

Efficient Tournaments within Teams*

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Abstract

We analyze incentive problems in team and partnership structures where the only available information to condition a contract on is a partial and noisy ranking which specifies who comes first in efforts among the competing partners. This enables us to ensure both first best efficient effort levels for all partners and the redistribution of output only among partners. Our efficiency result is obtained for a wide range of cost and production functions. (JEL *C7, D7, D8, L2*. Keywords: *Moral hazard, Teams and Partnerships, Tournaments*.)

1 Introduction

We study teams and partnerships in which risk neutral partners jointly produce output which they share among themselves. It is generally accepted that such partnerships are inefficient if the partners' actions are not verifiable. The argument is that partners shirk because they must share their marginal benefit of effort with others but bear the cost alone. The question is, then, why are there well publicized examples of extremely profitable partnerships which seem to have very little trouble to provide incentives to partners?

We provide an intuitive answer to this question by focusing on team compensation schemes. These schemes are based on a partial and noisy ranking which specifies who comes first in efforts among the competing partners. We find that full efficiency can be obtained under the assumption that such a partial ranking of the partners' efforts is verifiable. This ranking should be less costly to acquire than cardinal information on efforts or a full ranking of efforts. Our result gives very clear recommendations on how to structure efficient bonus compensation schemes: nearly equal bonuses should be given to all members in teams where the information on the ranking of individual efforts is very precise and a single, high bonus should be awarded in teams where effort monitoring is poor. In the motivating examples below, there are two main elements present on which we build our analysis: team output determines the pool from which

*Financial support from the German Science Foundation through SFB/TR 15 is gratefully acknowledged. We thank Elmar Wolfstetter, Anna Maria Reforgiato-Recupero, Benny Moldovanu, Motty Perry, Tymofiy Mylovanov, James Walker, Bengt Holmström, Mark Armstrong and two anonymous referees for helpful comments. [†]Department of Economics, University of Bonn, Lennéstraße 37, 53113 Bonn, Germany, Alex.Gershkov@uni-bonn.de, Paul.Schweinzer@uni-bonn.de. [‡]University of International Business and Economics, Beijing, China, and Humboldt University of Berlin, LiJianpei2007@gmail.com.

prizes are taken and information about relative performance is used as the means of allocating these prizes.

Partnerships are the dominant organizational form in several fields of the professional services industry, especially in law, accounting and, until recently, investment banking.¹ The perceived advantages of partnerships are highly valued by dominant firms in these sectors. For instance, when Goldman Sachs was converted into a public company in 1999 it retained important elements of the partnership that it maintained for 130 years previously. The \$20.2 billion that Goldman Sachs set aside during 2007 for salaries and bonuses was roughly 50% of its net revenue. (CNNMoney.com, 18-Dec 2007) As this pool is typically split into 40% salaries and 60% bonuses, this amounts to a bonus averaging \$400,000 for *each* of its 30,000 staff. New partners are elected every two years. Since their share of this compensation pool is disproportionately larger than the associates' share, there is a fierce promotion tournament going on among the lower ranks.

Similarly, in the tradition of Galanter and Palay (1991), Rebitzer and Taylor (2007) argue that law "firms are typically structured as partnerships. Attorneys become partners via up-or-out promotion contests." Promotions are indeed lucrative, as "at Sullivan & Cromwell, for example, according to the American Lawyer, the average partner earned \$2.35 million last year" while young lawyers at the same firm have to make do with a meagre \$145,000. (The Wall Street Journal, 1-April-06) A performance evaluation system which could be used by a partnership for its promotion or bonus award decisions is the popular 360° assessment where subalterns, peers and superiors of a given candidate are asked, typically on questionnaires, to assess the candidate. The partnership can use this noisy and at least partly qualitative information to decide on the ranking of partners, that is, on who to promote and who to fire.²

The model's most direct application is to partnerships in the professional services. However, since our setup is applicable to any partnership or team structure as long as there exist performance related bonuses paid from the joint product, there is a much wider area of application in virtually any form of cooperation. As Battaglini (2006) observes, "budget balance means that the mechanism cannot commit to 'throw away' surplus or to give it to someone who does not participate in production. Since budget balance can be interpreted as a constraint imposed by renegotiation-proofness, [our analysis] is relevant not only for those organizations in which budget balance seems a natural assumption from an institutional or empirical point of view (for example, a partnership), but for all the organizational forms."

In our examples, it is crucial that the tournament prizes are determined by joint output. This distinguishes our setup from the fixed prizes which are usually studied in the tournaments literature. We capture this feature by using final output as the total sum of prizes awarded in a tournament. In our model, the sharing rule which specifies the percentage of output allocated

¹ Greenwood and Empson (2003) list the percentage of partnerships as form of governance for the top 100 firms per industry as follows: Law 100%, Accounting 56%, Management consulting 17%, Architecture 18%.

² "Often (although not always) the objective and measurable criteria used in promotions, including billable hours in the context of law firms, and profits earned in the corporate context, are less indicative of future success than other subjective, but less measurable criteria." Baker, Choi, and Gulati (2006).

to the winner, second, etc, is proposed by an arbitrary player and the partnership is only formed if all players agree. A (subgame perfect) equilibrium consists of two elements: a sharing rule which specifies the prizes in the compensation tournament and a set of efforts which determines the output and the probabilities of winning these prizes. There are two main incentive effects from a player increasing effort. On the one hand, additional effort increases the total output of the team of which the agent only receives a share. On the other hand, however, increased effort also raises the agent's chance of winning the tournament. Relative to the socially optimal level of effort, the first effect leads to under-investment while the second gives the leverage to counter this adverse effect. For the offered sharing rule, these two effects exactly cancel out in symmetric equilibrium and we always obtain full efficiency. Moreover, for a sufficiently precise ranking technology (interpreted as the firm's capability to monitor efforts), the players who are not ranked first in the tournament will also get a positive prize in symmetric equilibrium. In such cases, a winner-take-all compensation scheme leads to overinvestment and a positive second prize has the effect of dampening incentives such that efficient efforts are obtained.

If we allow for limited liability of partners, there is the additional caveat that the (marginal) chance of winning the tournament must be reasonably responsive to changes in effort.³ Thus, relative performance compensation schemes under limited liability are only useful if the precision with which the partial ranking technology picks out the winner is not too low. This is arguably easier to achieve in partnerships where professionals share a certain specialization than in general corporations. Our model can therefore explain why partnerships emerge rather between similarly specialized colleagues than between professionals with complementary skills.

The remainder of the paper is organized as follows. We start by relating our contribution to the literature on partnerships and tournaments. In section 2, we introduce the model and our team game. Section 3 illustrates the main result through example and in section 4, we prove the efficiency result. We provide extensions to our model for teams with more than two members and to asymmetric partners in section 5 where we also discuss equilibrium existence issues. All proofs and technical details can be found in the appendix.

Related literature

Alchian and Demsetz (1972) and Holmström (1982) pose the original problem of unattainability of first best efforts in neoclassical partnerships when output is ex ante non-contractible and shared among partners. Legros and Matthews (1993) show that full efficiency can be obtained in some cases, for example, for partnerships with finite action spaces or with Leontief technologies. Nevertheless, they confirm and generalize Holmström's result that full efficiency is unattainable for neoclassical partnerships, ie. the case which we study where production and utility functions are smooth. They show that approximate efficiency can be achieved in mixed-strategy equilibria, where one partner takes an inefficient action with small probability. (These

³ Under limited liability, a partner's share of output is non-negative. This is important, because "since the introduction of legal forms such as the limited liability partnership and the limited liability company, unlimited liability partnerships are rarely seen in the professional services." Levin and Tadelis (2005)

results have been recently generalized by Rahman and Obara (2008).) However, sustaining such equilibria depends crucially on the partners bearing *full* liability. If the partners enjoy limited liability, the mechanism does not work since it is impossible to impose the large fines necessary to prevent deviations. In our result, full efficiency is attainable even with limited liability, provided that the ranking technology gives a sufficiently high marginal probability of winning for symmetric efforts. Battaglini (2006) discusses the joint production of heterogeneous goods. For multi-dimensional output he finds that implementing the efficient allocation is possible whenever the average dimensionality of the agents' strategy spaces is lower than the number of different goods produced. He confirms that efficiency is unattainable in the standard case.

Kandel and Lazear (1992) show that the existence of peer pressure can weaken the free rider problem in teams and partnerships. Their concept of peer pressure among partners captures factors such as guilt, norms, and mutual monitoring which all serve as disciplinary devices. Unlike our compensation scheme, they use constant shares of output rather than a tournament. Miller (1997) shows that whenever a single partner can observe and report on at least one other's actions, efficient efforts can be implemented. Strausz (1999) shows that when agents choose their efforts sequentially and observe the actions taken by their predecessors, there exists a sharing rule which implements efficient efforts. This sharing rule induces players to reveal a shirking partner by influencing final output in a particular way.

The classic reference on efficiency in tournaments is Lazear and Rosen (1981).⁴ They compare rank order wage schemes to wages based on individual output and find that, for risk-neutral agents, both allocate resources efficiently. Nalebuff and Stiglitz (1983) find that, in addition, efficiency can also be obtained if the individual outputs are correlated. The two papers share the feature that there exists a principal facing a perfectly competitive market. Assuming the presence of an optimizing principal, an efficiency result only requires risk neutrality in their setup. Indeed, as shown by Mookherjee (1984) in the setup of a principal with many agents, the optimal incentive scheme can sometimes be based solely on the ranking of the agents' outputs. In other words and using Holmström's terminology, the ranking of the agents' outputs can, under some conditions, be a *sufficient statistic* of the agents' outputs.⁵ In a similar setup of a principal with many agents, the advantages of ranking-based compensation schemes are discussed by Green and Stokey (1983) and Malcomson (1986), among others. However, existence of a principal immediately solves Holmström's moral hazard in teams problem as he can play the role of a budget breaker or residual claimant. Absent the optimizing principal, the presence of perfectly competitive and centralized markets is required in the setup of Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) to obtain efficiency. By contrast, we do neither require any markets nor a budget breaker, and therefore, our model is suitable for the analysis of incentive problems inside a partnership regardless of the industry market structure.

⁴ Kurshid and Sahai (1993) survey the measurements literature which lends support to the tournaments approach by arguing that ordinal statistics are inherently cheaper to produce than cardinal statistics.

⁵ For comparisons of different information structures in principal-agent environments, see Gjesdal (1982) and Amershi and Hughes (1989).

Contrasting with our view of a tournament as an incentive device, Holmström (1982) argues that the rationale for using relative performance evaluation is not to induce competition among the agents, but to reduce risk exposure of the agents. Nalebuff and Stiglitz (1983) find that depending on the degree of risk aversion, the agents may choose effort levels either higher or lower than the first best level. But neither they nor—to our knowledge—any other contribution studies the problem of designing an endogenous prize structure which encourages efficient expenditure of efforts in a limited liability team production problem.

There exists a sizable literature on contests with effort dependent prizes such as, for example, lottery contests, promotion tournaments, R&D, patent- or innovation races. Konrad (2008) surveys this literature in detail, so we focus on the contributions most relevant to our work. Chung (1996) discusses a Tullock-style rent seeking model where productive efforts increase the single rent which contestants compete for. Thus he considers a winner-take-all contest with linear costs. For this setup, he shows that the equilibrium efforts obtained for the simplified Tullock success function are always greater than socially optimal. He uses the exponent on efforts in the general Tullock success function to fine-tune the tournament in order to obtain efficient efforts.⁶ Baye and Hoppe (2003) explore the relationships between R&D races and innovation games in the similar analytical framework of rent seeking tournaments. In both cases, endogenised prizes arise naturally and the authors show the equivalence of certain formulations of these classes of games to rent seeking contests. For these games, they arrive at the conclusion of excessive investment. These contributions do not study the problem of designing an optimal prize (rent sharing) structure to achieve first best.

Cohen, Kaplan, and Sela (2008) characterize optimal effort dependent prize structures in an all-pay auction setup. Although they account for the optimal design problem, they do not allow for the possibility of a second prize, as in our paper. Depending on the designer's objective, they find that the optimal reward may decrease or increase in the players' efforts. Since there is no interpretation to their efforts in terms of social desirability, there is no optimality yardstick against which to measure efficiency or the over- or underprovision of effort. Heintzelman, Salant, and Schott (2006) discuss a problem where a single player's decision leads to the overuse of a common-property. Gathering agents together into a single group generates a free-riding problem, which in turn weakens the agents' incentives and decreases the excessive use of the common resource. The authors show that efficient efforts can be implemented when the number and sizes of partnerships are chosen appropriately. This contrasts with our paper which uses a contest in a single team to ensure decentralized efficiency of productive efforts.

When more than one prizes can be used, Nalebuff and Stiglitz (1983) find that the optimal fixed prize structure entails at most three prizes for a uniformly and normally distributed individual error term. Their second prize may not necessarily be positive. Clark and Riis (1998a) study an all-pay auction with multiple identical prizes and show that multi-prizes can be used to elicit extra efforts from the agents. Moldovanu and Sela (2001) characterize the

⁶ Baik and Lee (2007) are interested in the structure of (private) sharing rules employed in a rent seeking contest between two groups of players. Although they develop their inter-group contest over a fixed prize, the intra-group contests are over variable prizes.

optimal prize structures of all-pay auctions. They show that, for convex cost functions and a fixed prize pool, it is optimal to give positive prizes not only to the winner. One contribution of our analysis is to show that convexity per se does not optimally lead to multiple prizes in partnerships. In order to get multiple prizes, the informativeness of the ranking technology has a crucial role as well. In all discussed contributions, a second prize is used to elicit more efforts from the agents, in contrast to our view that a second positive prize can be used to soften the incentive for overexertion such that efficient efforts can be obtained.

2 The model

There are two identical, risk neutral agents exerting unobservable individual efforts $e_i \in [\delta, \infty)$, $i \in \{1, 2\}$, for some positive δ arbitrarily close to zero.^{7,8} However, some noisy and partial ranking of efforts of the partners is assumed to be observable and verifiable. For the ratio of efforts $x_{ij} = e_i/e_j$ (we drop subscripts on x_{ij} when there is no risk of confusion), the technology translating a partner's effort into his probability of being ranked first is described by⁹

$$\Gamma(e_i, e_j) = [f_i(x_{ij}), f_j(x_{ij})] \quad (1)$$

where $f_k(\cdot)$ is the probability that partner $k \in \{i, j\}$ gets the first place in the ranking given his effort e_i and the rival partner's effort e_j , with $f_i + f_j = 1$. We make the following assumptions on $f(\cdot)$

A1 Symmetry: $f_i(x_{ij}) = f_j(x_{ji})$, for $x \in (0, \infty)$;

A2 Responsiveness: $\frac{df_i(x)}{dx} > 0$, $\frac{df_j(x)}{dx} < 0$; $\lim_{x \rightarrow 0} f_i(x) = 0$ and $\lim_{x \rightarrow \infty} f_i(x) = 1$;

A3 $f(\cdot)$ is twice continuously differentiable.

Assumption **A1** captures the symmetry of the two partners. Assumption **A2** reflects the idea that the probability that one partner is ranked first in effort is dependent upon the relative performance of the two partners, measured by $x = e_i/e_j$. In particular, partner i 's winning probability is increasing in x , but partner j 's winning probability is decreasing in x .

If a partnership is formed, the output of the partnership is a function of the total efforts of the partners. Denote the production function as $y = y(\sum_i e_i)$. The production function is smooth and twice continuously differentiable, with $y(2\delta) = 0$, $y'(\cdot) > 0$ and $y''(\cdot) \leq 0$. A partner who receives a share s of final output, given his own effort e_i and the other partner's

⁷ The natural effort choice set would be $[0, \infty)$ but we avoid zero effort because of potential division by zero in the effort ratio.

⁸ We generalize our results to more than two players in section 5 but the full intuition can be understood from the two players case.

⁹ This class includes the Tullock success function $\frac{e_i^r}{e_i^r + e_j^r}$ for $r > 0$ where $f_i(x) = \frac{1}{1+x^{-r}}$. It has been axiomatized by Skaperdas (1996) and others who show that only variants of the Tullock contest success function satisfy a set of desiderata similar to our assumptions.

effort e_j , gets utility

$$u_i(e_i, e_j) = sy(e_i + e_j) - C(e_i)$$

where $C : [\delta, \infty) \rightarrow \mathbb{R}$ is a cost function with $C(\delta) = 0$, $C'(\cdot) > 0$ and $C''(\cdot) > 0$.

A partner's objective is to maximize his own expected utility. We investigate whether it is possible to induce the players to exert efficient efforts using a rank order compensation scheme.

Timing

At the first stage, an arbitrary partner initiates the partnership formation by making a proposal to the other partner, offering a sharing rule $(s, 1 - s)$. Without loss of generality let partner 1 be the proposer. Partner 2 then decides whether to accept the proposal or not. If he accepts, the partnership is set up, and the game proceeds to the next stage. If he rejects, the game ends and each player obtains his reservation utility which we normalize to zero. At the second stage, conditional on the formation of the partnership, the partners choose their efforts simultaneously to maximize their own expected utility. Some noisy ranking of efforts specifies a winner, final output realizes and is then distributed among the two partners. The winner obtains the share s of total output, and the other partner gets $1 - s$.

(In-)Efficiency benchmark

Efficient actions are those which maximize the total welfare of the two partners absent of any incentive aspects

$$\max_{(e_i, e_j)} w(e_i, e_j) = y(e_i + e_j) - C(e_i) - C(e_j).$$

The first best effort level is determined by $y'(2e^*) = C'(e^*)$ where $e_i^* = e_j^* = e^*$. Suppose the two partners fix the shares (s_i, s_j) ex ante, with $s_i + s_j = 1$. As shown by Holmström (1982), there is no sharing rule that achieves full efficiency and satisfies a balanced budget at the same time. Given the sharing rule (s_i, s_j) , the partners choose their efforts to maximize

$$u_i(e_i, e_j) = s_i y(e_i + e_j) - C(e_i).$$

Conditional on the formation of the partnership, partner i 's best response is given by

$$s_i y'(e_i + e_j) = C'(e_i),$$

where equilibrium efforts are dependent upon the share s_i received. The bigger the share received, the higher the effort. However, since $s_i + s_j = 1$, at least one of the partner always chooses suboptimal effort.

3 Example of efficient team production

We now use a specific example to illustrate that the proposed partnership tournament game achieves full efficiency. Let the production function be linear in total efforts

$$y = \alpha(e_i + e_j), \quad \alpha > 0$$

and assume cost functions to be quadratic¹⁰

$$C(e_i) = \frac{e_i^2}{2}, \quad i \in \{1, 2\}.$$

Let the technology specifying winning probabilities be the generalized Tullock success function.¹¹ Partner $i \in \{1, 2\}$ is ranked first with probability

$$\frac{e_i^r}{e_i^r + e_j^r} = \frac{1}{1 + x^{-r}}, \quad r > 0, \quad (2)$$

if he exerts effort e_i and the other partner exerts effort e_j . The partner who is ranked first receives share s of the final output, and the partner who is ranked second receives share $1 - s$. In this example, the efficient effort levels are $(e_1^*, e_2^*) = (\alpha, \alpha)$.

In our tournament game, given the shares agreed on at the first stage, partners choose efforts non-cooperatively at the second stage. Thus partner $i \in \{1, 2\}$ chooses effort e_i to maximize

$$u_i(e_i, e_j) = \underbrace{\frac{e_i^r}{e_i^r + e_j^r}}_{\text{pr winning}} \underbrace{s\alpha(e_i + e_j)}_{\text{payoff if win}} + \underbrace{\frac{e_j^r}{e_i^r + e_j^r}}_{\text{pr losing}} \underbrace{(1 - s)\alpha(e_i + e_j)}_{\text{payoff if lose}} - \underbrace{\frac{e_i^2}{2}}_{\text{cost}}. \quad (3)$$

The symmetric equilibrium efforts depend on s

$$e_i(s) = e_j(s) = \frac{\alpha - r\alpha + 2rs\alpha}{2}. \quad (4)$$

Equilibrium efforts are increasing in the share s . Notice that, for arbitrary $r > 0$, the agents choose efficient efforts if s equals

$$s^* = \frac{1 + r}{2r}. \quad (5)$$

The intuition of the efficiency result is straightforward. As one partner increases effort, given the other partner's effort level, he increases the final output, and at the same time increases his probability of being ranked first. This implies that he has a higher probability of receiving the winning share of an increased final outcome. The larger the winner's share s , the higher

¹⁰ Multiplying the cost function with a constant does not make the formulation more general as its effects can be assimilated into the coefficient α of the production technology.

¹¹ As δ is only a technical device to prevent division by zero in the effort ratio, we allow efforts in $[0, \infty)$ in this example and set a probability of $f_1(\cdot) = 1/2$ for $e_1 = e_2 = 0$. The generated discontinuity of the success function at zero efforts plays no role in the example.

the incentive for a partner to exert high effort.¹²

Comparing a partner's objective function (3) with that of a social welfare maximizer

$$w(e_1, e_2) = \alpha(e_1 + e_2) - \frac{e_1^2 + e_2^2}{2}$$

we see that in (3), each partner's incentive to exert effort consists of two parts. The first is that exerted effort increases total output $\alpha(e_i + e_j)$, which increases a partner's payoff no matter whether he is ranked first or second. This motive also exists in the social welfare maximization problem. In a partnership, an agent expects to receive only part of the output and thus does not internalize the positive externality of higher effort on the other partner. This is the usual incentive to free-ride leading to under-investment of effort in partnerships. In our game, however, a tournament is used to allocate the shares. Therefore, partners have an additional motive to exert effort, because higher effort increases the probability of getting a bigger share of the output and decreases the probability of getting the smaller share. This extra incentive brought about by the tournament exactly offsets the incentives to free ride from profit sharing if the prizes are chosen appropriately.

To illustrate that full efficiency is achieved, we still need to show that it is optimal for partner 1 to propose the share (5) at the first stage and for partner 2 to accept. Given the equilibrium efforts $e_i(s)$, partner 1 chooses share s at the first stage to maximize

$$u_1(s) = u_1(e_1(s), e_2(s)) = \frac{(3 - r(2s - 1))(1 + r(2s - 1))\alpha^2}{8} \quad (6)$$

subject to participation of the second player. Since choosing a minimal effort generates non-negative utility, this participation is ensured.¹³ This objective (6) is maximized at (5) and thus it is optimal for partner 1 to propose the efficient sharing rule at the first stage of the game.

For the case of $r = 1$ in the success function (2), the optimal allocation rule is to give the entire outcome to the partner who ranks first in efforts. But this is not a feature of the efficient mechanism in general. For a sufficiently precise ranking technology, ie. $r > 1$, the efficient mechanism shares output between the players such that each partner receives a positive prize. With higher ranking precision, however, the existence of pure strategy symmetric equilibrium becomes problematic. This phenomenon is well studied in the rent seeking literature where no such equilibria exist for $r > 2$. We discuss the lessened scope of this problem in our framework in section 5 and demonstrate that existence is ensured for $r \leq 5$ for the current example. In the case of $r = 5$, the resulting winning share is as small as 60%.

From expression (5), if $r < 1$ the loser needs to be punished (ie. $s > 1$) in order to achieve efficiency. Such a policy is not feasible if partners face a limited liability constraint. As the ranking technology gets more responsive, ie. $r > 1$, a winner-take-all tournament with $s = 1$ gives the agents too high incentives, and the agents overinvest $e_i = (1 + r)\alpha/2 > e_i^* = \alpha$, as

¹² This result is similar to (but in our case stronger than) the standard tournaments result with fixed prizes where incentives increase with the spread between the prizes.

¹³ For the case of unlimited liability, the second player's participation constraint has to be examined separately.

shown by Chung (1996) and Baye and Hoppe (2003). If the winner's share is $1/2$, the allocation mechanism is equivalent to a fixed equal sharing rule. It is well known that in this case the agents free ride and provide insufficient efforts. As, for any given r , function (3) is continuous in s , we can ascertain the existence of a winner's share $s^* \in (\frac{1}{2}, 1]$ ensuring positive shares of final output for both agents and efficient effort provision at the second stage.

4 Results

We now show that in the general setup, full efficiency is attainable for general concave (including linear) production functions and a large class of ranking technologies. Recall the production technology

$$y = y(e_i + e_j), \quad \text{with} \quad y(2\delta) = 0, \quad y'(\cdot) > 0, \quad y''(\cdot) \leq 0.$$

Given the sharing rule s and partner j 's effort of e_j , partner i 's expected utility from exerting effort e_i is

$$u_i(e_i, e_j) = f_i\left(\frac{e_i}{e_j}\right) sy(e_i + e_j) + \left(1 - f_i\left(\frac{e_i}{e_j}\right)\right) (1 - s)y(e_i + e_j) - C(e_i). \quad (7)$$

By restricting attention to this implicitly linear contract we do not actually lose generality as is shown for risk neutral agents by Nandeibam (2002).¹⁴ Assuming the existence of interior solutions, maximizing the above implies for $i = 1, 2$, that

$$\begin{aligned} & f'_i\left(\frac{e_i}{e_j}\right) \frac{1}{e_j} (2s - 1)y(e_i + e_j) + \\ & \left(f_i\left(\frac{e_i}{e_j}\right) s + \left(1 - f_i\left(\frac{e_i}{e_j}\right)\right) (1 - s) \right) y'(e_i + e_j) - C'(e_i) = 0. \end{aligned} \quad (8)$$

Given j 's effort e_j , (8) implies that marginally increasing effort e_i has three effects: 1) a marginal increase of final output, 2) a marginal increase of partner i 's winning probability, and 3) a marginal increase of effort cost.

When the symmetric Nash solution exists, then $e_i = e_j = e$ and $f_i(1) = \frac{1}{2}$. Substituting these, we obtain from (8) that

$$\frac{f'_i(1)}{e} (2s - 1)y(2e) + \frac{1}{2}y'(2e) = C'(e). \quad (9)$$

As equilibrium effort e is a function of s we write effort as $e(s)$ and the associated output as $y = y(2e(s))$. Intuitively, $f'_i(1)$ relates to the precision of the tournament's ranking technology. (In the example of section 3, for instance, $f'_i(1) = r/4$.) A high value of $f'_i(1)$ corresponds to a highly precise ranking technology involving a drastic change of the winning probability as the ratio e_i/e_j approaches 1.¹⁵ In the following lemma we begin the analysis of equilibrium effort

¹⁴ We are grateful to an anonymous referee for pointing out the generality of this formulation.

¹⁵ Output variance is necessary for pure strategy equilibrium existence in Lazear and Rosen (1981) or Nalebuff and Stiglitz (1983). In our case, this role is taken by the assumed differentiability of the ranking technology.

choice.

Lemma 1. *Second stage equilibrium effort e is increasing in s .*

This corresponds to the standard tournament literature result that a partner's incentive is increasing in the spread between prizes. Here we have replaced the fixed prizes with a fixed sharing of the final output. It is natural that effort is increasing in the share s since a larger share means a bigger prize for the winner.

Lemma 2. *Denoting the first best, efficient efforts by e^* , the sharing rule s which satisfies*

$$f'_i(1)\frac{1}{e^*}(2s-1)y(2e^*) = \frac{1}{2}y'(2e^*) \quad (10)$$

elicits the efficient effort choice at the second stage. Under unlimited liability, there always exists a share s^ such that (10) is satisfied.*

We proceed to show that first best can always be implemented as a subgame perfect equilibrium under unlimited liability.

Proposition 1. *Under unlimited liability, full efficiency is obtained. At the first stage, partner 1 proposes a sharing rule $(s^*, 1 - s^*)$ and at the second stage, each partner exerts first best efforts.*

The efficiency result of proposition 1 does not depend on the ratio-form of the success function, it extends to any symmetric, differentiable success function.

If partners face a limited liability constraint, their possible shares are restricted to $[0, 1]$. The next lemma establishes a threshold precision for the ranking technology for efficiency to obtain under limited liability.

Lemma 3. *Under limited liability with $s \in [0, 1]$, there exists an s^* that satisfies (10) if $f'_i(1) \geq \frac{1}{4}$.*

Notice that, when $f'_i(1)$ is sufficiently low (ie. below $1/4$), the equilibrium share which induces efficient efforts may exceed 1. Denote by \tilde{s} the solution to (10).

Proposition 2. *Under limited liability, if $\tilde{s} \in [0, 1]$, then full efficiency can always be obtained. If $\tilde{s} \notin [0, 1]$, then player 1 proposes shares $(s^*, 1 - s^*) = (1, 0)$ and the agents choose suboptimal efforts.*

For a sufficiently precise ranking technology it is always the case that $\tilde{s} \in [0, 1]$. Hence, there is a threshold precision above which efficiency is guaranteed.

Corollary 1. *If the ranking technology is sufficiently precise, that is if $f'_i(1) \geq \frac{1}{4}$, then full efficiency can always be obtained.*

The above propositions show that for the class of production functions studied, as long as the ranking technology is such that the marginal winning probability for symmetric efforts is sufficiently large, full efficiency can always be achieved, even under limited liability. There is no necessity for a budget breaker. The only requirement is the observability of some noisy and partial ranking of efforts picking out winning probabilities. This result does not depend on whether or not one can deduce the other partner's effort after output is observed. The efficiency result is robust to production functions of other forms, as long as the concept of symmetric equilibrium can be applied.

The condition on the marginal winning probability for symmetric efforts is critical for limited liability. In the symmetric equilibrium we consider here, a partner is only willing to increase his efforts if doing so significantly increases his probability of winning a bigger share of the final output. We emphasize that $f'_i(1) < 1/4$ is a necessary but not sufficient condition for inefficiency under limited liability. When inefficiency occurs, it depends on the curvature of the production function and the tournament ranking technology. In the example of section 3 with linear production function, $f'_i(1) = 1/4$ is exactly the critical value between full efficiency and inefficiency. If the production function is strictly concave, $y(e)$ is strictly larger than its linear approximation $y'(e)e$ and the required threshold on the marginal probability of winning decreases. Thus, the requirements on the precision of the ranking technology become weaker for more concave production functions. The reason is that, as the production technology becomes more concave, efficient efforts get smaller and the saved (convex) costs from free riding shrink quickly. Therefore, the power of the incentive scheme, ie. $f'_i(1)$ and s^* , needs to be less responsive to deviations from the efficient symmetric effort level because output itself gives ample reward for high effort provision.

Corollary 2. *If, under limited liability, efficiency can be achieved under linear production, then it can always be achieved with any strictly concave production technology.*

It is worth pointing out that whenever the partial ranking technology is precise enough, the player who comes out second also receives a positive share. The exact precision threshold depends on the used production function. Notice that (10) can be solved to obtain the efficient sharing rule corresponding to (5) as

$$s^* = \frac{e^* y'(2e^*)}{4y(2e^*) f'_i(1)} + \frac{1}{2}.$$

If $f'_i(1)$ indeed exceeds $1/4$, the loser's prize $(1 - s^*)$ is strictly positive.

Corollary 3. *If the ranking technology is sufficiently precise, that is if $f'_i(1) > 1/4$, then $s^* < 1$, ie. both players receive a positive share. Moreover, the winner's share s^* exceeds $1/2$ and decreases with $f'_i(1)$.*

For a sufficiently precise ranking technology, the players who are not ranked first in the tournament also get a positive prize in symmetric equilibrium because the added ranking pre-

cision allows to dampen incentives.¹⁶ This is a property which, together with limited liability, corresponds to actual compensation schemes. If we were to compare two similar limited liability partnerships, one with a very precise monitoring technology and one where the higher effort provider is identified rather imprecisely, we could obtain two very different sharing rules which nevertheless both implement first best efforts. In the case of the precise monitoring technology, all partners could get nearly the same share of output while for the imprecise effort ranking a sharing rule giving all output to the winner could be optimal. Thus, in our setting, decidedly egalitarian looking partnerships may actually arise from pure efficiency considerations.

5 Extensions

More than two partners

In this subsection we show that our full efficiency result is not an artifact of two member partnerships, where one can deduce the effort level of the other partner from observing output. We demonstrate that with only two types of prizes—one for the winner of the tournament and one for everyone else—efficiency can be reached. Assume that there are n players and notice that the efficient effort level e^* should satisfy

$$y'(ne^*) = C'(e^*). \quad (11)$$

In our model, success probabilities are functions of effort ratios. For a ratio of efforts $x_{ij} = e_i/e_j$, we denote the vector of n effort ratios of player i by $x_i = (x_{i1}, x_{i2}, \dots, x_{in})$, where the i^{th} element is 1. Denote by $f_i(x_i)$ the probability that player i is ranked first, if x_i is the vector of the agents' effort ratios. We need assumptions on $f_i(\cdot)$ similar to those made for the two player case.¹⁷

A1 Symmetry: For any two players $l \neq m$ and for any two vectors of efforts, (e_1, \dots, e_n) and $(\tilde{e}_1, \dots, \tilde{e}_n)$ with $e_k = \tilde{e}_k$ for $k \notin \{l, m\}$ and $e_l = \tilde{e}_m$ and $e_m = \tilde{e}_l$, we have

$$f_l(x_l) = f_m(\tilde{x}_m).$$

We also need an additional type of symmetry. For any player i , let the elements of an effort ratio vector x'_i be arbitrary permutations of those in x_i except for the element at the i th position. For these we require

$$f_i(x_i) = f_i(x'_i).$$

¹⁶ It is straightforward to check the performance of a winner-take-all contest as a special case of our second stage game. Such an allocation rule leads to overinvestment at the second stage if $f'_i(1) > \frac{1}{4}$.

¹⁷ The literature has treated n -player contests in two distinct ways: Either as fully discriminatory all-pay auctions as exemplified by Moldovanu and Sela (2001) or as a nested form of Tullock contest as formalized by Clark and Riis (1996) or Clark and Riis (1998b).

A2 Responsiveness: For any $l \in \{1, \dots, n\}$ and $l \neq i$, $\frac{\partial f_i(x_i)}{\partial x_{il}} > 0$.

A3 $f(\cdot)$ is twice continuously differentiable.

The new part of **A1** says that every opponent of player i affects the winning probability of i in a similar way. Thus, if players l and m exchange their effort levels, this does not affect the winning probability of player $i \notin \{l, m\}$. The interpretation of **A2** is that the probability of being ranked first should react positively to increased efforts.

When agents exert identical efforts, then **A1** implies that for any $i \in \{1, \dots, n\}$, $l, m \neq i$, we have that $\frac{\partial f_i(\mathbf{1})}{\partial x_{il}} = \frac{\partial f_i(\mathbf{1})}{\partial x_{im}}$ where $\mathbf{1}$ is the n dimensional vector of 1. Therefore, $\forall l \neq i$, we can use the following notation

$$\frac{\partial f_i(\mathbf{1})}{\partial x_i} = \frac{\partial f_i(\mathbf{1})}{\partial x_{il}}.$$

The timing structure and the rules of the game are similar to the two players case. At the first stage, partner 1 proposes a sharing rule consisting of a winner's share s and an identical prize $\frac{1-s}{n-1}$ to everyone else. If any of the n partners fail(s) to participate in the mechanism, the partnership is not formed and the game ends. For a given sharing rule, the expected utility of player i is

$$f_i(x_i)sy\left(\sum_{j=1}^n e_j\right) + (1 - f_i(x_i))\frac{1-s}{n-1}y\left(\sum_{j=1}^n e_j\right) - C(e_i). \quad (12)$$

The next proposition shows that full efficiency can be obtained if the success function is precise enough, ie. if a partner exerting higher effort than his opponents is selected as the winner with sufficiently high probability.

Proposition 3. *There exists a winner's share s , such that the sharing rule $(s, \frac{1-s}{n-1}, \dots, \frac{1-s}{n-1})$ elicits efficient efforts at the second stage. This rule is proposed at the first stage of the game. Moreover, it satisfies limited liability whenever*

$$\frac{\partial f_i(\mathbf{1})}{\partial x_i} \geq \frac{1}{n^2}. \quad (13)$$

As efficiency can be obtained with just two prizes, the use of the Tullock success function (which only picks out the winner) seems justifiable also in the n -players case.

Asymmetric partners

We now show that efficiency does not depend on the symmetric partners. For that purpose, let's consider the following 2-partner setup where asymmetric efficient efforts are given by

$$y'_i(e_i^*, e_j^*) = C'_i(e_i^*), \text{ and } y'_j(e_i^*, e_j^*) = C'_j(e_j^*) \quad (14)$$

where y'_i denotes the partial derivative of output with respect to player i 's effort. Let the winner's shares be identity-dependent, ie. a winning partner i gets s_i and a winning j gets s_j .

Thus, taking e_j^* as given, partner i maximizes

$$f_i(x_{ij})s_i y(e_i, e_j^*) + (1 - f_i(x_{ij}))(1 - s_j)y(e_i, e_j^*) - C_i(e_i)$$

which, when taking derivatives and inserting (14), gives the best response of partner i as

$$\frac{1}{e_j^*} f_i'(x_{ij}) y(e_i, e_j^*) (s_i + s_j - 1) + [f_i(x_{ij})(s_i + s_j - 1) - s_j] y_i'(e_i, e_j^*) = 0. \quad (15)$$

One can similarly solve for partner j 's best response to $e_i = e_i^*$. Solving the two simultaneous equations gives the pair (s_i, s_j) of winning shares which elicits efficient efforts from both partners. The construction of a first stage proposal game, however, where these shares are actually proposed in equilibrium proved thus far elusive as there always exists an incentive for the proposer to increase his (identity-dependent) winning share and to decrease the winning share of the opponent in order to increase the losing share of the proposer.¹⁸ Nevertheless, if one moves share proposals to a symmetric ex ante stage (where partners have common priors and do not yet know about cost or production specializations), one can construct a game where efficient sharing functions giving rise to full efficiency at the second stage are proposed.

Existence

Cornes and Hartley (2005) extend Baye, Kovenock, and de Vries (1994) in giving a necessary and sufficient condition for existence of symmetric, pure strategy equilibrium in the rent seeking case of the general Tullock success function with linear cost and fixed prizes $(P, 0)$. In the rent seeking case, the symmetric equilibrium payoff of $(1/2)P - e_i$ decreases in own effort and becomes negative for $e_i > (rP)/4$ or $r > 2$ (where $(rP)/4$ is the unique candidate symmetric equilibrium effort). For linear cost, this destroys all hope for equilibria using higher precision success functions than $r = 2$. Our approach of endogenising efforts into the prize structure and assuming convex costs differs from this rent seeking case and obtaining positive results for $r > 2$ is possible. (In particular, our symmetric equilibrium efforts are independent of r .)

Nevertheless, ensuring existence in our framework is problematic because, as the precision of the success function increases, the efficient winner's share $s(e^*)$ goes to one half. Therefore, as the contest becomes more discriminating, the two-player contest loser is certain to obtain nearly half the joint product. Therefore, if the cost function is such that the marginal cost gain of reducing effort below the efficient level is higher than the marginally increased payoff from winning, agents will underprovide effort. Chung (1996) analyzes the case of productive efforts with linear costs. He obtains a range of precisions r of the Tullock contest under which existence is guaranteed. As above, the case of $r = 2$ represents an upper bound.

The following proposition derives global sufficiency for a significant subclass of problems.

¹⁸ This problem is present with any identity-dependent sharing rule. If in the example of section 3 the proposer were allowed to offer an identity-dependent sharing rule, he would find it optimal to increase his own winning share while decreasing his opponent's winning share, relative to the efficient sharing rule.

Before turning to that general analysis, we illustrate the existence issue by looking again at the example from section 3 where player i maximizes

$$u_i(e_i, e_j) = \frac{e_i^r}{e_i^r + e_j^r} s \alpha (e_i + e_j) + \frac{e_j^r}{e_i^r + e_j^r} (1 - s) \alpha (e_i + e_j) - \frac{e_i^2}{2}. \quad (16)$$

Figure 1 shows the geometry of this problem. It plots partner i 's utility when unilaterally deviating from symmetric equilibrium e^* . It shows how first single-peakedness and then optimality of e^* is lost as r increases beyond a value of 5.¹⁹

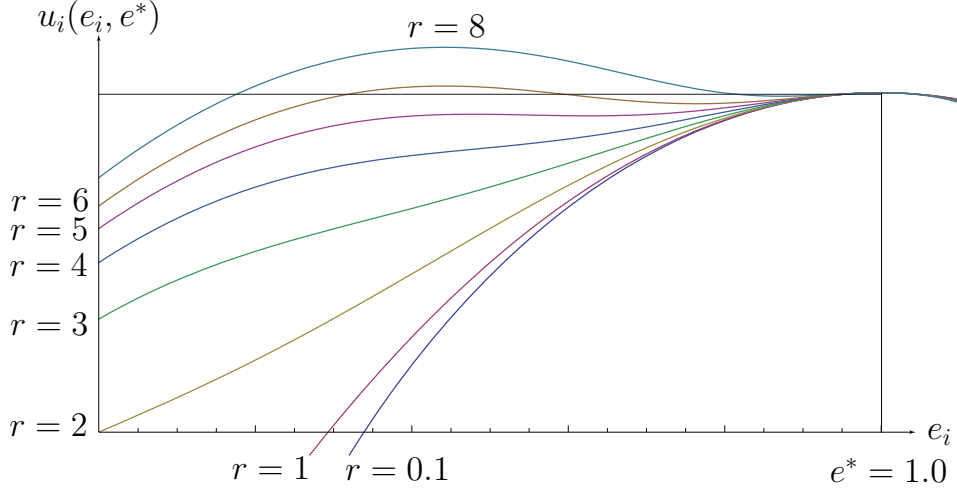


Figure 1: Checking player i 's objective (16) at its critical points ensures existence of the efficient symmetric equilibrium in the linear production case for $r \in (0, 5]$.

The general argument starts with the observation that the efficient solution is necessarily a maximum. We show quasiconcavity of partner i 's objective by restricting attention to the class of problems parameterized by r , p and z , ie. to the class comprising of Tullock success function with parameter r , the production function $(e_i + e_j)^{\frac{1}{p}}$ and the cost function $\frac{e_i^{z+1}}{z+1}$.

Proposition 4. *Existence of symmetric pure strategy equilibrium for efficient efforts is ensured if $pr \geq 1$ and $z \geq r$.*

This proposition shows that under linear production, $z \geq r \geq 1$ ensures existence. For $r \in (0, 1)$, the well-known results cited above indicate that existence is not problematic. Thus both a more concave production technology and a more convex cost function weaken the existence problem.

6 Concluding remarks

Other examples which match our model well are, for example, political contests where the partners in a coalition government work jointly on what may be viewed as maximizing the

¹⁹ As varying the coefficient α on the production technology does not significantly change the problem we set $\alpha = 1$ for simplicity.

countries' tax base. A follow-up election is a rank order tournament which may not confirm all coalition parties in office. Joint research among tenure track researchers at a university which may grant tenure based on the perceived quality of individual research is a further example. Apart from this promotion aspect, publishing itself can be viewed as a tournament: in the sciences and engineering, several researchers usually work together on one project. Although they share joint output, the most important contributor is typically made the first author of a resulting publication.

Warfare history and the vassal system are rich sources for anecdotes. In the Thirty Years' War, for instance, Albrecht von Wallenstein, a Bohemian nobleman, was rewarded for his services to the Catholic emperor Ferdinand II against the Danish King Christian IV: in 1628, Wallenstein received the duchy of Mecklenburg where combined forces of Wallenstein and Count Tilly had previously defeated the Danes.²⁰ Similarly, during the Napoleonic wars, a Royal Navy man-of-war capturing an enemy prize split the proceeds according to a fixed rule specified by the Cruizers and Convoys Act (1708). It granted three eighths to the ship's captain, one eighth each to the (increasingly numerous sets of) wardroom, principal warrant and petty officers, and the final two eighths to the crew.²¹ Promotion, thus, was more than a source of pride.

Finally, a team or partnership in which information quality is the key in the selection of projects is a natural application. Partners spending more effort in collecting information increase the quality of information, hence the quality of project selection, and thus expected output. Therefore, a partner with good information plays a more important role in decision making than the ones with bad information who would rather rubber stamp the suggestion of the former. The partners, as a matter of fact, engage in a contest in the collection of information about projects. In order to provide incentives to exert effort, a larger share of output should be granted to better informed partners.²²

Appendix

Proof of lemma 1. Rewrite expression (9) as

$$\frac{f'_i(1)}{e}(2s-1)y(2e) = C'(e) - \frac{y'(2e)}{2}.$$

Notice that the right hand side of this expression increases in e while the left hand side decreases in e . Thus, e must be an increasing function of s as claimed by the lemma. \square

Proof of lemma 2. Given that s satisfies (10), partner i chooses stage 2 effort such that (9)

²⁰ F. Schiller gives a literary but historically accurate account in his 1792 *History of the Thirty Years' War*.

²¹ Kert (1997) and Benjamin and Thornberg (2007) detail related incentive systems in both other navies and the private sector.

²² Incidentally, many TV game shows—for example the CBS reality show *Survivor*—have this format. Players start out in teams but the final prizes are awarded to individuals based on their earlier team performance.

is satisfied. Substituting (10) into (9), one obtains

$$y'(2e) = C'(e)$$

which determines the fully efficient effort level. Therefore, there always exists a unique $s \in (\frac{1}{2}, \infty)$ solving (10). \square

Proof of proposition 1. Expecting the symmetric equilibrium effort levels $e_1(s) = e_2(s) = e(s)$ determined by equation (9), partner 1's expected utility from choosing the share s is

$$\begin{aligned} u_1(s) = u_1(e(s), e(s)) &= f_1(1)sy(2e(s)) + (1 - f_1(1))(1 - s)y(2e(s)) - C(e(s)) \\ &= \frac{1}{2}y(2e(s)) - C(e(s)) \end{aligned}$$

subject to partner 2's participation constraint which we will verify later for the derived equilibrium. Solving partner 1's utility maximization problem gives the following foc

$$\frac{d}{ds} u_1(s) = (y'(2e(s)) - C'(e(s))) \frac{de(s)}{ds} = 0.$$

Partner 1 chooses \hat{s} such that

$$y'(2e(\hat{s})) = C'(e(\hat{s})).$$

Therefore, the sharing rule which implements efficient efforts maximizes player 1's utility. It is now easily verified that partner 2's participation constraint holds because in symmetric equilibrium both players expect the same utilities and by offering $s = 1/2$, the proposer can ensure non-negative utility. \square

Proof of lemma 3. Rewrite equation (10) as

$$4f'_i(1)(2s - 1)y(2e^*) = 2y'(2e^*)e^*. \quad (17)$$

Since $y(\cdot)$ is a concave function, $y(x) \geq y'(x)x$ for any $x \in [\delta, \infty)$. Therefore, whenever $4f'_i(1) \geq 1$, there exists a $s^* \in [0, 1]$ solving (17). \square

Proof of proposition 2. If $\tilde{s} \in [0, 1]$, then the proof is exactly as the proof of the previous proposition. If there is no $\tilde{s} \in [0, 1]$ which solves (10), then $\tilde{s} > 1$. In that case, since $de(s)/ds > 0$ limited liability equilibrium efforts are necessarily lower than the efficient levels e^* . Therefore we have, for the optimal sharing rule s^* ,

$$\frac{d}{ds} u_1(s^*) = (y'(2e(s^*)) - C'(e(s^*))) \frac{de(s^*)}{ds} > 0,$$

where $de(s^*)/ds > 0$ by lemma 1 and $y'(2e) > C'(e)$ for any $e < e^*$ from our curvature assumptions on production and cost functions. This implies that the optimal $s^* = 1$. \square

Proof of proposition 3. If an interior solution exists to the partners' maximization of (12), then it satisfies

$$\sum_{l \neq i} \frac{\partial f_i(x_i)}{\partial x_{il}} \frac{1}{e_l} \left(s - \frac{1-s}{n-1} \right) y \left(\sum_{i=1}^n e_i \right) + y' \left(\sum_{i=1}^n e_i \right) \left(f_i(x_i) s + (1 - f_i(x_i)) \frac{1-s}{n-1} \right) = C'(e_i).$$

Moreover, since $f_i(\mathbf{1}) = \frac{1}{n}$, if the symmetric Nash equilibrium exists, the equilibrium effort e satisfies

$$\frac{ns-1}{e} y(ne) \frac{\partial f_i(\mathbf{1})}{\partial x_i} + \frac{1}{n} y'(ne) = C'(e). \quad (18)$$

Since the efficient effort level satisfies

$$y'(ne^*) = C'(e^*),$$

the winner's share s that solves

$$\frac{ns-1}{e^*} y(ne^*) \frac{\partial f_i(\mathbf{1})}{\partial x_i} = \frac{n-1}{n} y'(ne^*) \quad (19)$$

elicits efficient efforts at the second stage. We will show now that, in equilibrium, the efficient winner's share will be proposed at the first stage of the game. In symmetric equilibrium, each agent expects the payoff $(1/n)y(ne(s)) - C(e(s))$. Thus partner 1 faces the following maximization problem at the first stage

$$u_1(s) = u_1(ne(s)) = \frac{1}{n} y(ne(s)) - C(e(s)).$$

Partner 1 chooses the s that satisfies the first order condition, which is

$$(y'(ne(s)) - C'(e(s))) \frac{de(s)}{ds} = 0.$$

The s solving this first order condition elicits the first best effort level. Finally, for concave production $y(ne^*) \geq ne^* y'(ne^*)$ there always exists $s \in (\frac{1}{n}, 1]$ that solves (19) if (13) holds. \square

Proof of proposition 4. Consider an auxiliary problem of the following type. The utility of player i when choosing effort e_i is given by

$$g(e_i, e^*) s^* (e_i + e^*)^{\frac{1}{p}} + (1 - g(e_i, e^*)) (1 - s^*) (e_i + e^*)^{\frac{1}{p}} - \frac{e_i^{z+1}}{z+1} \quad (20)$$

where $e^* = \left(\frac{2^{\frac{1-p}{p}}}{p} \right)^{\frac{p}{p+pz-1}}$ and $s^* = \frac{1}{2} + \frac{1}{2pr}$. We will show that this function is strictly quasi-concave in e_i if $g(e_i, e^*) = \frac{e_i^r}{e_i^r + e^{*r}}$, which is sufficient, since $\left(\frac{2^{\frac{1-p}{p}}}{p} \right)^{\frac{p}{p+pz-1}}$ is the efficient effort level and s^* is the sharing rule that leads to efficient efforts. Taking the first derivative of (20)

with respect to e_i and setting it zero gives

$$g(e_i, e^*) + \frac{\partial}{\partial e_i} g(e_i, e^*)(e^* + e_i)p = \frac{1}{2} + \frac{e_i^z p^2 r}{(e^* + e_i)^{-1 + \frac{1}{p}}} - \frac{pr}{2}.$$

Hence the plan is to show that, for $e_i < e^*$,

$$g(e_i, e^*) + \frac{\partial}{\partial e_i} g(e_i, e^*)(e^* + e_i)p > \frac{1}{2} + \frac{e_i^z p^2 r}{(e^* + e_i)^{-1 + \frac{1}{p}}} - \frac{pr}{2}$$

and the opposite holds for $e_i > e^*$. Notice that for $g(e_i, e^*) = \frac{e_i^r}{e_i^r + e^{*r}}$ we have

$$g(e_i, e^*) + \frac{\partial}{\partial e_i} g(e_i, e^*)(e^* + e_i)p = \frac{e_i^{r-1}(e_i^{r+1} + e^{*r}(e_i + (e^* + e_i)pr))}{(e_i^r + e^{*r})^2}.$$

1. Thus, we will to show that, for $e_i < e^*$,

$$\frac{e_i^{r-1}(e_i^{r+1} + e^{*r}(e_i + (e^* + e_i)pr))}{(e_i^r + e^{*r})^2} > \frac{1}{2} + e_i^z p^2 r (e_i + e^*)^{1 - \frac{1}{p}} - \frac{pr}{2}$$

or alternatively

$$\begin{aligned} & \frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pre_i^r e^{*r} + pre_i^{r-1} e^{*r+1} + \frac{pre_i^{2r}}{2} + \frac{pre^{*2r}}{2} > \\ & p^2 r (e^* + e_i)^{1 - \frac{1}{p}} e_i^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}). \end{aligned}$$

Notice that

$$p(e^* + e^*)^{1 - \frac{1}{p}} e^{*z} = 1.$$

Therefore, since $x^{1 - \frac{1}{p}}$ is increasing in x and $e_i < e^*$, we have

$$pr \left(\frac{e_i}{e^*} \right)^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}) > p^2 r (e^* + e_i)^{1 - \frac{1}{p}} e_i^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}).$$

Therefore, it is sufficient to show that

$$\begin{aligned} & \frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pre_i^r e^{*r} + pre_i^{r-1} e^{*r+1} + \frac{pre_i^{2r}}{2} + \frac{pre^{*2r}}{2} \geq \\ & pr \left(\frac{e_i}{e^*} \right)^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}). \end{aligned}$$

However, since $z \geq r$ and $e_i < e^*$, it is sufficient to show that

$$\begin{aligned}
\frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pr e_i^r e^{*r} + pr e_i^{r-1} e^{*r+1} + \frac{pr e_i^{2r}}{2} + \frac{pr e^{*2r}}{2} &\geq \\
pr \left(\frac{e_i}{e^*} \right)^r (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}) &\Leftrightarrow \\
\frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + pr e_i^r e^{*r} + pr e_i^{r-1} e^{*r+1} - \frac{3}{2} pr e_i^{2r} + \frac{pr e^{*2r}}{2} &\geq pr \frac{e_i^{3r}}{e^{*r}} \Leftrightarrow \\
\frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + pr e_i^r e^{*r} + pr e_i^{r-1} e^{*r+1} - \frac{1}{2} pr e_i^{2r} - pr e_i^{2r} + \frac{pr e^{*2r}}{2} - pr \frac{e_i^{3r}}{e^{*r}} &\geq 0 \Leftrightarrow \\
\left(\frac{e^{*2r}}{2} - \frac{e_i^{2r}}{2} \right) (pr - 1) + pr (e_i^r (e^{*r} - e_i^r) + e_i^{r-1} (e^{*r+1} - e_i^{r+1})) &\geq 0
\end{aligned}$$

where the last line follows since $\frac{e_i^{3r}}{e^{*r}} = e_i^{2r} \frac{e_i^r}{e^{*r}} < e_i^{2r}$. Finally, since x^n is an increasing function for $x > 0$ and $n > 0$, all elements of the last expression are positive.

2. Now we will show that for $e_i > e^*$,

$$\frac{e_i^{r-1} (e_i^{r+1} + e^{*r} (e_i + (e^* + e_i) pr))}{(e_i^r + e^{*r})^2} < \frac{1}{2} + e_i^z p^2 r (e_i + e^*)^{1-\frac{1}{p}} - \frac{pr}{2}$$

or alternatively

$$\begin{aligned}
\frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pr e_i^r e^{*r} + pr e_i^{r-1} e^{*r+1} + \frac{pr e_i^{2r}}{2} + \frac{pr e^{*2r}}{2} &< \\
p^2 r (e^* + e_i)^{1-\frac{1}{p}} e_i^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}). &
\end{aligned}$$

Recall that $p(e^* + e^*)^{1-\frac{1}{p}} e^{*z} = 1$. Therefore, since $x^{1-\frac{1}{p}}$ is increasing in x and $e_i > e^*$, we have

$$pr \left(\frac{e_i}{e^*} \right)^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}) < p^2 r (e^* + e_i)^{1-\frac{1}{p}} e_i^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}).$$

Therefore, it is sufficient to show that

$$\begin{aligned}
\frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pr e_i^r e^{*r} + pr e_i^{r-1} e^{*r+1} + \frac{pr e_i^{2r}}{2} + \frac{pr e^{*2r}}{2} &\leq \\
pr \left(\frac{e_i}{e^*} \right)^z (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}). &
\end{aligned}$$

However, since $e_i > e^*$ and $z > r$, it is sufficient to show that

$$\begin{aligned}
& \frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + 2pre_i^r e^{*r} + pre_i^{r-1} e^{*r+1} + \frac{pre_i^{2r}}{2} + \frac{pre^{*2r}}{2} \leq \\
& \qquad \qquad \qquad pr \left(\frac{e_i}{e^*} \right)^r (e_i^{2r} + 2e_i^r e^{*r} + e^{*2r}) \quad \Leftrightarrow \\
& \frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + pre_i^r e^{*r} + pre_i^{r-1} e^{*r+1} - \frac{3}{2}pre_i^{2r} + \frac{pre^{*2r}}{2} \leq pr \frac{e_i^{3r}}{e^{*r}} \Leftrightarrow \\
& \frac{e_i^{2r}}{2} - \frac{e^{*2r}}{2} + pre_i^r e^{*r} + pre_i^{r-1} e^{*r+1} - \frac{1}{2}pre_i^{2r} - pre_i^{2r} + \frac{pre^{*2r}}{2} - pr \frac{e_i^{3r}}{e^{*r}} \leq 0 \Leftrightarrow \\
& \qquad \left(\frac{e^{*2r}}{2} - \frac{e_i^{2r}}{2} \right) (pr - 1) + pr (e_i^r (e^{*r} - e_i^r) + e_i^{r-1} (e^{*r+1} - e_i^{r+1})) \leq 0
\end{aligned}$$

where the last line follows since $\frac{e_i^{3r}}{e^{*r}} = e_i^{2r} \frac{e_i^r}{e^{*r}} > e_i^{2r}$. Finally, since x^n is an increasing function for $x > 0$ and $n > 0$, all elements of the last expression are negative. \square

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