelihood (ML) and GMM. Since ML and GMM require the specification of instrumental variables whereas bias correction does not, bias correction is a practical and attractive alternative to GMM and ML.[8] In Section 4 we suggest using equation (17) to bias correct LSDV estimates of β.

2.5. Impulse Responses

Just as VARs are used to simulate the dynamic effects of exogenous shocks upon the state variables, so SpVARs may be used to simulate the spatial-temporal dynamic effects of exogenous shocks. Impulse response analysis in SpVARs is inevitably more complex than in VARs and spatial models because shocks propagate across space as well as over time. We begin by considering the case of a single-state variable ($K = 1$ and $P = 0$) in which the shocks are not spatially autocorrelated, for which the SpVAR is:

$$Y_{nt} = \beta Y_{nt-1} + \theta \sum_{i \neq n}^{N} w_{ni} Y_{it} + \lambda \sum_{i \neq n}^{N} w_{ni} Y_{it-1} + \varepsilon_{nt}. \tag{18}$$

The spatial lags are expressed once more by the scalars θ and λ. If these parameters are zero, equation (18) reverts to an autoregressive process.

Writing equation (18) in matrix form gives:

$$Y_t = \beta I_N Y_{t-1} + \theta W Y_t + \lambda W Y_{t-1} + \varepsilon_t, \tag{19}$$

where Y is $N \times 1$. Equation (19) may be written using the temporal lag operator (L) as:

$$(A + BL)Y_t = \varepsilon_t \tag{20}$$

$$A = I_N - \theta W$$

$$B = -(\beta I_N + \lambda W).$$

The impulse response profiles are obtained by deriving the Wold representation of equation (20), i.e. by expressing Y_t in terms of current and lagged values of ε. This is obtained by dividing both sides of equation (19) by $C = A - BL$, in which case the solution for Y_t is:

$$Y_t = C^{-1}\varepsilon_t + \sum_{i=1}^{N} a_i r_i^t, \tag{21}$$

where the eigenvalues are denoted by r and the as are arbitrary constants determined by initial conditions. Provided the data are stationary $|r_i|<1$, in which case the summation term tends to zero with time. The N eigenvalues are the solution to:

$$|C^{-1} - rI_N| = 0. \tag{22}$$

Since A and B depend upon θ and λ it is inevitable that the eigenvalues of the SpVAR depend upon the spatial lag coefficients. This also means that the stationarity conditions for VARs are different from their counterparts in SpVARs.

More generally, the number of eigenvalues is equal to NKq. Therefore, in a typical SpVAR this number will be large. By definition, $N = 1$ in VARs so that the number of eigenvalues in an SpVAR is N times larger than in its VAR counterpart. In the empirical example below $K = 4$, $N = 9$ and $q = 1$ so that there are 36 eigenvalues.

To illustrate, we set $N = 2$, $K = q = 1$ and $w_{12} = w_{21} = 1$, in which case the structural model is:

$$Y_{1t} = \theta Y_{2t} + \beta Y_{1t-1} + \lambda Y_{2t-1} + \varepsilon_{1t} \tag{23}$$

$$Y_{2t} = \theta Y_{1t} + \beta Y_{2t-1} + \lambda Y_{1t-1} + \varepsilon_{2t}. \tag{24}$$

The characteristic equation is:

$$ar^2 + br + c = 0 \tag{25}$$

$$a = 1 - \theta^2$$

$$b = -2(\beta + \theta\lambda)$$

$$c = \beta^2 - \lambda^2.$$

Equation (25) has two eigenvalues r_1 and r_2 given by:

$$r_1 = \frac{(1 - \theta)(\beta - \lambda)}{1 - \theta^2}$$

$$r_2 = \frac{(1+\theta)(\beta+\lambda)}{1-\theta^2}.$$

Stationarity requires these roots to be less than unity in absolute value. It is obvious that stationarity does not simply depend upon β as it does in the absence of spatial effects. Indeed, the absolute value of β may be less than unity, but Y may nonetheless be non-stationary. The following results are easily established:

(i) If $\beta = 1$ there are no values of θ and λ that induce stationarity. Therefore, if a variable is temporally non-stationary it remains so when spatial dynamics are present.
(ii) If $\theta = 0$ the eigenvalues are $r = \beta \pm \lambda$. Therefore, if $\lambda = 1$ the variable must be non-stationary regardless of β.
(iii) If $\beta = 0$ the eigenvalues are $r = \lambda(\theta \pm 1)/(1-\theta^2)$, in which case the variable may be non-stationary.
(iv) If $\theta = 1$ and $\lambda = 0$ there is a single eigenvalue with $r = 1½\beta$. In this special case the variable is stationary when $\beta < 2/3$.

Assuming stationarity, the general solution for Y_{1t} is:

$$Y_{1t} = \frac{\varepsilon_{1t} - \pi\varepsilon_{1t-1} + (\theta\varepsilon_{2t} + \lambda)\varepsilon_{2t-1}}{(1-r_1L)(1-r_2L)} + A_1r_1^t + A_2r_2^t, \tag{26}$$

where the As are determined by initial conditions. Since the roots lie within the unit circle these terms tend to zero over time. Inverting the lag polynomials by partial fractions in equation (26) gives the relationship between Y_{1t} and current and lagged εs:

$$Y_{1t} = \frac{1}{r_1 - r_2}\sum_{\tau=0}^{\infty}[r_1^{1+\tau}(\varepsilon_{1t-\tau} - \beta\varepsilon_{1t-\tau-1}) - r_2^{1+\tau}(\theta\varepsilon_{2t-\tau} + \lambda\varepsilon_{2t-\tau-1})] + C_1r_1^t + C_2r_2^t, \tag{27}$$

where the Cs are arbitrary constants determined by initial conditions. According to equation (27), current and lagged shocks in region 2 reverberate onto region 1. If, however, there are no spatial dynamics ($\theta = \lambda = 0$) equation (27) simplifies to:

$$Y_{1t} = \sum_{\tau=0}^{\infty}\beta^i\varepsilon_{1t-\tau} + C_1\beta^t. \tag{28}$$

2.6. *Spatial Weights*

We experiment with alternative spatial weights. However, the main results we present use:

$$w_{knit} = \frac{1}{d_{ni}}\frac{Z_{it}}{Z_{nt} + Z_{it}}, \tag{29}$$

where d_{ni} denotes the distance between regions N and i, and Z is a variable that captures scale effects. For example, if Z is represented by population, equation (29) states that the importance of region i to region N varies directly with the population in region i relative to region N. Spatial weights are therefore larger for bigger neighbours, and smaller for smaller neighbours. This spatial weighting scheme is asymmetric unless $Z_n = Z_i$ (i.e. the regions are of equal size). Other asymmetric weighting schemes include, for example, commuting weights, which reflect rates of commuting between regions N and i. Spatial weights are assumed to be the same across variables (do not depend on K), but may vary over time.

If the spatial lag coefficients are estimated by ML and the W matrix is symmetric, the estimated variance–covariance matrix of the parameters is symmetric. This result does not extend, however, to the case where W is asymmetric.[9] If, however, the spatial lag coefficients are estimated by IV rather than ML it does not matter that W is asymmetric.

2.7. Panel Unit Roots

In this section we discuss the econometrics of SpVAR estimation using regional panel data. The variables of interest in the SpVAR have to be stationary. Trending variables such as gross regional product (GRP) or GRP per capita cannot be stationary since their means and variances must increase over time. However, trendless variables will be non-stationary if their variance increases over time. This happens if their data-generating process (DGP) happens to be a driftless random walk, which necessarily contains a unit root. Panel unit root tests[10] have been reviewed by Maddala & Kim (1999). In a regional science context, the issue of spurious regressions produced by non-stationary data containing spatial unit roots has been addressed by Fingleton (1999) and Mur & Trivez (2003). We prefer the widely used panel unit root test proposed by Im *et al*. (2003) because it allows for heterogeneity across the regions in the panel.[11] Suppose, as seems likely, that GRP per capita and other regional variables of interest are non-stationary, but Im *et al*.'s panel unit root test shows that their first differences happen to be stationary. In this case the SpVAR must be estimated in first differences, and not levels.

Panel unit root tests typically assume that the panels are independent.[12] Baltagi *et al*. (2005) have investigated the power of panel unit root tests, such as Im *et al*.'s, when the residuals happen to be spatially autocorrelated. If the spatial autocorrelation coefficient is large (0.8) there is considerable size distortion in Im *et al*.'s as well as other unit root tests. However, if the spatial autocorrelation coefficient is small (0.4) there is little or no size distortion.[13]

2.8. Fixed vs Random Effects in Spatial Panels

The choice between fixed and random effects in spatial panel data models is not trivial. Several issues have been raised in the literature. First, if the data happen to be a random sample of the population, unconditional inference about the population necessitates estimation with random effects. If, however, the objective is limited to making conditional inferences about the sample, then fixed effects should be specified. Since researchers are usually interested in making unconditional inferences about the population the default option should be random effects. This line of reasoning[14] implies that if the sample happened to be the population, specific effects should be fixed because each panel member represents itself and has not been sampled randomly.

In household panels the sample is small relative to the population. However, in spatial panels the data set typically covers the entire population of spatial units. For example, the Penn World Tables cover all the countries in the world and NUTS2 covers all the regions in the EU. Our data cover all the regions of Israel. Since none of the regions are sampled randomly, estimation should be with fixed effects. 'For example, an intercountry comparison may well include the full set of countries for which it is reasonable to assume that the model is constant' (Greene, 2003, p. 293). Matters would be different if the spatial units in the data set were a random sample of the spatial units in the population, such as a sample of cities or counties.

A second issue raised in the literature concerns dependence between random effects and the covariates in the model. Such dependence, if it exists, typically induces bias in the parameter estimates of the model. Mundlak (1978) has argued in this case that the fixed effects estimator is observationally equivalent to the random effects estimator. Indeed, this is the line adopted by Wooldridge (2002), who suggests specifying fixed effects if the covariates and random effects happen to be dependent. This argument would only be relevant if the spatial panel data set were a sample rather than the population.

A third issue is practical. If the number of units in the panel is large, estimating fixed effects consumes degrees of freedom and reduces the variation in the data. Also, LSDV does not allow the estimation of parameters that vary in the cross-section but which do not vary over time. These problems do not arise when random effects are specified. In spatial data the number of spatial units tends to be relatively small, so that this issue is not of major importance. In our case the number of spatial units is nine.

3. Data

3.1. Data Sources and Definitions

For our empirical application of SpVAR we use annual panel data for nine regions in Israel (see Figure 2) for the period 1987–2004. The vector comprises four variables: real earnings, population, real house prices and the stock of housing. The latter is measured in 1000s of square metres. Hence, $T=18$, $N=9$, and $K=4$. Since these observations are too few to estimate individual models for each region, we pool the time series and cross-section data for purposes of estimation. We note that the panel unit root tests proposed by Im *et al*. (2003) report critical values for $T=10$, in which case we feel that it is meaningful to use 18 years of data. Calculations by Im *et al*. show that when $T=18$ and $N=9$ the size of the unit root test is about 0.05 and its power is about 0.2. This means that the probability of incorrectly rejecting the null hypothesis when it is true is about 5%, and the probability of correctly rejecting it when it is false is about 20%. The latter would have been 26% with $T=25$ and 75% with $T=50$. In our opinion what matters is the length of the observation period and not merely the number of data points. Eighteen monthly or even quarterly data points would not have been adequate because the observation period would have been only a year and half in the former case and four and half years in the latter. These periods would have been too short for observing convergence phenomena, whereas 18 years is, in our opinion, a sufficiently long period for these purposes.

At this stage we do not present a formal economic model, which relates these four variables. Such a model might predict that house prices vary directly with the

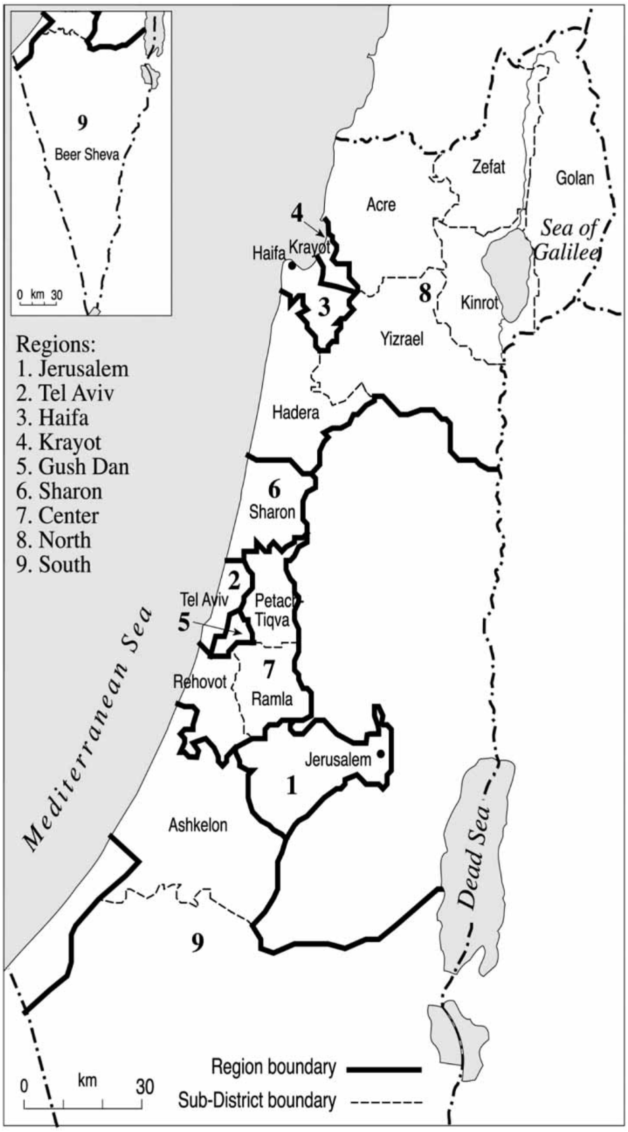

Figure 2. Geographic regions of Israel.

demand for housing services in the region, which in turn varies directly with income and population, and they vary inversely with the supply of housing services as measured by the stock of housing.[15] It might also predict that the regional distribution of the population depends upon house prices and earnings; people prefer to live in regions where earnings are higher and housing cheaper. It might further predict regional spillover effects. For example, if house prices happen to become more expensive in neighbouring regions house buyers will prefer to move into the region where it is cheaper, so that house prices increase there. Therefore, there is sufficient reason to believe that the SpVAR will not be vacuous. However, we stress that we do not use the SpVAR to test structural hypotheses about regional housing and labour markets. Our main motivation is to apply SpVAR, and to illustrate the methodology presented in Section 2.

Real earnings in region N at time t (W_{nt}) have been constructed by us from the Household Income Surveys (HIS) of the Central Bureau of Statistics (CBS) and are deflated by the national consumer price index (CPI). The population in region N at the beginning of time t (POP_{nt}) is published by CBS. CBS also publishes indices of house prices for the nine regions, which are based on transactions data and which we deflate by the CPI. Finally, we have constructed the stock of housing in region N at the beginning of time t (H_{nt}), which is measured in (gross) square metres. We use data on housing completions in the nine regions measured in square metres, published by CBS. The change in the stock of housing is defined as completions minus our estimates of demolitions. The level of the housing stock is inferred from data in the 1995 census.

3.2. Panel Unit Root Tests

The data are plotted in Figure 3. Not surprisingly, all four variables have grown over time, hence they cannot be stationary. It should be noted that the 1990s witnessed mass immigration from the former USSR, which had major macro-economic implications, especially for labour and housing markets (Beenstock & Fisher, 1997). The population grew in all regions, but particularly in the south where housing was cheaper. In Table 1 we report panel unit root tests (t-bar) due to Im *et al*. (2003), which is the average of the first-order augmented Dickey–Fuller statistics for variable j in the nine regions. When $d=0$ the absolute value of t-bar is below its critical value in the case of earnings and the housing stock, so these variables are clearly non-stationary. Surprisingly, however, Table 1 suggests that population and house prices are stationary in levels. When $d=1$, absolute t-bar is greater than its critical value for all variables, hence all four variables are difference stationary. Although Table 1 suggests that earnings and the housing stock are $I(1)$ while population and house prices are $I(0)$, we specify SpVAR in log first differences.

Spatial dependence in the data may distort the empirical size of the IPS test, as noted in Subsection 2.7. However, the data plotted in Figure 3 are clearly trending, so the conclusion that $d=1$ is not controversial despite potential size distortions in Table 1.

4. Results

Our main objective is to estimate equation (16a). However, since Y_t^* and v_t are correlated we cannot estimate equation (16a) directly. The 'reduced form' of equation (16a) is:

$$Y_t = \Gamma_0 + \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-1}^* + u_t, \tag{30}$$

where $\Gamma_0 = (I_{NK} - \Pi_2 W^*)^{-1}\Pi_0$, $\Gamma_1 = (I_{NK} - \Pi_2 W^*)^{-1}\Pi_1$, $\Gamma_2 = (I_{NK} - \Pi_2 W^*)^{-1}\Pi_3$, $u = (I_{NK} - \Pi_2 W^*)^{-1}v$ and $W^* = I_K \otimes W$. Provided ε_t is temporally independent, Y_{t-1} and Y_{t-1}^* will be independent of u_t in equation (30), i.e. they are weakly exogenous instruments for Y_t^*. We begin by estimating equation (30). Then we use the predicted values of Y_t to serve as instrumental variables for Y_t^* in equation (16a), i.e. we estimate equation (16a) using $\hat{Y}_{kt}^* = W\hat{Y}_{kt}$ instead of Y_{kt}^*.

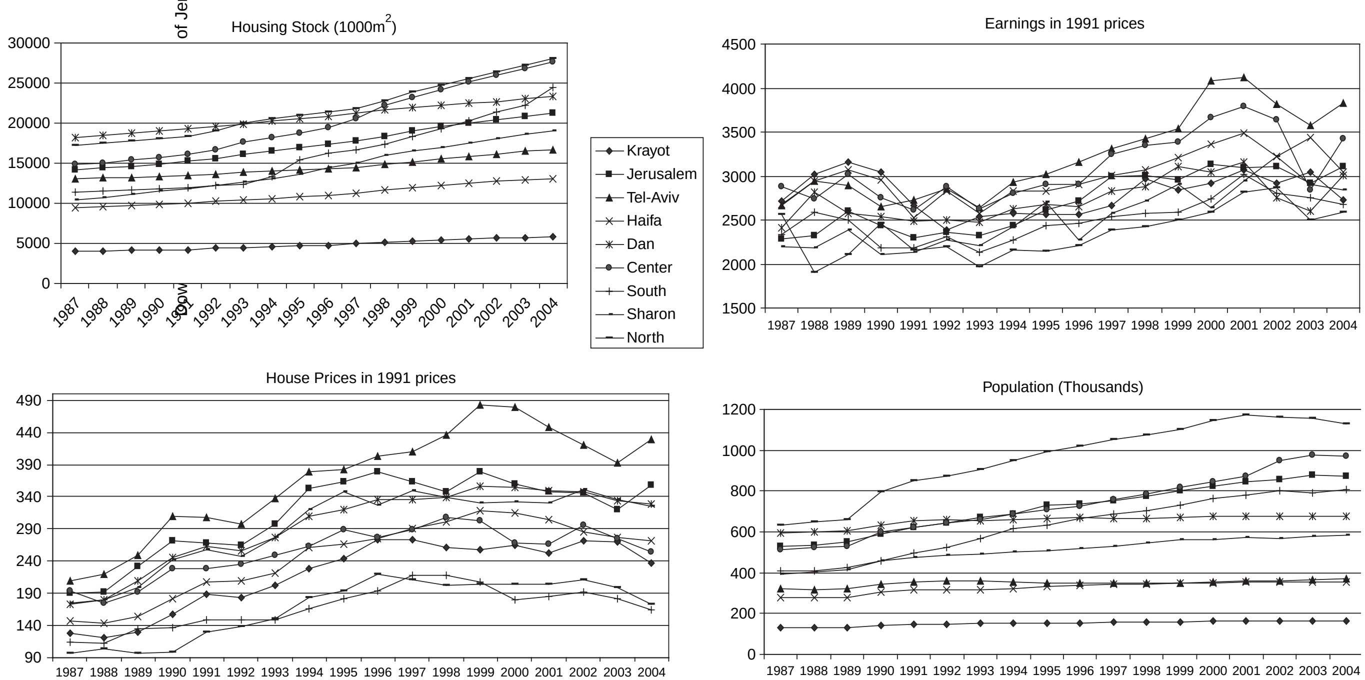

Figure 3. Regional panel data: Israel 1987–2004.

Table 1. Panel unit root tests (t-bar)

$\text{Ln}(Y_j)$	$d=0$	$d=1$	$d=2$
Earnings	−1.205	−3.503	−5.079
Population	−2.707	−2.531	−6.603
House prices	−3.030	−2.537	−5.321
Housing stock	−0.092	−2.227	−3.410

Notes: Auxiliary regression: $\Delta^d \ln Y_{knt} = \alpha_{kn} + \lambda_{kn}\Delta^{d-1}\ln Y_{knt-1} + \delta_{kn}\Delta^d \ln Y_{knt-1} + \varepsilon_{knt}$. The critical values of t-bar with $N=9$ and $T=18$ are -2.28 at $p=0.01$ and -2.17 at $p=0.05$.

4.1. Estimating the SpVAR's Reduced Form

Since $T=18$ the SpVAR is limited to first-order spatial and temporal lags. There are insufficient degrees of freedom to justify higher order lags. In any case the panel Darbin Watson (DW) statistics and other tests do not suggest that higher order temporal lags are required. There are also insufficient degrees of freedom to estimate heterogeneous models in which the parameters vary by region and/or by time period since $N=9$ and $T=18$. Therefore, the SpVAR is homogeneous. For example, in equation (15) $A_{nt} = A$. Due to the paucity of degrees of freedom we do not test for poolability.

Because the SpVAR includes 36 separate relationships (nine regional relationships for each of the four variables), we simplify by estimating each variable as a separate bloc consisting of nine regional panel relationships. For example, the earnings bloc specifies first-order temporal and lagged spatial lags for earnings, but it also specifies first-order temporal and lagged spatial lags for each of the other three variables. This implies, for example, that current earnings in Jerusalem may be affected by lagged house prices in Tel Aviv as well as in the neighbourhood of Jerusalem. Each of the four blocs is estimated separately, which implies that spatial correlation exists within variables but not between variables. For example, earnings shocks in Jerusalem may be correlated with earnings shocks in Tel Aviv, but they are assumed to be uncorrelated with population shocks in Tel Aviv. As mentioned in Subsection 2.3, to have specified spatial correlation between variables would have greatly increased the burden of estimation.

Each region in the bloc is specified to have a specific effect and the regional shocks within the bloc are assumed to be contemporaneously correlated (spatially correlated). The method of estimation in each of the four blocs is therefore SUR, which provides estimates of the spatial correlation coefficients for the bloc. Finally, the regional-specific effects are assumed to be fixed for the bloc and are estimated by LSDV.

As mentioned in Subsection 2.6, we experimented with various spatial weighting schemes. However, for reasons of space we only report in Table 2 the results obtained using equation (15), i.e. the spatial weights are asymmetric reflecting distances and relative population sizes.

We estimate an unrestricted first-order SpVAR as reported in Table 2. In the unrestricted SpVAR the panel DW statistics do not indicate the presence of first-order temporal serial correlation in the residuals. In the unrestricted model several parameters are not statistically significant. We applied the 'general-to-specific' methodology (Hendry, 1995) to estimate a restricted model, which is also reported in Table 2. Restrictions are acceptable when the multivariate SBC (Schwarz

Table 2. Parameter estimates of the SpVAR's reduced form

	Earnings		Population		House prices		Housing stock	
	Unrestricted model	Restricted model	Unrestricted model	Restricted model	Unrestricted model	Restricted model	Unrestricted model	Restricted model
Temporal lag								
Earnings	−0.357	−0.332	0.038	0.037	0.104	0.102	0.006★★	–
Population	−0.311★	–	0.112★★	–	0.678	0.672	0.059	0.060
House prices	−0.148	−0.104	0.0004★★	–	−0.006★★	–	0.016	0.018
Housing stock	0.970	1.019	−0.078★★	–	0.0003	–	0.396	0.389
Lagged spatial lag								
Earnings	0.131★★	–	0.018★★	–	0.233	0.235	0.0003★★	–
Population	−0.314★★	−0.497	0.037★★	–	−0.593★	−0.605★	−0.064	−0.068
House prices	0.205★★	0.196★★	0.104	0.103	0.493	0.403	0.003★★	–
Housing stock	1.836	2.174	−0.359	−0.458	−0.790	−0.810	0.172	0.170
R^2 adjusted	0.146	0.148	0.297	0.312	0.091	0.107	0.464	0.474
Panel DW	2.235	2.176	2.116	1.866	1.843	1.861	1.641	1.639
SAC (δ)		0.794		0.836		0.853		0.952
Lagged SAC γ		0.118★★		−0.040★★		−0.007★★		−0.060★★
TAC (ρ)		−0.147★		−0.034★★		0.009★★		0.044★★
Det Ω		0.0049		0.0003		0.0001		0.0014
F-statistic		0.847		0.393		0.000		0.019

Notes: All variables are first differences in logarithms. Bloc estimation by SUR with fixed effects and residual covariance matrix Ω. The number of observations (NT) per bloc is 144. The estimation period including lags is 1987–2004. All parameter estimates have p-values < 0.05. Asterisked parameters have p-values between 0.05 and 0.1. Double asterisked parameters have p-values > 0.1. SAC and TAC, respectively, denote the spatial and temporal autocorrelation coefficients for the residuals. The F-statistic is a Wald test of the restricted model within blocs. SBC (the Schwarz Bayesian Criterion for testing the restricted model within and between blocs) has the following values: SBC unrestricted = −814.88; SBC restricted = −818.97.

Bayesian Criterion) is minimized and the residuals remain serially independent. Both of these conditions are fulfilled for the restricted model reported in Table 2.

Recall that all the variables that feature in the SpVAR are first differences of logarithms. In the case of the first difference in the logarithm of earnings all temporal lags, with the exception of population, are statistically significant in the unrestricted model. The autoregressive coefficient for earnings is negative (-0.357). As discussed in Subsection 2.3, this coefficient is biased downwards. Using equation (17) to bias correct this estimate implies that the true estimate is about -0.31. Additionally, earnings vary directly with lagged housing stock and inversely with the lagged house prices. None of the lagged spatial lags are significant, with the exception of housing stock which is positively related to current earnings. The restricted model tells very much the same story, the only exception being the (negative and significant) effect of spatially lagged population. This implies that population growth in neighbouring regions reduces current wage growth. The opposite applies to the rate of growth in the housing stock in neighbouring regions.

In the restricted model for the rate of population growth there is a small temporal lag on the rate of growth of earnings, but no autoregressive effect. Equation (17) suggests that when the estimated autoregressive coefficient is zero, the bias-corrected coefficient is approximately 0.06. Two lagged spatial lag coefficients are statistically significant, implying a spillover effect to population growth from the growth in house prices in neighbouring regions. The opposite applies to the rate of growth in the housing stock in neighbouring regions.

The current growth in real house prices varies directly with the lagged rates of growth in earnings and population, but as in the case of population growth, there is no autoregressive effect, hence the bias-corrected coefficient is 0.06. All four lagged spatial lag coefficients are statistically significant. The growth in house prices varies directly with lagged house price growth in neighbouring regions, and inversely with the growth in the housing stock in these regions. There is also a lagged spillover effect from earnings growth in neighbouring regions. Finally, the rate of growth of the housing stock varies directly with its own lag. Equation (17) suggests that the bias-corrected autoregressive coefficient is about 0.48. There is also a positive spillover effect from lagged housing growth in neighbouring regions.

We make no systematic attempt at interpreting the coefficients of the SpVAR's reduced form in terms of economic theory. Our view is that VAR modelling does not constitute a sound methodological basis for hypothesis testing, especially when the data happen to be non-stationary as here. The main reason for this is that economic theory refers to relationships between levels of variables whereas VARs typically refer to changes in their levels. Establishing empirically that Y and X happen to be related in first differences does not necessarily mean that they are related in levels (Hendry, 1995). We think that hypothesis testing with non-stationary panel data such as ours should be carried out using panel co-integration (Kao, 1999). Nevertheless, VAR modelling requires no methodological justification and should be viewed as a statistical tool for understanding the dynamic structure between variables, especially when economic theory is often vague about the nature of these dynamics (Sims, 1980). This applies a fortiori in the case of SpVARs when economic theory is vague about spatial dynamics as well as temporal dynamics.

4.2. Spatial Correlation

Table 2 indicates that the residuals are spatially correlated. Indeed, the SAC coefficient ranges between 0.794 and 0.952. Since Table 2 refers to the reduced form these SAC coefficients are not a major concern; they do not affect the consistency of the reduced-form parameter estimates. More important is the fact that the lagged SAC coefficients and the temporal autocorrelation coefficients are not significantly different from zero, for otherwise the variables in the model could not serve as weakly exogenous instrumental variables for estimating equation (16a) and identifying the contemporaneous spatial lag coefficients. Finally, Table 2 reports the determinant of the residual variance–covariance matrix estimated by SUR (det $\mathbf{\Omega}$). If the residuals between regions are independent det $\mathbf{\Omega} = 1$. The greater the regional dependence between residuals the closer to zero will be det $\mathbf{\Omega}$. The estimates of det $\mathbf{\Omega}$ are clearly less than unity, and quite close to zero, suggesting a high degree of correlation between the residuals of different regions for all four variables in the model.

In Table 3 we report the spatial correlation coefficients estimated by SUR. These are the SC coefficients referred to in Subsection 2.3. For example, the correlation between earnings shocks in Tel Aviv and Jerusalem is 0.4689, while the correlation between population shocks in these two regions is 0.0592. Table 3 reveals that almost every element in the spatial correlation matrix is statistically significant. Most of the spatial correlations are less than 0.5 in absolute value. However, a few exceed 0.8 and the largest in absolute value is 0.9057 (between house price shocks in Jerusalem and Dan). We make no attempt at interpreting these coefficients. Recall that these spatial correlations have been estimated by SUR within blocs but not between them. Therefore the spatial correlations between variables are zero by construction. This means, for example, that earnings shocks in Jerusalem are uncorrelated with population shocks in Tel Aviv.

4.3. Impulse Responses

We illustrate the properties of the reduced-form SpVAR by reporting impulse response simulations. In a temporal VAR the impulse responses refer to the dynamic effects of shocks to a certain variable upon itself as well as the other variables that feature in the model. In an SpVAR the impulse responses refer to the effects of shocks that occur in a specific region to a certain variable upon the following:

(i) The shocked variable in the region in which the shock occurred.
(ii) Other variables in the region in which the shock occurred.
(iii) The shocked variable in other regions.
(iv) Other variables in other regions.

The impulse responses in SpVARs therefore include the temporal dynamic effects as in a regular VAR as well as the ricochet effect between regions and across variables that is induced by the spatial specification of the model. The latter include the spatial lag structure of the restricted model as given in Table 2 as well as its spatial autocorrelation structure as given in Table 3. For these purely illustrative purposes we use the autoregressive coefficients as reported in Table 2 rather than their bias-corrected counterparts.

Table 3. Spatial correlations (SCs): SUR estimates

	Jerusalem	Tel Aviv	Haifa	Krayot	Dan	Centre	South	Sharon
Tel Aviv								
Earnings	0.4689							
Population	0.0592							
Housing	0.4681							
Prices	0.8367							
Haifa								
Earnings	0.5258	0.4885						
Population	0.6395	0.3769						
Housing	0.4465	0.1443						
Prices	0.5760	0.7259						
Krayot								
Earnings	0.3261	−0.0986	0.3123					
Population	0.3571	0.7532	0.6699					
Housing	0.3628	−0.0947	0.7005					
Prices	0.1686	0.1560	0.4088					
Dan								
Earnings	0.4624	0.6346	0.2150	−0.1596				
Population	0.4381	0.7662	0.6846	0.6268				
Housing	0.1188	0.2435	0.0275	−0.0042				
Prices	0.9057	0.8092	0.7621	0.3445				
Centre								
Earnings	0.6940	0.7720	0.4029	−0.0672	0.7591			
Population	0.3192	0.4450	0.6501	0.6314	0.3945			
Housing	0.5693	0.5025	0.5410	0.6675	0.4096			
Prices	0.4371	0.3631	0.2653	0.1329	0.4384			
South								
Earnings	0.3180	0.6475	0.5510	0.1060	0.3680	0.5494		
Population	0.2908	0.2860	0.2584	0.5066	0.2959	0.2491		
Housing	−0.3851	−0.2398	−0.4762	−0.2845	−0.4985	−0.4704		
Prices	0.3490	0.1425	0.2024	0.1480	0.4834	0.4808		
Sharon								
Earnings	0.1975	0.0748	0.2110	0.1117	0.0491	0.2969	0.6222	
Population	0.3651	0.6995	0.7510	0.7970	0.7944	0.4116	0.3496	
Housing	−0.1399	0.1156	0.2709	0.4803	0.3213	0.5398	−0.2150	
Prices	0.6307	0.5167	0.6013	0.4715	0.7682	0.5781	0.3371	
North								
Earnings	0.4529	0.2913	0.3333	0.1053	0.2991	0.4946	0.2078	0.2438
Population	0.6555	0.4359	0.8813	0.7927	0.6439	0.5445	0.5638	0.7686
Housing	0.6104	0.5999	0.4791	0.4058	−0.0860	0.5896	−0.2150	0.0463
Prices	0.1364	−0.0331	0.3159	0.5648	0.1499	−0.1297	0.1607	0.1653

We begin by temporarily shocking earnings in Jerusalem and investigating the effects of this shock upon the four dimensions mentioned in the previous paragraph. At first we assume that regional shocks are uncorrelated (i.e. we ignore the spatial autocorrelation structure of the model as given in Table 3). This means that the earnings shock in Jerusalem is entirely idiosyncratic. It also means that the impulse responses stem entirely from the spatial and temporal lag structures featured in Table 2. Subsequently, we assume that regional shocks are spatially correlated (i.e. we calculate the impulse responses using the parameters in Tables 2 and 3).

This means that earnings shocks in Jerusalem are not entirely idiosyncratic, in which case an earnings shock in Jerusalem is accompanied by earnings shocks elsewhere through the model's spatial autocorrelation structure.

To compute the impulse responses we first carry out a dynamic simulation of the entire SpVAR starting in 1990 and terminating in 2004, which takes as its initial conditions the values of the variables as of 1989. This provides base-run values in levels for all the variables during 1990–2004. Because the SpVAR is estimated in the first differences of logarithms we shock earnings in Jerusalem by 0.02 (2%) in 1990 followed by an antithetic shock of −0.02 in 1991, so that the level of the variables in the model is preserved in the long run. We compute new dynamic solutions for all the variables in levels during 1990–2004. The impulse responses are defined as the differences between these new solutions and their base run values. We expect the impulse responses to die out over time. Since the model is log-linear the impulse responses are not base dependent (i.e. they are independent of when they occur). In these simulations the connectivity matrices W_t are assumed to remain unchanged.[16]

Figure 4 plots the impulses generated by a 2% earnings shock occurring in the region of Jerusalem in 1990 for all four variables in the model in three of the nine regions. (To have included all nine regions would have been too confusing.) The upper left panel in Figure 4 plots the impulse for earnings in the three regions. Note that in all panels the local impulses are measured on the left vertical and the external impulses are measured on the right vertical, which has a smaller scale. Initially the level of earnings in Jerusalem necessarily increases by 0.02 (2%), but in 1991 it falls by nearly 1% relative to the base run. This overshooting happens because the temporal autoregressive coefficient for earnings is negative (Table 2). The impulses die down quite rapidly. The spatial lag structure implies that the increase in earnings in Jerusalem in 1990 spills over into other areas in 1991. The spatial lag coefficient is 0.131 in Table 2, and hence we expect these spillover effects to be positive. However, we do not expect them to be identical across regions since the spatial weighting matrix is not uniform. Jerusalem has a greater impact on the south than it does on the Dan region. This is why from 1993 onwards, the earnings impulse in the south is more positive than it is in the Dan region. Subsequently these impulses oscillate but eventually die out, as expected.

These regional spillovers cannot, of course, arise in a standard VAR. They are the distinctive contribution of SpVARs. Shocks that occur in one region spill over to other regions provided that the spatial lag coefficient is non-zero. The differential force of these spillovers depends on the spatial weighting matrix. The force will be stronger in regions in which Jerusalem is relatively more important. Recall that we have defined these spatial weights asymmetrically using equation (15). Therefore the force will be stronger in regions closer to Jerusalem and in which the population is smaller than Jerusalem's.

The other three panels in Figure 4 plot the impulses for the three other variables in the three regions. The impulses for Jerusalem are standard because they would arise in a standard VAR. For example, the top right panel shows that following the earnings shock in Jerusalem in 1990, house prices rise in Jerusalem in 1991, reflecting the positive (0.104) temporal lag, reported in Table 2, of earnings on house prices. Subsequently these impulses die out as expected. The novel feature in this panel is the spatial spillover of earnings shocks in Jerusalem onto house prices in other regions. These spillovers are positive because the spatial lag for earnings on

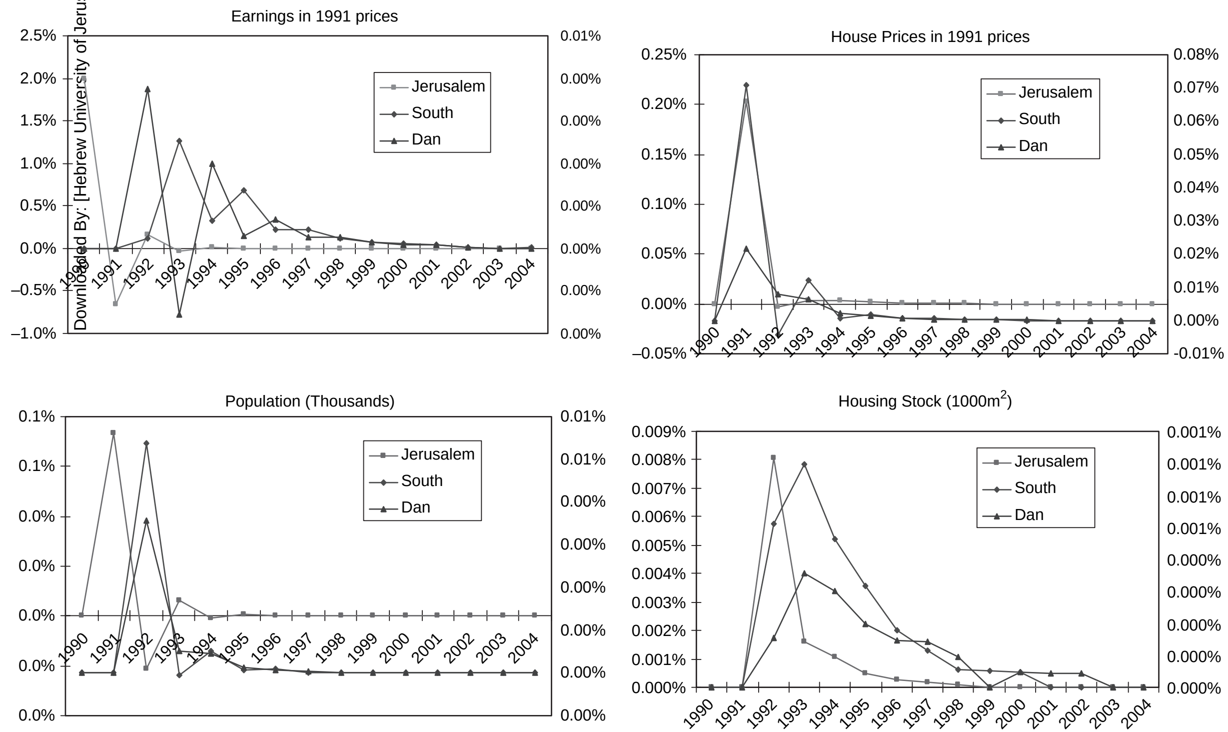

Figure 4. Impulse responses: 2% earnings shock in Jerusalem.

house prices is positive (0.233) in Table 2. The spillover is greater in the south than in the Dan region because the spatial weight for the south is larger. Subsequently, the impulses on both regions die out as expected.

The remaining (lower left and right) panels in Figure 4 plot the impulses for population and housing stock. They show that earnings shocks in Jerusalem spill over onto population and the housing stock in other regions. In general the impulses in Figure 4 die out quite rapidly, within about 4 years for population and slightly longer for housing stock. In the short run, however, they are non-zero within and between regions. Note that had we shocked earnings in say Tel Aviv instead of Jerusalem we would not have got the same impulses reported in Figure 4, because the spatial spillovers are not independent of where the shocks occur. The spillover from Tel Aviv to the south is not the same as the spillover from Jerusalem to the south. Indeed, because of the use of asymmetric spatial weights, the spillover from Jerusalem to Tel Aviv is not the same as the spillover from Tel Aviv to Jerusalem. In short, direction matters, as does the geographical distribution of shocks.

In Figure 5 we plot the impulses for a 2% population shock in Tel Aviv. The upper right panel plots the population impulses for Tel Aviv, Dan region and the Krayot. Note that according to Table 2 the spatial lag for population is positive (0.037), in which case we expect population shocks in Tel Aviv to spill over positively onto population in other regions. This expectation is confirmed. The spatial lag coefficient on population for earnings is negative (-0.314), so that the increase in the population in Tel Aviv in 1990 should spill over negatively onto earnings elsewhere in 1991 (but positively in Tel Aviv). This upper left panel shows that this is what happens in 1991. The same applies to both house prices in the lower left panel and housing stock in the lower right panel. In both cases, the impulses for the Dan region and the Krayot are negative, because the population spatial lags are negative (-0.593 and -0.064, respectively).

In the interest of space we do not report impulses for shocks to house prices and the housing stock. Here, too, there are spatial spillovers. The nature of these spillovers may be inferred from Table 2. For example, in the case of house prices the spatial spillover onto house prices elsewhere is positive (0.493), and the spatial spillover onto population elsewhere is also positive but smaller (0.104).

4.4. Impulses with Spatially Correlated Shocks

Recall that the impulses in Figures 3 and 4 refer to uncorrelated shocks and ignore the spatial correlation structure in Table 3. In this section we calculate impulses in which the shocks are assumed to be regionally correlated according to Table 3. The basic theory has already been discussed in Subsection 2.2. It may be shown that the correlated impulses are a simple matrix transformation of their uncorrelated counterparts. Let $\varepsilon_t = A\varepsilon_t + e_t$ be the spatial autocorrelation model in which ε is a column vector of regional shocks, e is a vector of idiosyncratic shocks, and A is a lower triangular matrix of spatial correlation coefficients. In our SpVAR, A is given by Table 3. We may solve for the εs in terms of the es as: $\varepsilon_t = (I - A)^{-1} e_t$. Figures 2 and 3 are calculated assuming $A = 0$, in which case $\varepsilon_t = e_t$. Therefore to transform uncorrelated impulses into correlated impulses we simply have to multiply the former by $(I - A)^{-1}$. This means that the correlated impulses are a weighted average of their uncorrelated counterparts.

Table 3 implies that when there is a positive shock to earnings in Jerusalem of 2% there is a positive shock to earnings in Tel Aviv of 0.9378% ($= 2 \times 0.4689$)

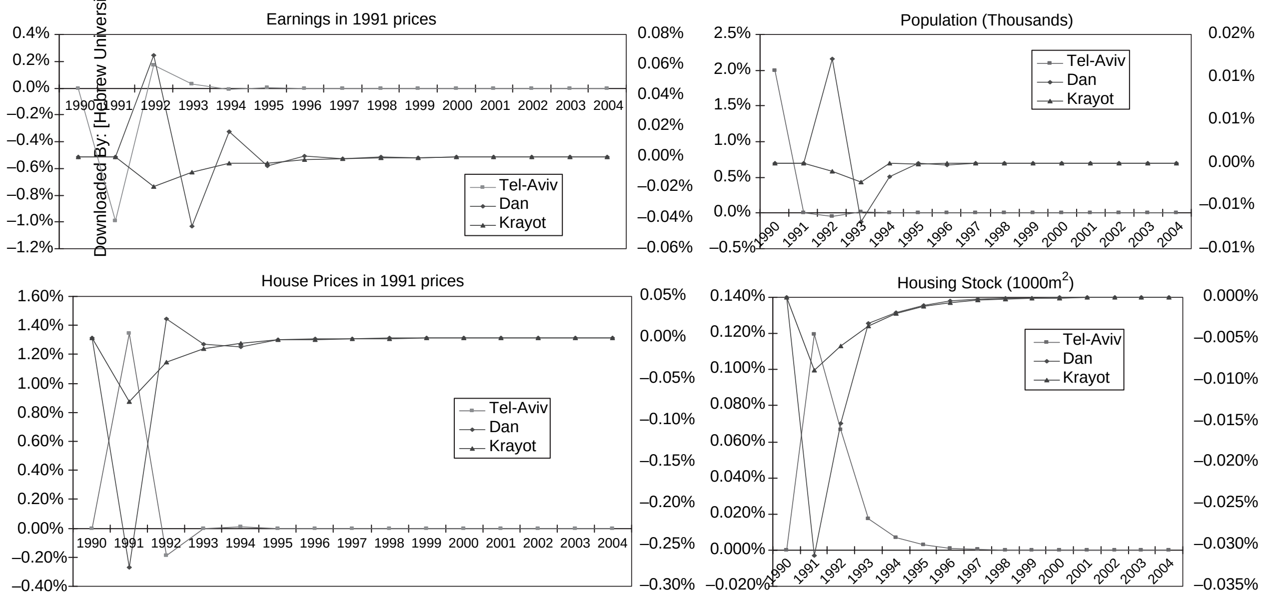

Figure 5. Impulse responses: 2% population shock in Tel Aviv.

and a positive shock in Haifa of 1.0516%, etc. For example, the correlated impulses for an earnings shock in Jerusalem on earnings in the south are just a weighted average of the uncorrelated impulses in the nine regions. We compare the impulses generated in 1991 from a 2% shock inserted a year earlier in both Jerusalem and Tel Aviv. The Jerusalem simulation addresses an earnings shock and the Tel Aviv simulation relates to a population shock (see Table 4). We limit ourselves to reporting the correlated and uncorrelated impulses on the same three regions represented in Figures 3 and 4.

Table 4 shows that allowing for spatial correlation can make a difference to the magnitudes of the impulses. For example, in the case of a 2% earnings shock in Jerusalem, the correlated impulse effect on house prices in all other regions is consistently larger than in the uncorrelated case. The same can be seen for the impact of a 2% population shock in Tel Aviv. The uncorrelated impulse response with respect to housing supply and house price in other regions is consistently smaller than in the correlated case. In some instances spatial correlation can even reverse the sign of the impulse. This can be seen with respect to the correlated impulse response on both house prices and housing supply in the Krayot region. These results come as no surprise, however. The spatial autocorrelation structure may obviously reinforce or offset the impulses obtained when the shocks are assumed to be spatially uncorrelated.

4.5. Estimating the Contemporaneous Spatial Lag Coefficients

Finally, we turn to the estimation of equation (16a), which includes contemporaneous spatial lag (SAR) coefficients for the state variables. The spatial lag variables (Y^*) are instrumented using their predicted values from the restricted model in Table 2. As mentioned, these predicted values are weakly exogenous because the residuals in Table 2 are neither temporally autocorrelated nor is there lagged spatial autocorrelation. Results are reported in Table 5.

Table 4. Comparing impulses in 1991 with and without spatial correlation: (a) 2% earnings shock in Jerusalem, (b) 2% population shock in Tel Aviv

	Earnings	Population	Prices	Housing
(a)				
Jerusalem	− 0.00664	0.00073	0.00421	0.00000
	− 0.00664	0.00073	0.00203	0.00000
Dan	− 0.00307	0.00043	0.00370	0.00000
	0.00000	0.00000	0.00021	0.00000
South	− 0.00211	0.00023	0.00328	0.00000
	0.00000	0.00000	0.00071	0.00000
(b)				
Tel Aviv	− 0.00994	0.00000	0.01968	0.00155
	− 0.00993	0.00000	0.01345	0.00119
Dan	0.00630	0.00000	− 0.00801	− 0.00053
	0.00000	0.00000	− 0.00272	− 0.00031
Krayot	− 0.00098	0.00000	0.00083	0.00004
	0.00000	0.00000	− 0.00078	− 0.00008

Notes: The upper number refers to the spatially correlated case and the lower number refers to the spatially uncorrelated case. Impulses have been multiplied by 100.

Table 5. Parameter estimates of the SpVAR

	1. Earnings		2. Population		3. House prices		4. Housing stock	
	Coefficient	Standard error	Coefficient	Standard error	Coefficient	Standard error	Coefficient	Standard error
Temporal lag								
Earnings			0.0348	0.0114				
Population							0.0889	0.0204
House prices							0.0258	0.0060
Housing stock	0.4830	0.0995					0.5265	0.0760
Lagged spatial lag								
Earnings								
Population					−0.4955	0.2325	−0.1038	0.0250
House prices			0.1012	0.0301	0.5163	0.0557		
Housing stock			−0.4638	0.1420			0.2408	0.0431
Spatial lag	0.7833	0.0530	0.0229	0.1566	0.5844	0.0600	−0.3973	0.1280
R^2 adjusted	0.1677		0.3081		0.1170		0.4761	
Panel DW	2.2608		1.8710		1.7962		1.6661	
Det Ω	0.001325		0.00004		0.00011		0.002	
SAC (δ)	0.7974	0.0026	0.8364	0.0023	0.8491	0.0015	0.9705	0.0140
Lagged SAC γ	0.1883	0.0659	0.0401	0.0734	−0.0118	0.0661	−0.1094	0.0830
TAC (ρ)	−0.2458	0.0827	−0.0347	0.0874	0.0215	0.0778	0.0699	0.0844

Note: See notes to Table 2.

The contemporaneous SAR or spatial lag coefficient is statistically significant for three of the four variables, the exception being population growth. In the case of the housing stock the SAR coefficient is −0.397 and for earnings it is 0.7834. Note that despite the specification of spatial lags, the SAC coefficients are statistically significant. This means that SAC does not result from dynamic spatial misspecification. Note also that the determinant of the residual correlation matrix (det **Ω**) is close to zero for all four variables even after allowing SAC. This means that the residual correlation matrix is not simply due to SAC and that residuals are correlated between regions because shocks happen to be correlated for reasons unrelated to SAC. With the exception of earnings, the lagged SAC and temporal autocorrelation coefficients are not statistically significant. Many temporal lag coefficients are not significantly different from zero,[17] with the exception of the housing stock. Finally, an F-test shows that the fixed effect coefficients are not statistically significant for earnings and house prices, but they are very significant for population and the housing stock.

5. Conclusions

Curiously, whereas spatial econometricians have shown a growing interest in time series data, time series econometricians have shown little or no interest in spatial data. Our paper joins a small but expanding literature on the integration of time series and spatial data. VAR models do not have a spatial dimension. In this paper we consider how space may be introduced into the VAR framework. Conversely, spatial models have traditionally been devoid of time. This is obviously true for cross-section data, but it also applies to spatial panel data where the focus has been upon such phenomena as spatial lags and spatial autocorrelation, without any temporal dimension, such as temporal lags and temporal autocorrelation.

A rare exception is Badinger *et al*. (2004), who suggested a two-stage procedure in which they filter away the spatial dimension of the data in the first stage and then apply dynamic panel econometric techniques to these spatially filtered data in the second stage. This procedure treats the spatial relationships in the data as nuisance parameters, which can be 'concentrated out' in the first stage. Our view is that spatial relationships are not nuisance parameters and that spatial and temporal dynamics should be estimated jointly.

In this paper we have tried to integrate time series econometrics with spatial econometrics by estimating spatial and temporal dynamics jointly. Moreover, we use vectors of variables rather than single variables. We refer to this kind of modelling of spatial panel data as SpVARs, or spatial vector autoregressions. SpVARs contain such features as temporal lags, spatial lags, lagged spatial lags, spatially autocorrelated errors and spatially correlated error that are not autocorrelated. The latter are estimated by SUR and measure the correlation between shocks in different regions. Spatial autocorrelation imposes restrictions on the spatial correlation matrix. Whereas in cross-section data only spatial autocorrelation can be identified, in spatial panel data both types of correlation may be estimated.

We have illustrated these issues by estimating an SpVAR using annual data for Israel over the period 1987–2004 for nine regions and four variables. We show that in addition to temporal lags there is evidence of lagged spatial lags as well as spatially correlated errors.[18] We use the estimated SpVAR to simulate impulse responses which propagate within and between regions and within and between variables. These impulse responses show that innovations propagate over time and across space. For example, an innovation in a single region not only propagates over the variables in that region but also between regions and over time. In turn these reverberations feed back onto the source region. As expected for stationary spatial panel data, these shocks eventually die out after about 4 years.

We distinguish between correlated and uncorrelated shocks. In the former case, innovations in one region are correlated with innovations elsewhere according to the spatial correlation matrix estimated in the SpVAR. Such correlated shocks inevitably induce more regional turbulence than their uncorrelated counterparts.

Without formulating a formal economic model, we have statistically tested the temporal and spatial dynamics relating to those leading variables that contribute to disparities between regions: earnings, house prices, housing demand (represented by population distribution) and housing supply (regional housing stock). We estimate SpVAR models with first-order temporal and spatial lags. Spatial effects are estimated using asymmetric spatial weights based on distances and population sizes. For inter-regional impulse effects, these give more weight to closer and larger, more populated regions.

Finally we use the estimated SpVAR to estimate contemporaneous spatial lag coefficients by the method of instrumental variables, having first established that the latter are weakly exogenous. This model incorporates temporal lags, contemporaneous spatial lags, and lagged spatial lags.

Notes

1. For an early pioneering discussion of the problem in a univariate context see Pfeifer & Deutsch (1980).
2. Not SVAR since these mnemonics refer to structural VARs.
3. Stationarity here is defined temporally rather than spatially, as in Fingleton (1999).
4. See, for example, Enders (1994).
5. Elhorst (2004) considers the estimation of static panel models in which there is both spatial and temporal autocorrelation in the residuals.
6. As might have been expected according to the statistical theory for dependent observations (e.g. Gleser & Moore, 1983).
7. Yu *et al*. (2006) have recently suggested that the non-spatial parameters be concentrated out of the likelihood function, and that the spatial parameters be estimated from the concentrated likelihood function. This proposal, like that of Badinger *et al*., is equally problematic.
8. Hahn & Kuerstiener (2002) show that a simpler bias correction than equation (17) is not outperformed by GMM in finite samples.
9. See Anselin (1988, p. 79, fn. 14).
10. A more general panel unit root test would allow for spatial autocorrelation (Mur & Trivez, 2003). However, since such a test has yet to be developed we rely on conventional panel unit root tests.
11. This test was used, for example, by Mäki-Arvela (2003). Drennan *et al*. (2004) also tested for unit roots in spatial data. The issue of non-stationarity is sometimes ignored, however, in the regional science literature (e.g. Badinger *et al*., 2004).
12. However, Moon & Perron (2004) and Pesaran (2005) allow for some cross-section dependence.
13. For example, when $N = 50$ and $T = 25$ the empirical size for IPS is 5.6% at the 5% level when W is contiguous. The distortion varies inversely with N and T.
14. See, for example, Hsiao (1986, p. 43), Maddala (2001, p. 576), Baltagi (2005, p. 14) and Cameron & Trivedi (2005, p. 717).
15. See Bar Nathan *et al*. (1998).
16. This is a simplifying approximation because, according to equation (29), W_t depends on Z_t.
17. There is only one autoregressive coefficient in Table 5 (for the housing stock). Its bias-corrected counterpart is 0.64. The other bias-corrected autoregressive coefficients are 0.06, since their biased counterparts are zero.
18. Spatial lags are not estimated separately here. They are implicitly estimated in the lagged spatial lag coefficients. See equation (15).

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