

Analysis of Variance (ANOVA)

Review of:

- t-test
- ANOVA
- MANOVA
- ANCOVA
- MANCOVA

Review of concepts and definitions

- Population Vs. sample
- A Statistic
- A normal population : $y \sim N(\mu, \sigma)$ (mean and variance)
- Average and Standard deviation:

$$\bar{y} = \sum_i \frac{y_i}{n}$$

$$s_y^2 = \frac{\sum_i (y_i - \bar{y})^2}{(n-1)}$$

$$s_y^2 = SS_y / df_y$$

- Null Hypothesis: $H_0 : \mu_y = \mu_0$

One Sample t-test

The null hypothesis $H_0 : \mu_y = \mu_0$ is tested against the alternative hypothesis $H_1 : \mu_y \neq \mu_0$

If the null hypothesis is true at .05 significance level then 95% of the time a mean is computed it would fall within the limits $\mu_0 \pm t_{.025;n-1} s_y^-$

Stated differently we can compute the t statistic: $t = \frac{\bar{y} - \mu_0}{s_y^-}$ and compare it to tabled critical values. If observed t exceeded the critical $t_{\alpha/2;n-1}$ (where (1-alpha) represents the confidence level of the test and n-1 the degrees of freedom), we could reject the null hypothesis to be plausible.

Note that t-test is a function of three components: 1) the “effect size” 2) the sample variability and 3) the sample size. The relations between those components can be understood better in the following equation:

$$t = \frac{\bar{y} - \mu_0}{s_y^-} = \frac{\bar{y} - \mu_0}{s_y / \sqrt{n}} = \frac{\sqrt{n}(\bar{y} - \mu_0)}{s_y}$$

Two sample t-test

Imagine now that the observations are drawn from two independent populations (e.g., men and women, heavy Vs. not heavy buyers) with different means and standard deviations (e.g., on some “likelihood to purchase” .

The null hypothesis is $H_0 : \mu_1 = \mu_2$ or $(\mu_1 - \mu_2 = 0)$

The t-statistic is: $t = \frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{s_{(\bar{y}_1 - \bar{y}_2)}}$ on $(n_1 - 1) + (n_2 - 1)$ df,

where:

$$s_{(\bar{y}_1 - \bar{y}_2)}^2 = s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right) \quad \text{and} \quad s_p^2 = \frac{\sum_i (y_{1i} - \bar{y}_1)^2 + \sum_i (y_{2i} - \bar{y}_2)^2}{(n_1 - 1) + (n_2 - 1)}$$

Note that the two sample t-test has the same basic form as the one sample t-test: it is a comparison of data to “theory” relative to some indication of variability. The numerator is the “effect size” and these *between-group* differences can be judged large or small (I.e., significant or not) only relative to the *within-group* differences, which appear in the denominator.

Q: Give an example for a possible use of t-test

One-Way ANOVA

The logic of a t-test can be easily extended to three or more independent populations. Imagine, for example, comparing three test markets in which different pricing strategies have been implemented. In this example, price is manipulated by the researcher and we define it as an *experimental factor* or *independent variable*. Sales in this case is the variable measured by the researcher and it is expected to be affected by the price variation, so it is called the *dependent variable*. One-way ANOVA refers to the fact that only one *factor* was manipulated.

Definitions:

The null hypothesis: $H_0 : \mu_1 = \mu_2 = \mu_3 = \dots \mu_a$

Assumption: $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$

Where y_{ij} is the score of the j^{th} observation ($j=1,2,\dots,n$) in group i ($i=1,2,\dots,a$) (“ n is the sample size in each group so the total sample is $a*n$). μ is the grand mean (across all $a*n$ observations), α_i is the incremental difference between groups i 's mean and the grand mean. α_i (“alphas”) are usually the focus of the analysis. ε_{ij} is the error term (assumed random; independent, normally distributed, and each group share a common variance: $\sigma_1 = \sigma_2 = \dots$).

Estimation of groups effects: $\alpha_i = (\bar{y}_{i.} - \bar{y}_{..})$

The error term is the variability within the group - even though all subjects in a condition are treated similarly, they are unlikely to yield identical response $(y_{ij} - \bar{y}_{i.})$

The F-test in ANOVA

$$\sum_i \sum_j (y_{ij} - \bar{y}_{..})^2 = \sum_i \sum_j (\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$$

SST

SSB

SSW

SST: The total variability of the dependent variable.

SSB: The variability between each group relative to the grand mean

SSW: The variability within each group relative to the group mean.

SSB is explainable by the effect of the manipulated factor and SSW is attributed to chance.

Degrees of freedom: The number of data points less the number of parameter estimated (or constraints on those estimations).

In our case: $df_{total} = a*n-1$, $df_{between} = a-1$, and $df_{within} = a(n-1)$

MS=SS/df

$$F_{(df_{between}, df_{within})} = MSB/MSW$$

Illustrative example for one-way ANOVA

$$\bar{y}_{1.} = 5$$

$$\bar{y}_{2.} = 6$$

$$\bar{y}_{3.} = 10$$

ID	Factor A	y
1	1	4
2	1	5
3	1	6
4	1	5

5	2	5
6	2	6
7	2	6
8	2	7

9	3	9
10	3	10
11	3	11
12	3	10

$$\bar{y}_{..} = 7$$

$a=3$, $n=4$ (too small to be realistic!), $SST=11$, $SSB=56$, $SSW=6$, $dftotal=11$, $dfbetween=2$, $dfwithin=9$.

$MSB=56/2=28$, $MSW=6/9=.667$: $F=28/.667=41.98$.

From the tables: $F(.05;2,9)=4.26$.

We can reject the null hypothesis and say that at least one group's mean is significantly different.

Run ANOVA for the data above in SPSS or SAS

Further analysis in ANOVA

In the example, at this point, all the analyst knows is that the group means (5,6,10) are not statistically equal. It may be that 5 is approximately equal to 6 and only 10 is different, or it could be that all three means are distinct. More specific information is needed.

Way to “separate between the groups:

- Post-Hoc Statistics (e.g., scheffe)
- Contrasts analysis

Two-way ANOVA

Two factors are manipulated.

Random assignments of subjects (drawn from the same population) to one (and only one) of the experimental conditions (when the subjects are exposed to more than one condition, the proper analysis is “repeated measures” ANOVA.

Two factorial means that the two factors are being manipulated simultaneously, thus creating all possible combinations of the levels of the independent variables.

If, for example, in addition to varying the price levels as one factor (e.g., low, medium and high), the test marketers might add the factor ad copy (e.g., ad A and B). If every test market is randomly assigned to one of the six combinations of this design it would be a “three by two factorial” design.

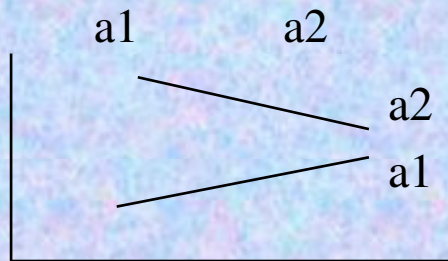
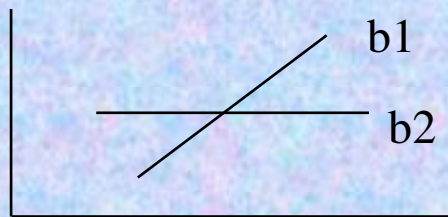
Advantages: 1) Economical design, 2) Interactions

Main effects and Interactions

Consider 2x2 designs:

With interaction:

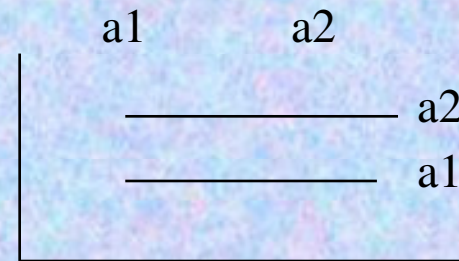
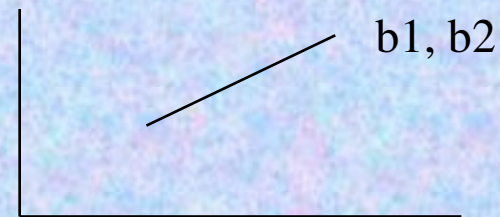
	b1	b2	$\bar{y}_{.1} = 5$	$\bar{y}_{.2} = 9$
a1	3	7	$\bar{y}_{.1} = 7$	$\bar{y}_{.2} = 7$
a2	11	7		



b1 b2

With no interaction:

	b1	b2	$\bar{y}_{.1} = 7$	$\bar{y}_{.2} = 7$
a1	5	5		
a2	9	9		



b1 b2