Meaning and Use in Wittgenstein's Tractatus

Among the tasks of a philosophical theory of meaning for a language is the elucidation of the very concept of meaning, and related concepts upon which that theory of meaning is founded.[1] Two of the most important approaches to these problems have been associated with Frege and Wittgenstein. Frege's fundamental idea is that a theory of meaning and the concept of meaning itself are based on the notion of truth, so that the meaning of an indicative sentence is captured by its truth-conditions, and the meaning of sub-sentential expressions is captured by their systematic role in determining these truth-conditions. Wittgenstein's fundamental idea – the predominant alternative to Frege's – is that the meaning of an expression is to be captured in terms of its linguistic use.

Wittgenstein's writings are traditionally divided roughly into two periods: in the earlier period, whose central expression is his Tractatus Logico-Philosophicus (hence Tractatus), he still upheld, according to this tradition, the main tenets of the Fregean approach. In the later period, whose central expression is his *Philosophical Investigations* (hence *Investigations*), he rejected the Fregean position and replaced it with his own, in which the concept of use became predominent.

In the discussion below I suggest that this traditional view is misleading – the concept of use is at the heart of Wittgenstein's theory of meaning already in the Tractatus. It is essential in explaining the concepts of meaning and in determining the meanings of the expressions of language. The actual use of a sign is what makes it into a symbol and determines its "mode of signification". The use of a proposition determines its "logical structure", which is common to it and the state of affairs it presents. The use and application of a proposition shows its sense; they do it by determining the projection rules, by dint of which a proposition has its sense and is a picture,

A central claim in the following is that these notions, in terms of which the notion of meaning is explicated are intra-linguistic, and do not appeal to external, ontological concepts. They are rather governed and expressed by "logico-syntactic rules". In this respect the ontology of the Tractatus should be seen as a transcendental framework for the possibility of language. It is not operative in the theory of meaning of the book. This trend in Wittgenstein's thought is intensified in later writings of the thirties, particularly in *Philosophical Grammar*, but its seeds are clearly rooted in the Tractatus.

In light of the intra-linguistic nature of the principles governing the notion of meanintg in the Tractatus it seems that it is a mistake to regard its conception of meaning as representational, or as Fregean. On the other hand, the central role of the notion of use in the theory of meaning of the Tractatus, underlined here, is one of the lines of continuity in the philosophy of Wittgenstein.

The Concept of Use in the Tractatus
In 3.326 Wittgenstein says:

> Um das Symbol am Zeichen zu erkennen, muss man
> auf den sinnvollen Gebrauch achten.
> In order to recognize a symbol by its sign we must
> observe how it is used with a sense.

Recognizing the symbol in the sign is at the heart of the philosophy of meaning in the Tractatus: this is what constitutes understanding. And the above proposition teaches us of the utmost importance of use here. The above translation, by Pears and McGuiness, leaves unclear the consequent of the sentence: How is anything "used with a sense"? Is a sense something attached to the use? If so, how? And what does "with a sense" relate to - the sign or the use? This vagueness in the translation seems to me associated with a widespread misconception on a matter crucial to the understanding of the Tractatus – the relationship between meaning and use - to which we shall turn

shortly. But let it be mentioned that the Pears-McGuiness translation, "how it is used with a sense", is unnecessarily cumbersome, and indeed misleading: Wittgenstein talks simply of "sinnvolen Gebrauch" – meaningful use.

The general problem with which we are faced pertains to the relation between use and sense, or meaning, in the Tractatus: Can there be a use without a sense? Can there be a use of a senseless sign? Can there be a meaningful expression, which has no use? The answer to all these questions is – no! The reason for this answer emerges from a fundamental position of the Tractatus on our general problem: it is that use constitutes sense and meaning. It is their heart and soul. Through the use of a sign, the sign obtains meaning and becomes a symbol. Hence, a sign in a particular use is ipso *facto* meaningful – it is a symbol. I shall expand later on the reasons for this position and its significance, as well as on some textual support for it.

Accordingly, proposition 3.326 cannot be taken to suggest the possibility of a use of a meaningless sign. Such a use were possible only if use, which is a criterial expression of concepts, could be described and comprehended in terms which do not presuppose the meaning inherent in it, or the concepts and meanings of which it is a criterial expression. I have elsewhere[2] labeled this conception of use "raw use". Such a conception is typical of reductivist theories of meaning, which Wittgenstein opposed also in his later writings, especially in Investigations.

The Tractatus conception of use, in which use constitutes meaning and is what turns a sign into a symbol, is not the 'thin' conception of 'raw use', some token action or behavior described in meaning-neutral terms. It is rather a 'thick' conception, imbued with meaning and concepts, by which the use is described and comprehended. Proposition 3.326 points out that by such use the sign becomes a meaningful symbol, which is recognized by observing how the sign is meaningfully used. That is the import of the phrase "meaningful use" (sinnvolen Gebrauch). When one uses a certain word as a mere phonetic object, e.g. to practice correct pronunciation, this would not be 'meaningful use', in which the symbol is recognized in the sign.

In PR, section 14 Wittgenstein illustrates the Fregean context-principle, which is adopted in the Tractatus (3.3), by saying that "**only in use is a rod a lever. Only the application makes it into a lever**." This illustration indicates the importance of use (or application) for understanding an expression's meaning. In the terms of our 3.326 it may be put thus: to recognize the lever by the rod we must observe its meaningful use – we must observe how it is actually used in acts of measuring, and related applications. These are not 'thin' acts of 'raw use', which can be grasped independently of the notion of measurement. It is rather a 'thick' use, made in acts, which are conceived and described in terms of measurement.

The 'thick' conception of use is also at the base of Wittgenstein's response to two problems related to a theory of meaning founded on the concept of use: (a) the problem of individuation – what sets apart a type of use of an expression? What groups different tokens of use of a word into a type of use? (b) The problem of normativity – how, and in what sense, can a use be mistaken?[3]

The problems of individuation and normativity of use concerned Wittgenstein throughout his career. Main stages here can be broadly outlined thus: in the Tractatus Wittgenstein attempted to answer these problems in terms of logico-syntactical categories, which 'show themselves' in the use of a language. In *Philosophical Investigations* this approach seemed to him mistaken. He also rejects attempts to answer the problem in psychologistic terms, and in behavioristic terms. His positive account of meaning in *Philosophical Investigations* is carried out by describing various types of use and the concepts inherent in them – concepts whose criterial expression are the types of use and behavior concerned. Later, Wittgenstein emphasized another thread of his thought in *Investigations*: the typical use of a word gives it a character, a 'face', a 'soul', which remains intact from one token-use to the next. The ability to grasp this aspect of a word, to 'experience' its meaning, is conditional on experiential, mental and even psychological factors. These may seem to go

beyond the actual use of the word, whereas, I suggest, they are actually incorporated into the 'thick' conception of use.

The following discussion centers only on the first of these stages, articulated in the Tractatus.

These remarks suffice, I believe, to cast doubt on a prevalent conception of the Tractarean theory of meaning and its relationship to Wittgenstein's later views. On the prevalent conception Wittgenstein presented in the Tractatus a theory of meaning, which is basically representational, according to which the meaning of an expression is a thing in the world, of which the expression is a linguistic representation. A name, accordingly, represents an object, and a proposition depicts and presents a state of affairs. Later, beginning with the writings of the early thirties and culminating in *Investigations*, Wittgenstein rejected this view, and presented instead the conception that the meaning of an expression is basically its use – the way it is used. There is, according to this prevalent conception, a great gap between the two views, where the first deals in a very abstract manner with the relationships between language and the word, while the latter, in setting the notion of use at its base, is more mundane and humane. Instead of founding meaning on abstract relations between language and the world, it construes it in terms of the use and the linguistic behavior of concrete speakers in concrete situations.

There is, of course, much truth in this presentation, but it can be misleading and needs qualifications and clarifications in both parts. In a very general way I would put it thus: With regard to the Tractatus, the ontological concepts and the ontological framework of the Tractatus is a sort of a "transcendental condition" for the possibility of language; they are not operative in the theory of meaning of the book. As will be emphasized below, a name or a symbol is not identified as such in terms of the object it means. The main reason for that is that the ontological concepts of objects, states of affairs, etc. are not given to us and are not identifiable independently of language. Moreover, in contrast to the prevalent conception, it should be noted that the Tractatus emphasizes the role of use in

constituting the meaning of an expression and the "logic" of language. With regard to the *Investigation*s, use is indeed a criterial manifestation of meaning, but Wittgenstein objects to a reduction of meaning to a "thin" notion of "raw use". He argues instead for a "thick" notion of use, which is itself conceptually loaded and imbued with meaning.

<u>Frege and Wittgenstein on Reference and Meaning (*Bedeutung*)</u>
In the Tractatus Wittgenstein maintained with Frege that the meaning of a name is the object it denotes. But the nature of these relata, names and objects, raises grave difficulties: For Frege objects are given to us, they are accessible to perception and cognition, and the manner in which they are given determines the sense of the names denoting them. But Wittgenstein explicitly opposes these views, and seems to leave obscure the nature of objects and the manner in which they are given. The nature of objects, and the manner in which they are given are not included among the actual data, but are taken to be a transcendental condition on the possibility of language.

Much was written on this issue in attempt to explain the nature of objects and states of affairs in the Tractatus. I will not venture here to provide such an explanation, but to say that in view of the transcendental nature of the ontological framework (objects and states of affairs), many of these attempts seem to go against the grain of Wittgenstein's position. In the case of objects, for example, the transcendental nature of the exposition implies that objects, which "make up the substance of the world" (2.021), must be granted in order to ensure a realistic concept of truth, and to serve as the points of application of simple predicates in the ultimate analysis of the language. But exploring the ontic nature of objects is not a meaningful philosophical enterprise. Below, we shall clarify and substantiate this claim with regrad the relationship between sign and symbol, and the concept of 'mode of signification' of symbols.

In Frege, reference can be taken as an actual relation between identifiable *relata*. But in the Tractatus it is a fairly obscure 'formal category', which cannot be explicitly indicated,

or described. The entire explanatory burden is laid upon the other term – the name. What, then, is a name, and how is it individuated? The gist of the following discussion purports to show that Wittgenstein does not answer this question in semantic-ontological terms (e.g. in terms of the relation to the object denoted by the name), but in terms of use (logico-syntactic rules of use included). This is not a claim against the obvious fact that the Tractatus is steeped in ontological concepts – world, object, state of affairs. It is a claim as to the status of these concepts: they form, to repeat, a transcendental framework for the possibility of language, but do not serve as criteria by which to identify words and uses, as they are not given to us independently of these. Wittgenstein expresses this by saying that in logical syntax the meaning of a sign should never play a role (3.33). This idea is connected of course with the "ineffability of semantics", that many scholars ascribed to the Tractatus[4]. But this thesis may imply a too narrow and technical view. The point is not only that semantical relations are ineffable (in language) but that they cannot be conceived at all and are not operative in the workings of language.

Sign, Symbol, and Mode of Signification (Bezeichnungsweise)

What, then, is a name, and how is it individuated? In the technical jargon of the Tractatus one may answer that a name is a symbol with its mode of signification, both of which are determined by the logical-syntactical rules of the language. Wittgenstein distinguished between sign and symbol: a sign is "**what can be perceived of a symbol** (3.32)", and a symbol is "**any part of a proposition that characterizes its sense** (3.31)." Note that, by this definition, a symbol is a linguistic expression, a part of a proposition. And yet, 3.31 continues to stress that it is essential to the proposition's sense. But how can a linguistic expression, a part of a proposition, be essential to sense? Is not the sense a distinct entity, which exists and is grasped without essentially depending upon any specific expression? - Apparently not. Apparently, the Tractarian notion of sense closely associates the sense of a proposition with its linguistic

expression, so that the parts of the proposition, as well as the proposition itself, are essential for grasping the sense. Wittgenstein does not say here (perhaps consciously countering the Fregean position) that an expression has a sense, but that it is essential to the sense. This may seem to pull sense down to the mundane level of linguistic expression, but it is just as well elevating these expressions to be essential to sense, and not just a casual means for its expression.

The sign, the material manifestation of the symbol, is insufficient for the identification of the symbol, because the same sign may serve to express different symbols. A plausible response here might be that a sign serves different symbols, according to the different objects they symbolize. Significantly, Wittgenstein avoids this response: Identification of symbols is indeed all-important in his theory, but he believed that the attempt to define the concept of symbol and its various categories in ontological terms is nonsensical. Instead, he turns to aspects of the use of a word (in the context of a sentence). These aspects have to do, as we shall see, with logico-syntactical concepts (themselves aspects of use of the language), and other concepts of use. The same sign can serve different symbols, says Wittgenstein, when their mode of signification (Bezeichnungsweise) is different (3.322). But the mode of signification is not elucidated in terms of the signified objects (nor in terms of other ontological concepts), and a sign can signify different objects in the same mode (as implied by 3.322). Mode of signification (and the "characteristic of a symbol", constituted by it) is, rather, a logico-syntactic category:

> What signifies in a symbol is what is common to all the symbols that the rules of logical syntax allow us to substitute for it. (3.344)

This proposition is typical of Wittgenstein's avoidance of characterizing the notions of signification and naming – the foundation of his theory of meaning – in ontological terms. Instead, he speaks of a substitution-class defined by the rules of logical syntax. This general idea is exemplified with respect to

the logical connectives in 5.514: The rules forming negation, disjunction, etc. are equivalent to these symbols, and "**in them their sense is mirrored**". The mode of signification is, of course, what Wittgenstein calls a 'formal concept' (4.126), inexpressible as a function, but 'showing itself' in the use of what it is a mode of, i.e. a name. Hence Wittgenstein says that "**the rules of logical syntax must go without saying (sich von selbst verstehen) once we know how each individual sign signifies**" (3.334)

What if the same sign signifies different objects in the same mode of signification (e.g. when different people share the same proper name)? Is it then the same symbol? It might seem natural to think so, but, quite typically again, Wittgenstein does not bring up this possibility. Instead, appeal to equivalence classes and substitution is manifested in Wittgenstein's explanation of the sentential variable, and the stipulation of its values:

> To stipulate values for a prepositional variable is to give the propositions whose common characteristic the variable is. [...] And the only thing essential to the stipulation is that it is merely a description of symbols, and states nothing about what is signified. (3.317)

According to this, a symbol is simply a sign used in a particular mode of signification, whose meaning is given by an equivalence class of expressions, which can substitute for it by the rules of logical syntax. Wittgenstein does not yet tell us what these rules are and what are the bounds of the substitution: what is the invariant to be preserved – truth value? grammatical correctness? In any case, the essential point for our concerns is that the identity of a symbol is not defined ontologically by the object it signifies, but rather in terms of the logico-syntactical rules of use.

In the proposition by which we opened our discussion (3.326), the phrase "meaningful use" (sinnvollen Gebrauch) functions only to lay emphasis on what is already included in the notion of 'use', and not to disengage it from some "meaningless use", which is absurd. In 3.328 he explains "Ockham's razor" by

claiming that "**if a sign is useless, it is meaningless**." He repeats this claim in 5.473 and 5.47321. All this unmistakably shows that meaning is constituted by use – only in use is the symbol recognized in the sign, and without it, there is no meaning.

In 5.4733 Wittgenstein says that if a sentence has no sense, this can only be because "**we have failed to give a meaning to some of its constituents. (Even if we think we have done so.)**" How do we give meaning, and how do we fail to give meaning, even when we think we do? - In actually putting words and sentences to use.

Alternatively, one might think that we give meaning by rules of interpretation, which associate a certain meaning (e.g. a certain object) and a certain word. Indeed, many have thus understood the 'pictorial relationship' (abbildende Beziehung) and rules of projection in the Tractatus. But if this were correct, Wittgenstein's claim that we can fail to give meaning, even if we think we have, would have made no sense. Neither would his claim that there are no meaningful useless sentences. It is precisely the 'Platonist' view Wittgenstein attacks, by which meanings are 'there', signs are 'here', and they are coupled to create symbols, and to make language representational. But this is a "mythological" picture attacked already in the Tractatus. Tractarian rules of projection are not assignment functions of model-theory. Rather, they are determined only by the use being made of the sentences.

It is clear, on this presentation, that in claiming that meaning is grounded in use "the later" Wittgenstein does not break from the position of the Tractatus, but rather that already in the Tractatus Wittgenstein holds that meaning is deeply rooted in use. The use is expressed in the 'logic', or logical syntax, which Wittgenstein considers all- important. But in the Tractatus, the logical syntax is not severed from the use and meaning of the sentences of a language, as some logical-positivists and a venerable tradition of interpreters believed. On the contrary, it expresses them, and is determined by them. All this implies that use establishes the meaning and mode of signification of an

expression, and functions in determining the language's rules of logical syntax. That is the reason why use cannot go against these rules, and cannot be without sense.

Intra-linguistic and Extra-linguistic Use: Use and Rules of Projection

I have spoken of the "intra-linguistic" character of the Tractatus conception of meaning. In this I intend mainly to dissociate this conception from the "representational", ontological conception in which it is often understood. But the term may be misleading and needs qualification. A source of confusion in understanding the connection between use and meaning is that use can be taken, either as intra-linguistic (usage), or as extra-linguistic (use), i.e. an actual activity in the world. The citations from Wittgenstein above are inconclusive. Commentators have considered the ambiguity, or obscurity, of this distinction in Wittgenstein to be characteristic of his writings in the 30's, in which the notion of use is sometimes explicated in connection with a use in a game, such as chess, and at other times in connection with a use of a tool. The former notion of use is naturally taken to be intra-linguistic, defined by a set of rules; the latter notion is taken as an actual activity in the world[5].

From what has been said in the preceding sections it appears ,though, that at least with regard to the theory of meaning of the Tractatus, the seemingly intra-linguistic notion, and the set of logico-syntactical rules which define it, are basically an expression of the extra-linguistic notion of use, the actual use of the language.

Proposition 3.327 may serve to bring out the difficulty:

> A sign does not determine a logical form unless it is taken together with its logico-syntactical employment (Verwendung) (3.327).

What is a logico-syntactical employment, or use? It might seem as if this is an obvious intra-linguistic term, unrelated to actual use. It might seem to be defined by the rules of logical syntax, which determine the linguistic compositions in which the sign

may be put in relation to others. Proposition 3.3, of which the above is a comment, and which states the Fregean context principle, may also be understood in this light. It also seems to accord with 3.263, which states that elucidations (Erläuterungen) explain the meaning of primitive signs.

But this seems wrong. For, what are these linguistic compositions in which the sign may be incorporated, and what determines and constrains them? One may argue that this is the function of the logico-syntactical rules of the language, much like the rules of chess serve to determine the possible compositions of pieces on the board. But even if this is indeed an adequate account of the game of chess, it is certainly not an adequate account of language. Logical syntax is not an independent high authority imposed on our applications of language 'from above'. Neither is it a set of rules arbitrarily set by convention. Rather, the logical syntax of a language is a regulated expression of the actual use of its sentences. This is the gist of Wittgenstein's remarks that "**logic takes care of itself**", and that, in a sense, one cannot be mistaken in logic. One cannot be mistaken in logic because logic – the logico-syntactic rules – constitutes meaning. To talk of mistakes in logic means that meanings are independent of these logical rules, so that the rules do not accord with the meanings; but this is impossible when the meanings are seen as constituted by the rules. And since the rules express the use of the expressions in language, it is use that constitutes meaning. Thus, what can be superficially seen as a mistake in use is either a change of subject-matter, or senseless wording.

Furthermore, Wittgenstein famously distinguished between what is said and what only "shows itself". To see how this pertains to the matter of use, note that "**a proposition shows its sense** (4.022)", and that "**what signs fail to express, their application (Anwendung) shows** (3.262)." A sentence is the sign of a proposition, so these statements entail that the sense of a sentence is what its application shows. The application of a sentence shows something additional to what is said by the sentence itself. In this addition lies the preeminence of

proposition over sentence, of symbol over sign, i.e. its sense or meaning.

How is this brought about? How can an application of a sentence show its sense?

I have expanded on this central issue elsewhere and shall not repeat it here, but I suggest the answer to be essentially this: the use of a proposition, its actual application for describing a state of affairs, determines the rules of projection (as well as the point of projection, from which are spread the lines of projection according to the rules) from the state of affairs to the proposition presenting it. Thus, it establishes the pictorial relationship – "**the correlations of the picture's elements with things** (2.1514)" – which is included in the picture, and which makes it the picture it is, as well as a picture at all, as Wittgenstein says in 2.1514.[6] It thus also determines the structure of the proposition (2.15). It is seen, then, that the elements of a sentence-picture, the components which make something a picture of something, and hence a picture at all, are rooted in the application, or use, of the sentence-picture: the use determines the rules of projection; the sense of the sentence; the pictorial relationship between its components and those of the state of affairs it depicts; the structure of the sentence and the manner in which it presents the structure of that state of affairs.[7]

This interpretation of the role use has in determining the projection, and hence the sense, in the Tractatus, finds some corroboration in paragraph 51 of Wittgenstein's *Investigations*. Here Wittgenstein refers to the language-game described in §48, which closely resembles the Tractarian view. The language has four words, which are organized in sequences of nine places to describe compositions of colored squares. Wittgenstein asks "**what does this correspondence consist in? [...] what does it mean to say that in the praxis of language certain elements correspond to signs?**" It is plain that in a language game such as this the use, the praxis, is all-important. It is the praxis, which sets the correspondence between the signs and the features of the world which constitute their meaning – what is

termed in the Tractatus the "pictorial relationship", or rules of projection.

The conception of use in the Tractatus, often neglected by commentators, is therefore of great consequence. In the Tractatus, as in Wittgenstein's later writings, use is at the core of the theory of meaning. It is sufficient to guarantee sense, and without it neither a sentence nor its parts can be meaningful. It constitutes the mode of signification, which makes a name, indeed any symbol, what it is. The major difference between the Tractatus and Wittgenstein's later view is therefore not that the former was a representational, ontologically-based theory, replaced later by a use-based conception: in both meaning is constituted by use. The difference is rather that in the former Wittgenstein believed that the main features of the use of an expression can be expressed and systematized in a set of "logico-syntactical' rules – a belief criticized and abondaned in his later writings.

One implication of the Tractarean conception is the incorporation of the subject into the account: a use is someone's use, a use of a word by a certain subject. This subject is the point of projection from a state of affairs to a sentence, the projection being set by the subject's use of the sentence.[8]

Still, central aspects of these notions of use and subjectivity remain obscure. Use is presented in the Tractatus in the solipsistic framework of a single subject. It is a general, abstract and simple notion, and does not, perhaps cannot convey the diversity of uses of words and sentences. Moreover, the Tractarian notion of use seems to lack a normative dimension: the question of the correctness of an application of a certain phrase is never raised. These issues are uppermost in Wittgenstein's thought from the 30's onward, and serve as the focal point to his later criticism of his own positions in the Tractatus. Still, these differences and developments, significant as they are, ought not to distract us from the features common to Wittgenstein's entire oeuvre. Our concern in this discussion – the role of use in establishing meaning – is among the most important.

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Notes

[1]   I use 'meaning' here in a rather loose and genral way, neutral, in particular, as to the sense/reference distinction. The term has often been used for translating Frege's and Wittgenstein's technical use of 'Bedeutung'. For Frege's notion I shall use 'reference'; for Wittgenstein's, I shall stick to the standrd translation, and speak of objects as the meanings of names.

[2]   XXX, 2003

[3]   I have expanded on this in XXX 2003.

[4]   See, e.g. Hintikka and Hintikka, 1986, ch.1

[5]   See, for instance, Kenny, 1975, pp.167-8

[6]   Cf. XXX, 1995

[7]   I find some similarity between the basic claims about the intrinsic intentionality of the picture theory argued for in XXX1995 (as well as their textual evidence) and those in Ostrow 2002 (see in particular pp. 35, 37, 41). I suppose that Osrow didn't know of that paper, which was published in Hebrew.

[8]   Cf. XXX 1998