The Strategic Sequential Bargaining Model is Generically Incompatible with the Cooperative Bargaining Theories

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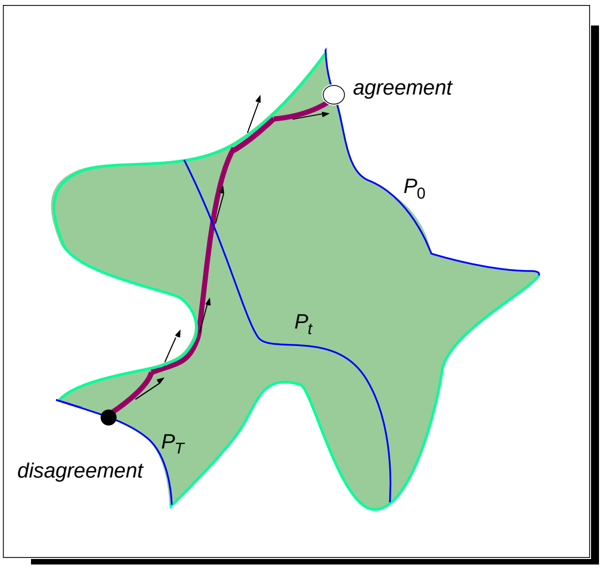
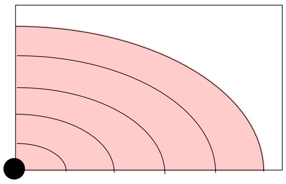
Extended Abstract (Prepared for the Game Theory Conference, Antalia, Turkey, 1996)

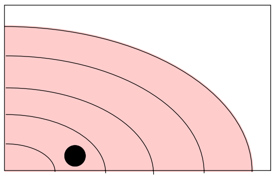
In a widely quoted work, Binmore, Rubinstein, and Wolinsky (1986) (henceforth, BRW) have shown that the unique subgame-perfect equilibrium of strategic (complete information) alternating offers bargaining models—one with risk of breakdown; the other with time preference—yield, at the continuous time limit, the same allocations as does the cooperative Nash solution. This result is interpreted by BRW as demonstrating a close compatibility between the strategic alternating offers model and Nash's cooperative bargaining theory.



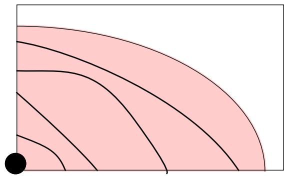
In the current work, the strategic alternating offers bargaining model is extended to account for non-stationary physical pies, preferences, protocols, constrains, outside options, etc.; in discrete time and in the continuous time limit. Once this general framework is set up, it is used to show that BRW's result is due to the strict stationarity of all elements comprising their model. This stationarity translates to the fact, that all shrinking contested pies of BRW's model have a very special geometry. They are called in the paper *centered dilatational Pareto-collections*, i.e., pies in the utility plane such that the Pareto-frontier of the set of feasible utility pairs at any time during bargaining is the image under a shrinking dilatational transformation (which preserves directions), with center at the disagreement-point, of the Pareto-frontier at the time of bargaining commencement.



Centered dilatational Pareto-collection



Non-centered dilatational Pareto-collection



Non-dilatational Pareto-collection

It is then shown that, in this special class of centered dilatational Pareto-collections, and given that the alternating offers bargaining protocol is stationary in a certain sense, Nash's four axioms and the hidden (zeroth) Welfarist axiom—that states that the bargaining solution depends on the bargaining-set, the disagreement-point, and nothing else—hold true as propositions at the continuous time limit. In other words, Nash's axiomatic system has an interpretation (in the logic-theoretical sense) within the strategic alternating offers bargaining deductive theory when restricted to centered dilatational Pareto-collections. BRW's result then follows by the Law of Deduction of formal logic. This provides a satisfactory explanation for BRW's surprising result.

On the other hand, when the domain of the strategic alternating offers bargaining deductive theory is extended to include generic non-stationary bargaining situations, which generate Pareto-collections that generically are not centered dilatational (or even when the latter are centered dilatational, but the bargaining protocol is non-stationary), only one of Nash's axioms, namely, Pareto-Optimality, survives as a true a proposition; the other three and the Welfarist (zero) axiom are no longer true. For example, the Welfarist axiom fails because in Nash's theory, beside the disagreement-point, what determines the solution is the geometry of the single outer Pareto-frontier, while in the strategic model the determinant of the solution is the geometric structure of the whole Pareto-collection of which the outer frontier is but one member, that becomes totally insignificant at the continuous time limit.

Due to the failure of all of Nash's bargaining axioms except for Pareto-Optimality, compatibility between the strategic bargaining allocation and the Nash solution should not be expected. Indeed, it is shown that on the domain that includes both stationary and non-stationary strategic bargaining situations, the strategic alternating-offers bargaining allocation and Nash's solution generically disagree. Although it is an empirical question, a priori it seems safe to believe that a bargaining situation selected at random is much more likely to be non-stationary than stationary.

An analogy within the domain of cooperative bargaining theory may help sharpen the preceding argument. All the cooperative theories that employ the axiom of Symmetry, like Nash's, Kalai-Smorodinsky's, and Perles-Maschler's, obviously yield the same symmetric solution when the respective theories are restricted to the limited domain of symmetric bargaining problems. Clearly, this does not mean that those different cooperative bargaining theories are compatible. This is so, because the said agreement is achieved on the extremely rare symmetric bargaining problems. When the domain of those theories is extended to include generic asymmetric bargaining problems, the different theories generically yield different solutions. It goes without saying then, that the different cooperative bargaining theories are incompatible; they were, indeed, designed to be so. The analogy to the argument above is clear.

Nash's solution is not unique in its generic incompatibility with the strategic alternating offers bargaining model. The fact that the Welfarist axiom is not true as a proposition on the extended domain of general Pareto-collections is sufficient to imply the incompatibility of the strategic model with all cooperative bargaining theories. (In fact, Kalai-Smorodinsky's Individual Monotonicity axiom does not hold true as a proposition even on centered dilatational Pareto-collections, where all of Nash's axioms do hold, which explains why the Kalai-Smorodinsky solution is incompatible with the strategic model even on that rare set.) Dissatisfaction with the Welfarist axiom led Rubinstein, Safra, and Thomson (1992) to propose a cooperative bargaining theory without the Welfarist axiom. They also show a compatibility of their theory with the strategic alternating offers model. But again, they only consider a strategic model that features a centered dilatational Pareto-collection, which results from their restrictive stationarity assumptions.

Interestingly, though, the generalized strategic alternating offers bargaining theory does imply a variant of the Welfarist property: Given an alternating offers bargaining protocol (not necessarily stationary), any two bargaining situations, even with different physical pies and different pairs of bargainers' preferences, which, nonetheless, yield the same Pareto-collection and disagreement-point, generate the same strategic bargaining outcome. The resultant structure in the utility plane is all that matters; the particulars of the underlying physical context do not.

References

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Key Words: *Nash Program; Nash bargaining solution; Axiomatic bargaining theory; Sequential Bargaining*

JEL Numbers: *C7*