# Modern censuses: The examples of the integrated census and the rolling census 

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## Census of population

"the operation that produces at regular intervals the official counting (or benchmark) of the population in the territory of a country and in its smallest geographical subterritories together with information on a selected number of demographic and social characteristics of the total population"
(United Nations Economic Commission for Europe UNECE, 2006, p. 6, no. 19).

## Essential features

## Universality

A well-defined census population

## Simultaneity of information

A reference date for all census data (Census Day)

## Individual enumeration:

Accurate data pertaining to individuals with regard to place of residence and other sociodemographic characteristics on Census Day

## Alternative approaches to census-taking

## Traditional census

Traditional with yearly updates

Rolling census

Register-based census
Combination of register with survey data

Register-based and complete enumeration

Why change the traditional approach?

Availability of massive administrative data
Growing strength of the information technolog
Growing demand for timely and detailed data

Changes in public opinion and decline in survey response rates

Increasing financial burden on national budgets

## Outline

IC:
The extended coverage model
Design of the coverage surveys
Example
$R C$ :
Concept and implementation

## The Integrated Census (IC)

Replaces the traditional field enumeration by enumeration of the Population Register (PR)

But... the PR is exposed to coverage errors
Uses sample surveys to
(a) estimate coverage errors of the PR
(b) set a reference date

## The problem

Let $N$ be the number of eligible people in a closed population $E$ in a given geographical area
$N$ is fixed but unknown
Aim: Estimate $N$
Data: A list that fails to capture every individual (undercount) and also counts individuals that do ${ }_{8}$ not belong to $E$ (overcount/false captures).

## Model overview



Undercount parameter
$p_{1+}=E Z_{1+} / N$

Overcount parameter
$\lambda=E X / N$

## Multinomial model (DSE, Peterson, 1986)

Suppose that List 1 is exposed only to undercount. Then, consider a capture-recapture experiment.

Capture history of individual $k$ is multinomial, that is

$$
\mathbf{Z}(k) \sim \operatorname{Mult}(\mathbf{p}(k))
$$

where $\mathbf{Z}(k)=\left(Z_{11}(k), Z_{12}(k), Z_{21}(k), Z_{22}(k)\right)$ and
$\mathbf{p}(k)=\left(p_{11}(k), p_{12}(k), p_{21}(k), p_{22}(k)\right)$.

Let $\quad Z_{1+}(k)=Z_{11}(k)+Z_{12}(k)$ and $p_{1+}=p_{11}(k)+p_{12}(k)$
Then $p_{1+}(k)=E\left(Z_{1+}(k)\right)$ is the capture probability of $k$ in List 1.

## Multinomial model (2)

Main assumptions
Homogeneity. Capture probabilities within each list are equal, that is

$$
p_{1+}(k)=p_{1+}, \quad p_{+1}(k)=p_{+1} \quad k=1, \ldots, N
$$

Independence. Captures are independent for each $k$, that is

$$
p_{11}(k)=p_{1+}(k) p_{+1}(k) \quad k=1, \ldots, N
$$

## Multinomial model (3)

Under the above assumptions, the MLEs of the capture probability of List 1 and of the population size are given by

$$
\hat{p}_{1+}=\frac{Z_{11}}{Z_{+1}} \quad \text { and } \quad \hat{N}=\frac{Z_{1+}}{\hat{p}_{1+}}
$$

where $Z_{a b}=\sum_{k=1}^{N} Z_{a b}(k), a, b=1,2,+$.

|  | List 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| List 1 <br> (PR) |  | In | Out |  |  |
|  | In | $Z_{11}$ | $Z_{12}$ | $Z_{1+}$ |  |
|  | Out | $Z_{21}$ | $Z_{22}$ | $Z_{2+}$ |  |
|  |  | $Z_{+1}$ | $Z_{+2}$ | $N$ |  |

## Multinomial-Poison model (Glickman,Nirel,Ben Hur 2003)

In addition, assume that the number of false captures $X$ in List 1 is Poisson, that is

$$
X \sim \operatorname{Poisson}(\lambda N)
$$

and adding the assumption:
Capture history and false captures are independent, that is $\{\mathbf{Z}(k)\}$ and $X$ are independent

## Multinomial-Poison model (2)

Then under the above assumptions, the MLEs of the parameters of interest are given by

$$
\hat{p}_{1+}=\frac{Z_{11}}{Z_{+1}}, \hat{N}=\frac{Z_{1+}}{\hat{p}_{1+}} \text { and } \hat{\lambda}=\frac{X}{\hat{N}} .
$$

Denote $Z=Z_{1+}+X$ then $p_{1+}+\lambda=E Z / N$.
Hence the estimate of the population size is

$$
\hat{N}=\frac{Z}{\hat{p}_{1+}+\hat{\lambda}}
$$

where $Z$ is the (known) size of List 1.

## Census Weight



## Bias and Variance

Taylor approximations yield

$$
\mathrm{E} \hat{N} \doteq N+C \quad \text { and } \quad \operatorname{Var}(\hat{N}) \doteq N C
$$

For a one-stage sample of $m$ clusters out of $M$

$$
\begin{gathered}
C=A+\frac{M-m}{m} B \\
A=\frac{\left(1-p_{1+}\right)\left(1-p_{+1}\right)}{p_{1+} p_{+1}} \quad B=\frac{1-p_{1+}}{p_{1+} p_{+1}}+\frac{\lambda}{p_{1+}+\lambda}\left(\frac{p_{1+}}{p_{1+}+\lambda}-\frac{1-p_{1+}}{p_{1+}}\right)
\end{gathered}
$$

## Coverage Surveys

Estimates of the coverage parameters are obtained through two coverage surveys:

The U-survey is based on an area sample and estimates the undercount rates

The O-survey is based on a sample of people from the PR and estimates the overcount rates

Independence between U-enumeration and the PR data is kept to avoid bias

## Sample design

Overall sample size. Typically large, e.g. about $1 / 5$ of the population in both samples

Sampling plan. Typically cluster sampling, e.g. select a simple random sample of $m$ enumeration areas (EAs) out of $M$ in each statistical area

Sample allocation. Based on model and sampling variance

## Implementation



U sample


O sample

## Fieldwork Process

## For an EA in the sample:



## The first Israeli experiment (Beit Shemesh, 2002)



## Census weights: $N(I C) / N(P R)$

|  | Age Groups |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| SA | $\mathbf{0 - 1 9}$ | $\mathbf{2 0 - 2 9}$ | $\mathbf{3 0}-\mathbf{3 9}$ | $\mathbf{4 0 +}$ |
| $\mathbf{2}$ | 0.97 | 0.89 | 0.86 | 0.90 |
| $\mathbf{3}$ | 0.91 | 0.98 | 0.84 | 0.85 |
| $\mathbf{4}$ | 0.90 | $\mathbf{1 . 0 0}$ | 0.73 | 0.96 |
| $\mathbf{5}$ | 1.18 | 1.26 | 1.17 | 1.02 |
| $\mathbf{6}$ | 0.91 | 0.88 | 0.94 | 0.84 |
| $\mathbf{7}$ | 0.85 | 0.77 | 0.85 | 0.83 |
| $\mathbf{8}$ | 1.03 | 1.21 | 0.97 | 0.98 |
| $\mathbf{9}$ | 0.96 | 1.02 | 0.92 | 0.90 |
| $\mathbf{1 0}$ | 0.98 | 1.09 | 0.98 | 1.04 |
| $\mathbf{1 1}$ | 0.99 | 0.85 | 0.89 | 0.83 |
| 13A | 1.09 | 0.94 | $\mathbf{1 . 0 0}$ | 1.01 |
| 13B | $\mathbf{1 . 0 5}$ | $\mathbf{1 . 0 0}$ | 0.95 | 0.88 |

## Rolling samples (Kish, e.g. 1990)

Jointly select a set of $k$ mutually exclusive (not overlapping) and representative periodic samples

Each with a sampling fraction $\quad f=1 / F$

One sample is interviewed at each time period

Accumulation of $k$ periods yields a sample with a fraction $k / F$

## A rolling census

For $k=F$ the entire population is covered over $F$ time periods (e.g., years)

## Advantages

- Information on temporal variation: frequent (annual) estimates for national and large domains levels; less frequent estimates for smaller domains
- Uniform expenditure over time

Drawbacks

- Limited information on spatial and demographic variation: no "snapshot" data
- Higher risk of bias: harder to estimate coverage errors


## Variants

## Cumulated representative sample (CRS) design

 Same PSUs in all $k$ samples, with a rolling sample within a PSU - inclusion of main units in all samplesPanel design
Overlap between subsequent samples - more efficient for comparisons

## French example (since 2004)

Rolling "census" - coverage of $k / F=5 / 7$ of the population over five years
A two-stage annual sample:
Large communes stratum - CRS design - all communes are included in the sample, and 0.08 of the dwellings are selected for enumeration.
Small communes stratum - Rolling sample approximately 0.20 of the communes are sampled, and all dwellings in the sampled communes are included in the sample.

$$
5(0.5 \cdot 0.08+0.5 \cdot 0.2)=5 \cdot 0.14=0.70
$$



## Estimation - general (Kish)

Let $Y$ be the outcome of interest
$\hat{Y}_{i} i=1, \ldots, F$ the annual estimator

$$
\begin{aligned}
& \hat{Y}(W)=\sum_{i=1}^{F} W_{i} \hat{Y}_{i} \\
& W=\left(W_{1}, \ldots, W_{F}\right), \sum_{i=1}^{F} W_{i}=1
\end{aligned}
$$

Examples:

$$
\begin{aligned}
& W_{F}=1 \\
& W_{i}=1 / F \\
& W_{1} \leq \ldots \leq W_{F}
\end{aligned}
$$

## Population estimates for year F-2

For a large commune, let $x_{i}$ be an auxiliary variable (number of dwellings) during year $i$, and $\bar{X}=\sum_{i=F-4}^{F} X_{i} / 5$.
Let $\hat{Y}_{i}$ be the expansion count estimator for year $i$,

$$
\overline{\hat{Y}}=\sum_{i=F-4}^{F} \hat{Y}_{i} / 5 .
$$

The estimate for year $F-2$ is the ratio (synthetic) estimate

$$
\hat{Y}_{F-2}=\overline{\hat{Y}} \frac{X_{F-2}}{\bar{X}}
$$

## Population estimates for year F-2 (2)

For a small commune, the estimator depends on the year it was enumerated. Denote by $Y_{i}$ the population count of a commune that is fully enumerated in year $i$.

$$
\begin{gathered}
\hat{Y}_{F-2}^{F-4}=\hat{Y}_{F-4} \frac{X_{F-2}}{X_{F-4}} \quad \hat{Y}_{F-2}^{F-3}=\hat{Y}_{F-3} \frac{X_{F-2}}{X_{F-3}} \quad \hat{Y}_{F-2}^{F-2}=\hat{Y}_{F-2} \\
\hat{Y}_{F-2}^{F-1}=\alpha_{F-1} Y_{F-1}+\left(1-\alpha_{F-1}\right) Y_{F-6} \frac{X_{F-2}}{X_{F-6}} \text { and } \\
\hat{Y}_{F-2}^{F}=\alpha_{F} Y_{F}+\left(1-\alpha_{F}\right) Y_{F-5} \frac{X_{F-2}}{X_{F-5}},
\end{gathered}
$$

where $0 \leq \alpha_{i} \leq 1 i=F-1, F$, and is typically no smaller than 0.5 .

## Estimates for current year

Each annual survey is a representative sample comprising about eight million people.
Hence, usual survey methods (e.g., expansion estimates) enable reliable national and regional estimates for the current year. These estimates are used to calibrate the commune estimates.

תכנון מדגם תאי פקידה עבור מפקד משולב מבוo על אומדים לשונות המוצגים בשקף 16. בתהליך התכנון "מנחשים" ערכים של הפרמטרים הלא ידועים כדי לחשב את גודל המדגם הדרוש עבור רמת טעות מסוימת. $a=\sqrt{\operatorname{Var}(\hat{N})} / N$ א. רשמו את גודל מדגם התאים הדרוש $m$ כפונקציה של הטעות היחסית של האומד

$$
\text { עבור המקרה } \lambda=0 \quad p_{1+}=p_{+1}=p
$$

ב. ניתוח רגישות. עבור התרחיש בסעיף א., ישוב בגודל משוער N=50000 אנשים, M=300 תאי פקידה, וטעות יחסית a=0.01 ציירו גרף של גודל המדגם הדרוש m כפונקציה של p, עבור ערכי p בתחום (0.5,1] בקפיצות של 0.05.

ג. מהי מסקנתכם לגבי רגישות התכנון להערכות מוקדמות של p?

