STAT 4015 Q
"Final", August 13, 2009, 6:15pm - 7:50pm

This final is a closed book exam. Answer **four** out of the five problems. Mark clearly the problems you submit for scoring. You may provide answers in a formula form. Numerical evaluations are not required, but the use of a particular formula should be given a short and precise justification. Equal weights are given to all the five problems. Equal weights are given to all questions within each problem.

Problem 1. European roulette wheel has 37 numbers on it, which we assume to be 1, 2, ..., 37. The roulette is rolled 4 times and the number obtained is recorded. Let X be the largest of the four numbers.

   (1) Compute $P(X = 20)$.
   (2) Compute the $\mathbb{E}(X)$.

Problem 2. The density of a random variable X is given in the form $f_X(x) = ce^{-3x^2+7x}$, for x real.

   (1) Evaluate c.
   (2) Find the value x with the property that $P(X > x) = 0.1$

Problem 3. The joint density of X and Y is given in the form $f_{XY}(x,y) = ce^{y-x}y^2x^{2.3}$, for $0 \le y \le 1$ and $x > 0$.

   (1) Evaluate c.
   (2) Compute the marginal density of X. Are X and Y independent or not?
   (3) Compute the probability $P(X < Y)$.

Problem 4. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed according to the $U(0,1)$ distribution.

   (1) Identify the distribution of $Y_i = -\log X_i$ and compute its mean and its variance.
   (2) Use the Central Limit Theorem in order to give and approximation to the probability $P(\prod_{i=1}^{n} X_i \le x)$, for $0 < x < 1$.

Problem 5. An urn contains 200 black and 200 white balls. They are removed from the urn in pairs. Let X be the number of pairs where both balls are black.

   (1) Compute $\mathbb{E}(X)$.
   (2) Compute (X).