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Modern Facet Theory: Content Design and Measurement in Behavioral Research

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Facet Theory is a research strategy that integrates formal analysis of research contents (using the tool of the mapping sentence) with intrinsic data analysis (multivariate procedures such as Faceted Smallest Space Analysis (FSSA) and Multiple Scaling by Partial Order Scalogram Analysis by base Coordinates (POSAC)). The integration of content design and data analysis is argued to be essential for theory construction and meaningful measurement in the behavioral sciences that typically study complex systems with many, possibly infinitely many, variables. Mapping sentences permit the balanced selection of observational variables from the content universe studied. Focusing on observed variables (columns of the data matrix), FSSA creates a geometric representation of the concept-space and partitions it into regions, thus enabling the systematic investigation of the concept structure. Focusing on the observed individuals (rows of the data matrix), POSAC provides optimal measurement scales for the individuals' assessment. Traditional prediction models assume the existence of factors (underlying variables), by which both variables and individuals can be evaluated, which determine empirical observations. Modern facet theory shows that two kinds of factors are at work: facets (partitionings) of the concept-space, for evaluating variables; and scales (coordinates) of the measurement space, for evaluating individuals. The two factor types point to two roads for attaining the goal of prediction. Mathematical results in Multiple Scaling, and especially the introduction of the newly devised coefficient of structural similarity, contribute to the convergence of the two roads.

Keywords: Facet Theory, content design, mapping sentence, faceted smallest space analysis, multi-dimensional scaling, complex systems.

What is Facet Theory?

Facet Theory (FT) is a strategy for research in psychology and other sciences that study complex behavioral systems. Facet Theory centers on the formalization of research contents and on intrinsic data analysis for the purpose of discovering stable laws and conducting theory-based measurements in those sciences. The formalization of research contents is attained through the use of mapping sentences, essentially a generalization of R.A. Fisher's design of experiments to the design of theories, while procedures for intrinsic data analysis, notably Faceted Smallest Space Analysis and Multiple Scaling, have evolved from factor theories and Guttman scaling. These developments have been motivated by the need to adapt research methods to the nature of theories and data in the social and psychological sciences. Consequently, facet theory advocates the following:

1. Conceptual aspects of the investigated domain should be treated with formality and care compatible with those typically accorded (e. g., by statistical procedures) to data analysis. Facet Theory is critical of research practices that place undue emphasis on the intricacies of the *quantitative* aspects of empirical observations, while relinquishing the definitional framework of these observations to intuitive understanding and communication. The tool offered by facet theory for the formalization of the definitional framework for collecting data, is the *mapping sentence*, which highlights conceptual differentiations considered essential for the hypotheses and theories to be tested.
2. Data analysis to be conducted must form an inseparable part of any hypothesis formulated, and as such should be *intrinsic*, that is, adhere only to the defining features of the data and to the intended hypotheses. This means that Facet Theory aims to exclude extraneous constraints from the data analysis (e. g., linearity – unless linearity is explicitly rationalized).

In the social and psychological sciences data often have the basic feature that the variables observed on a particular occasion are not of interest in and of themselves, but rather as representatives of a wider (typically complex) multivariate concept such as intelligence, positive attitude, and leadership. In fact, the observed variables (e. g., specific intelligence items) should be regarded as constituting a *sample* from the intended "content universe" (intelligence) that may comprise innumerable variables. Traditional statistical techniques treat observed variables as though *they* were the object of the study (rather than the content universe they represent). Intrinsic data analysis, especially the procedure known as Faceted Smallest Space Analysis (Shye, 1991, in press; Shye & Elizur, 1994) uses the sample of observed variables to make inferences concerning the entire content universe, which is indeed our intended object of investigation. (This is in analogy with making inferences from a sample of respondents – who are of no interest to us individually – to the entire population under investigation.) Another major manifestation of the adherence to intrinsic data analysis is the use of measurement scales that have been logically derived from a structural theory for the attribute assessed. This contrasts with the mere score summation often practiced for reaching a "total score." These points are explained in the sections that follow.

3. More generally, Facet Theory offers a totally different scientific imagery for the behavioral sciences: Concepts are depicted as having extensions, in analogy with a geometric space; and their constituent constructs are depicted as regions within that space. This imagery has far-reaching consequences for the research questions asked, the hypotheses tested, and the kind of theories developed in substantive domains of research. For example, it implies that empirical affinity between conceptual constructs cannot be represented by a single number, i. e., statistical correlation coefficient, but rather by the mutual orientation of their regions in space. This imagery also sheds new light on the question of defining and conducting measurement in behavioral research.

The Problem of Facet Theory

Facet Theory was originally motivated by the problem of reproducing and predicting empirical observations (Guttman, 1959). Consider observations taken on a set P of N persons, with respect to a set Q of n questions (variables). Such observations are often represented in the form of a *data matrix:*

	Variables				
Person	v_1	v_2	v_3	...	v_n
p_1	a_{11}	a_{12}	...	...	a_{1n}
p_2	a_{21}	a_{22}	...	...	...
p_3	a_{31}	a_{32}	...	...	...
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
p_N	a_{N1}	...	...	...	a_{Nn}

Observational design for a system of observations can be conceived of as a *mapping*

$$P \times Q \rightarrow R \qquad (1)$$

from the cartesian product of population P of subjects and the set Q of questions into the set R of all acceptable responses to these questions. Guttman (1959) proposed the following definition:

Definition: A *facet* is a set that is a component of a cartesian product. For example, in the above, P is the population facet and Q is the question facet.

Example 1: Suppose we want to study the kind of talents people have. For every member p_i of the population P of interest, the following question may be asked:

> Consider the three kinds of mental material: numerical, verbal, spatial. In which one of them does p_i excel (by being able to perform mental tasks involving objective rules)? Set of acceptable responses: {1. numerical, 2. verbal, 3. spatial, 4. none of the three}.

If more than one choice out of the first three responses is to be permitted, the question may be broken down to three questions:

- Is p_i good at mental tasks that concern objective rules in numerical material? {yes / no}
- Is p_i good at mental tasks that concern objective rules in verbal material? {yes / no}
- Is p_i good at mental tasks that concern objective rules in spatial material? {yes / no}.

Given the overall structure of the questions, the mapping is, in effect, $P \times Q1 \rightarrow R$, where Q1 is the material *content facet:* {numerical, verbal, spatial}, which differentiates among the three questions. Additional content facets of intelligence that cut across the material facet, such as the kind of mental task required of the respondent – whether memory (rule recall), following instructions (rule application), or inductive ability (rule inference) – may be included to expand the observational design. The new mapping would be: $P \times Q1 \times Q2 \rightarrow R$, where Q2 = {rule recall, rule application, rule inference}. Each of the content facets Q1 and Q2 classifies the expanded set of nine questions by a content criterion. The *mapping sentence* (Guttman 1959; Schlesinger, 1978; Shye, 1978, in press; Shye & Elizur, 1994) for the observational design would be:

A Mapping Sentence for Observing Mental Abilities

$$\text{Is respondent } p_i \ (p_i \in P) \text{ good at mental tasks in } \overset{\textit{Q1: Material}}{\left\{\begin{array}{l}\textit{numerical}\\ \textit{verbal}\\ \textit{spatial}\end{array}\right\}} \text{ material requiring } \overset{\textit{Q2: Cognitive task}}{\left\{\begin{array}{l}\textit{rule-recall}\\ \textit{rule-application}\\ \textit{rule-inference}\end{array}\right\}} \rightarrow \{y_1, n_1, y_2, n_2, \ldots, y_9, n_9\},$$

where the range contains the possible responses y (yes), and n (no), for each of the nine questions. For example, that respondent p_3 is not good at recalling number sequences would be recorded by mapping the point [p_3, numerical, recall] of the domain into the appropriate point (say, n_1) in the range. In symbols: [p_3, numerical, recall] $\Rightarrow n_1$.

Mapping sentences (and the investigated contents they represent) may be expanded systematically in two ways: by *extension* (i. e., adding a new element to an existing domain facet, e. g., "social material" may be added to Q1); or by *intention* (i. e., adding a new domain facet; for example, in the above mapping sentence, we can add the facet {oral, written} to differentiate between mental abilities expressed in oral tests and in written tests). Of course, the range of the mapping sentence could be refined to any number categories to allow for finer degrees of mental abilities. Finally, note that specific test items pertaining to the content universe of mental abilities (e. g., "Can p_i correctly recall his or her car license number?" or "Can p_i correctly infer the shape of a physical object from two of its geometric projections?") can be classified by each of the content facets. The extent to which the testee responds correctly to such an item is recorded in the item range contributing to the overall assessment of "being good at" the mental abilities indicated. Thus, a mapping sentence, which may have any number of content facets, serves as a general definitional framework for a system of observations. According to Guttman (1959) a facet design is *complete* to the extent that it permits effective reproducibility and prediction of empirical observations. More generally, we are interested in identifying that decomposition Q1 × Q2 × . . . × Qk of the content universe into content facets that would lead to stable lawfulness in empirical data. This is indeed the challenge for scientists in substantive domains of research.

The Original Prediction Paradigm

A basic assumption of psychometrics is that there exists a limited set of underlying variables or "factors" that determine all possible observations in a given domain. This is a general scientific assumption. Specifically, it is also often assumed that the same set of, say, m factors can efficiently assess:

1. Each observed person, p_i ($i = 1, \ldots, N$); these assessments are presented in an N × m matrix of factor scores U;
2. Each observed variable, v_j ($j = 1, \ldots, n$); these assessments are presented in an m × n matrix of weights V.

These assumptions have led to the general prediction paradigm

$$U \times V = D \tag{2}$$

where D is the prediction for the observed N × n data matrix. The column and row specification fit the rule of matrix multiplication:

$$\underset{\text{Persons}}{\text{N}}\ \overset{\text{m factors}}{\boxed{\ \mathbf{U}\ }} \times \underset{\text{factors}}{\text{m}}\ \overset{\text{n variables}}{\boxed{\ \mathbf{V}\ }} = \underset{\text{persons}}{\text{N}}\ \overset{\text{n variables}}{\boxed{\ \mathbf{D}\ }}$$

This equation is to be understood as a general prediction paradigm that may have to be taken not literally, but "metaphorically." Recent developments in the foundations of Facet Theory show this to be the case; moreover, even the prediction paradigm itself requires some modification, to allow for two different kinds of factors. This will become evident below.

To reproduce, and make predictions about, observations of complex behavioral systems is indeed a big order – a formidable task. How can it possibly be tackled?

The Two Roads to Prediction: Rows & Columns

Two major roads are open to tackle the prediction problem. The one road retains individuals' scores as its object of prediction, but restricts attention to specific families of scalograms which can be easily interpreted and generalized. (A scalogram is now defined as a data matrix the variables of which have a common meaning range; it is regarded as a set of profiles – the matrix rows; Shye, 1976; 1985a.) The other road, abandons the observed scores as the object of prediction and turns instead to correlations and to other meaningful transformations of the original observations which are more stable and more predictable than the raw scores. Significantly, the two roads correspond to the two perspectives of the data matrix: the rows, which represent the score-profiles of persons observed; and the columns, which represent the variables observed. Based on the same data matrix we expect these two roads to eventually converge, yielding an *integrated* approach to behavioral theory construction and measurement (Shye, 1985a,b).

Road 1: Predicting Scores in Scalogram Configurations

The first step: Guttman Scale. If empirical data support the scale hypothesis (Guttman, 1950), simplicity attains a great feat: The hypothesis implies that, in the studied population, every person has a *meaningful* single score that can reproduce that person's scores in all the variables observed. Although a unidimensional yardstick, backed by theory, is a nice outcome, the importance of the scale for Facet Theory lies elsewhere: If a scale is consistently found in empirical data over replications with different samples of variables from a defined content universe (and different samples from a defined population), then we may infer, by way of generalization, that the content universe *itself* is scalable (for that population), even if that content universe has infinitely many variables.

As an example of a Guttman scale consider the following scalogram and its mapping to scale-score *f*:

v1 v2 v3 v4 v5 v6	*f*
0 0 0 0 0 0	0
1 0 0 0 0 0	1
1 1 0 0 0 0	2
1 1 1 0 0 0	3
1 1 1 1 0 0	4
1 1 1 1 1 0	5
1 1 1 1 1 1	6

Figure 1. The 1-dimensional Guttman Scale in six variables. The *f* scale indicates how high a subject is with respect to the general attribute represented by set of accumulating traits.

Here *f* (which indicates how high a subject is with respect to the general attribute represented by set of accumulating traits) serves as the "factor," or underlying variable, from which specific observations can be reproduced or predicted.

The essential thing to note here is that the inference from samples of variables to the entire content universe relies on the ability to recognize and generalize the scale profile-pattern and its interpretation to any number of variables. The said inference *does not* rely on the unidimensionality of the scale, as such. Hence, the scaling approach to reproducibility and prediction can be extended to higher dimensionalities, provided we can identify families of scalograms which can be clearly generalized with respect to both, their profile patterns and the interpretation of their axes.

Further Steps: Multiple Scaling. Such an extension is indeed one objective of Multiple Scaling Theory

Y					
5	10000	11000	11100	11110	11111
4		01000	01100	01110	01111
3			00100	00110	00111
2				00010	00011
1	(00000)				00001
	1	2	3	4	5
			X		

Figure 2. The 2-d diamond scalogram in 5 variables: When applied to "rise & fall" processes, scale Y measures how early the rise begins; scale X measures how late it ends. Note that both X and Y share meaning with the common range of the observed variables ("extent of 'rise'").

(Shye, 1976; 1985a). For example, the diamond family of scalograms (empirically found, e. g., in "rise & fall" processes) is a *2-dimensional* configuration that easily generalizes to any number of variables (see Figure 2). Knowledge of the "factor scores" x and y enables prediction of the scalogram scores observed.

Multiple Scaling, with the POSAC/LSA program, has been used to reveal regular patterns in the scalogram and interpret its coordinate scales (e. g., Russett & Shye, 1994). A multiple scaling hypothesis, once established for a particular domain of investigation, can be used to make predictions about observable scores attributable to individuals.

Road 2: Predicting Transformed Scores in Content Configurations

The first step: Simplex. The other road to cracking the prediction problem was pointed to by the original simplex. The simplex proposed by Guttman (1954) is an insightful modification of Spearman's g-factor theory of intelligence. Spearman's theory posits a hierarchy that distinguishes between the general factor of intelligence and the specific tests observed, subordinate to it: All that any two specific tests, i and j, have in common derives from g, their common ancestor. This idea translates statistically into the annulment of the partial correlation $r_{ij.g}$ between i and j, given g. Whence it may be derived that:

$$r_{ij} = r_{ig}\, r_{jg}. \qquad (3)$$

That is, *the correlation between two observed tests equals the product of the correlations of each of them with the general factor.* Given a set of intelligence tests, the single-common-factor theory implies a hierarchy among the tests according to their factor loadings and hence a particular pattern in their correlation matrix.

Once it was realized that this theory was not supported by empirical intelligence data, most psychometricians concluded, naturally, that a single factor is not enough to account for the data and turned to multiple-factor theories (Thurstone, 1947). Guttman, however, had a radically different idea and came up with the ingenious simplex theory.

The simplex theory for mental ability tests hypothesizes as follows: Let the set of n tests $t_1, t_2, \ldots, t_n$ be simply ordered with respect to their cognitive complexity (a psychological concept). Then, for every three tests t_i, t_j, t_k, $(i < j < k)$ the partial correlation $r_{ik.j} = 0$. This implies that in the formula for partial correlation the numerator $r_{ik} - r_{ij}\, r_{jk} = 0$, or

$$r_{ik} = r_{ij}\, r_{jk}. \qquad (4)$$

That is, *the correlation between any two observed tests equals the product of their correlations with another observed test that lies between them in the order of complexity* (see Figure 2). Thus, the simplex theory refers to observable tests only; it is ordinal, not hierarchical; and it relies on a substantive concept for the determination of the order among tests. The substantive concept (test complexity, in this case) was first called "order factor", then *facet.*

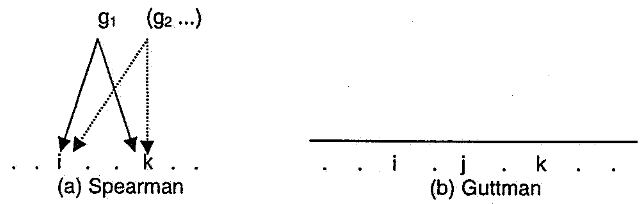


Figure 3. Depiction of two correlational approaches: (a) Spearman's common factor, which developed to multiple-factor analysis; and (b) Guttman's simplex, which developed to Facet Theory.

Taking the –log of Eq. (4) we obtain an additive relationship:

$$-\log r_{ik} = (-\log r_{ij}) + (-\log r_{jk}) \qquad (5)$$

If we let the distance between any two tests t_i, t_j be $d_{ij} = -\log r_{ij}$, we have $d_{ik} = d_{ij} + d_{jk}$: The tests can be mapped as points on the line so that correlations between them bear an explicit functional relationship to the distances:

$$r_{ij} = e^{-d_{ij}} \qquad (6)$$

Facet Theory takes the line to represent a content facet, i. e., an underlying conceptual differentiation (e. g., test complexity facet). Eq. (6) reproduces the observed correlations from the distances: *Once a simplex theory has been established, correlations can be predicted from distances implied by the content-facet analysis.* Hence, it is now the content facet that plays the role of "factor."

Ordinal simplex and other topological configurations. Since the metric version of the simplex, illustrated above, was too stringent for psychological data, ways to "soften" the model were proposed (Guttman, 1954; Shepard, 1962, 1966, 1978). For example, the *ordinal simplex* requires that intertest distances along the line would reproduce only the rank order of the n(n–1)/2 correlations between the n variables. But in cases where facet analysis can predict order among facet *elements* only, and not among their pairwise correlations, a weaker condition suffices for mapping tests on the line – and a weaker prediction is possible (Shye, 1976, 1985a):

$$r_{ik} \leq \min(r_{ij}, r_{jk}) \text{ iff } d_{ik} \geq \max(d_{ij}, d_{jk}) \ (i < j < k) \qquad (7)$$

Additional correlational structures were proposed and discovered in empirical data. The circumplex was defined as the pattern of circularly ordered tests and was identified, e. g., in the study of color perception (Shepard, 1966, 1978). The radex was defined as a two-dimensional configuration of tests, combining concentric circumplexes with simplexes emanating from a common origin (Anderson, 1959; Guttman, 1954; van den Wollenberg, 1978). These parametrized formulations, too, gave way to the less stringent ordinal or nonmetric formulations that led to better fit with behavioral data.

Further Steps: Multidimensional Scaling (MDS). For data analysis the problem was this: Given a set of n objects (typically variables) with a matrix of pairwise similarity coefficients, r_{ij} (typically correlations) between them, find a mapping of the objects into a suitable geometric space, where dij is the distance in that space between objects i and j, such that:

(i) If $r_{ij} > r_{kl}$ then $d_{ij} < d_{kl}$ for all $1 \leq i, j, k, l \leq n$; (8)

(ii) The dimensionality of the space is the smallest possible.

Further specifications on the mapping (e. g., the kind of distance function) as well as variations of condition (8) have been discussed in Guttman (1968) and Shye (1985a).

Following work by Coombs (1954) on interstimulus similarity and pair comparisons, algorithms for solving this problem were devised and programmed by Kruskal (1964a,b) and Guttman (1968). These computer programs have been successfully applied to research data in psychology, sociology, and other fields. For example, see the edited volumes by Canter (1985), Hox, Mellenbergh, and Swanborn (1995), and Shye (1978a).

Subuniverses in SSA Concept Space. Experience with mapping sentences has led to interpreting points in their domain as defining *classes* of variables rather than single variables. In principle, then, there could be infinitely many variables that belong to a single content profile. This led to redefining Q as the set of *all* variables (observed or not) that conform to the mapping sentence semantic structure (including, of course, its range facet). When the variables all have a *common-meaning range* (CMR), Q is referred to as the "content universe." Thus, for example, the set of all variables with a given CMR define a content universe such as intelligence (Guttman & Levy, 1991; Schlesinger & Guttman, 1969), commitment to work (Aranya, Jacobson, & Shye, 1976), or quality of life (Shye, 1989). Concomittantly, the contiguity hypothesis, which originally referred to *proximity between variables*, was replaced by the *regional meta-hypothesis* (Shye, 1978, 1985b), which concerns contiguity in the sense of *adjacency between regions*, in the SSA concept space:

> *The Regional Meta-Hypothesis (Shye):* Given a decomposition of Q into facets Q1, Q2, . . ., Qk ($Q = Q1 \times Q2 \times \ldots \times Qk$) with $Qi = \{q_{i1}, \ldots, q_{iki}\}$, then
> (i) The SSA concept representation space can be partitioned into connected regions each containing all observed variables pertaining to a single class $[q_{1j1}, q_{2j2}, \ldots, q_{kjk}]$.
> (ii) If two classes of variables have the same facet designation in all but one, say, the h-th facet, and if the two distinguishing facet elements, q_{hl} and q_{hl+1} are specified to be conceptually adjacent, then their two regions would be adjacent.

The challenge for researchers in substantive domains then is to identify the empirically relevant content facets, those for which specific regional hypotheses hold. Indeed, evidence for lawfulness cast in specifically continuous-regional terms has been accumulating in diverse fields ever since the early 1970s (Aranya, Jacobson, & Shye, 1976; Elizur & Shye, 1990; Galinat & Borg, 1987; Guttman & Levy, 1991; Shye, 1971, 1978c; Shye et al., 1994).

Simple, replicable partition patterns were defined in n-spaces (Shye, 1978b). Thus, in 2-space three main partition patterns were noted:
1. Partition into ordered stripes, by parallel straight lines (a 2-d generalization of the simplex);
2. Partition into circularly ordered sectors, by radii emanating from a common center (a 2-d generalization of the circumplex);
3. Partition into ordered concentric rings, by concentric circles.

Content facets found to conform to these partitions are called, respectively, axial, angular, and radial facets. The computerized procedure for testing regional hypotheses, in Faceted SSA (Borg & Shye, 1995; Shye, 1991; Shye & Elizur, 1994) and in the more recent FSSAWIN depicts the concept space with observed variables marked as points and then draws, for a prespecified facet, the best partition of the space according to each of the above models. For those partitions it computes the Facet Separation Index (a measure of facet-to-partition goodness-of-fit).

In empirical research, certain content facets (e. g., "life areas") have been identified which play a similar role (e. g., angular) across changing experimental contexts and even across different research domains (Levy, 1985).

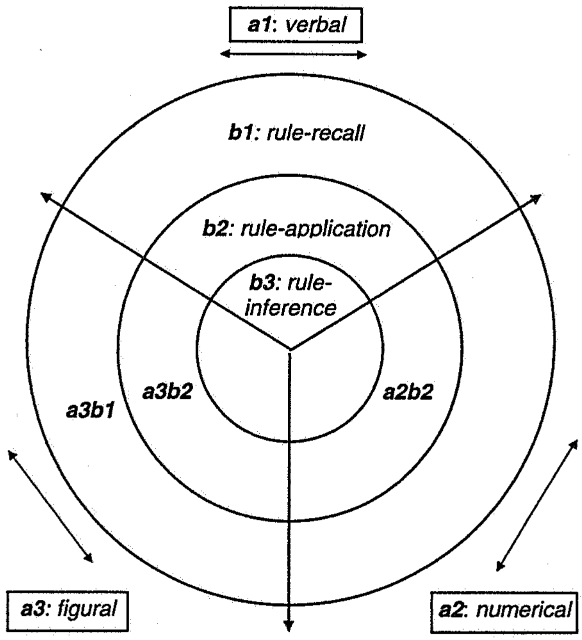
A particular content universe may be decomposed into a number of independent facets. Thus, the radex is a combination of a radial facet and an angular facet. The duplex is a combination of two axial facets; the cylindrex, a combination in 3-space of the radex and an axial facet, etc. (for a systematic description of partitions patterns see Shye, 1978b).

Facet Theory: A New Scientific Imagery. Given an investigated behavioral concept (such as intelligence, positive attitude towards an object, and adjustability) that may be attributed to investigated subjects, Facet Theory conceives of the concept as the semantic space of all the variables that assess it. In this imagery, each of the variables that comprise the concept is represented by a point in an analog geometric space. The finite set of observed variables is but a *sample* from the entire set comprising the concept. If that sample is taken with the aid of a mapping sentence, it is "stratified" relative to the *a priori* content facets. SSA enables inferences about the structure of the concept from the sample of observed variables to the "population" of variables that comprise the concept. This portrayal is encapsulated in the basic assumption of modern Facet Theory:

> *The Continuity Principle (Shye).* The mapping of the universe of variables to a connected subset of a geometric space is one-to-one and "onto,"

(e. g., Shye & Elizur, 1994). This means that every point in the concept space represents a possible variable of that concept (more precisely, a possible variable-equivalence-class or v.e.c., with v.e.c. being defined here as an entity of either of two forms: (i) a set of all observable variables that are similarly classified by a complete facet design, or (ii) a variable that is (definitionally) equivalent to a set of variables). The spatial imagery of concepts implies that spatial orientation between regions – rather than correlations between variables – is the way Facet Theory assesses affinity between conceptual components. Moreover, partitionability of the concept-space may now be regarded as a new kind of statistic, whose "values" are the particular partition patterns. Being a more general ("softer") aspect of the data than correlations, or even correlation ranking, partitionability leads to more stable lawfulness and better predictions.

Example: In the radex of intelligence (Schlesinger & Guttman, 1969), rule-recall and rule-application are definitionally specified to be adjacent (reflected in the content profiles, e. g., ***a3b1*** and ***a3b2***). Here, in accordance with a specific (substantive) hypothesis, the *definitional* adjacency corresponds to *regional* adjacency:



Integrated Facet Theory: The Convergence of the Two Roads

Based on the same data matrix, Faceted SSA concept space and POSAC measurement space are mathematically related. Proved mathematical relationships between them hinge on the introduction of a new kind of coefficient, E*, the coefficient of structural similarity (Shye, 1976, 1985a). While E* assesses pairwise similarity between variables, it does depend on variations in the remaining n–2 variables observed. That is, in the spirit of Facet Theory, E* depends on the sampled *contents* as well as on the sampled *population.* LSA1 procedure, within POSAC/LSA, is a special version of SSA with E* as its similarity coefficient, and with lattice ("city block") distance function. This suggests a plausible solution to a Guttman's (1982) challenge: What correlation coefficient should one use in mapping the content space by SSA, Pearson, monotone or some other? E*, which links the two spaces, is the natural candidate.

The Wife and the Secretary

The following analogy would serve to clarify the difference between common statistical coefficients such as Pearson or monotonicity, and the content-oriented coefficient of structural similarity, E*:

Suppose we were concerned with social proximity between people, rather than between variables. If we took the (waking) time two people spend together as a measure of how close they are, we would have a measure which is (1) quantitative, (2) strictly pairwise, i. e., involving no persons other than the ones whose social proximity is being assessed.

According to this measure a man who spends daily 8 hours with his secretary and 2 hours with his wife would be said to be "closer" to his secretary than to his wife. And for some (quantitative) purposes this may well be the appropriate measure. In our analogy this measure parallels the quantitative statistical coefficients.

But there is a more qualitative sense of "social proximity," which is missed by the quantitative measure proposed. In that qualitative sense, that same man may be much closer to his wife than to his secretary. How can that quality be captured in a coefficient? Consider a measure of social proximity between two persons which would increase with the diversity of social occasions in which these two persons are together; that is, the variety (of subsets) of other persons with whom they spend time together, *regardless of the duration of these occasions*. Such a measure would be (1) qualitative (structural), (2) pairwise but dependent on the social context (i. e., on the personal composition of other people present).

According to *this* measure our man may well be socially closer to his wife than to his secretary, for the personal compositions with whom he and his wife spend time together are more varied; whereas the time spent with his secretary is in the company of fixed (or not so varied) set of persons (e. g., office workers). It is *this* structural notion of proximity (applied to variables rather than people) that is captured by E*, the coefficient of structural similarity.

A new insight gained from mathematically relating POSAC and SSA is that *the nature and the number of factors that are required for assessing observed variables differ from those required for assessing the observed persons.* Hence, the role of "factors" (in the sense of underlying variables) is played by *two* different sets, one for the N persons, another for the n variables investigated. This means that the meaning of "factor" is ambiguous, and that the original prediction paradigm $U \times V = D$ will not do. The reason is this: Initially we assumed that there is a set of, say, m "underlying entities" against which both persons and variables can be assessed. m would then be the number of columns of U, and the number of rows of V, which permits their algebraic multiplication, to produce estimates of the observations. Now, however, we have found that the role of "underlying entities" is played by *two* different sets – one (of measurement scales) for the N persons and another (of content facets) for the n variables investigated. The two sets may be of different sizes, so the matrices U and V simply cannot be multiplied.

To deal with this impasse, I have recently proposed (Shye, 1997) a modification of the Facet Theory prediction paradigm along the following lines: Let p be the number of coordinate scales that span the POSAC measurement space, and q the number of facets that span the SSA content space. Let U be an $N \times p$ matrix that assesses the N persons with respect to the p scales, and let V be a $q \times n$ matrix that assesses the n variables with respect the q facets. Now let us introduce a mediating $p \times q$ matrix, M, which associates the p scales with the q facets. Then the Facet Theory Prediction Paradigm can be stated as:

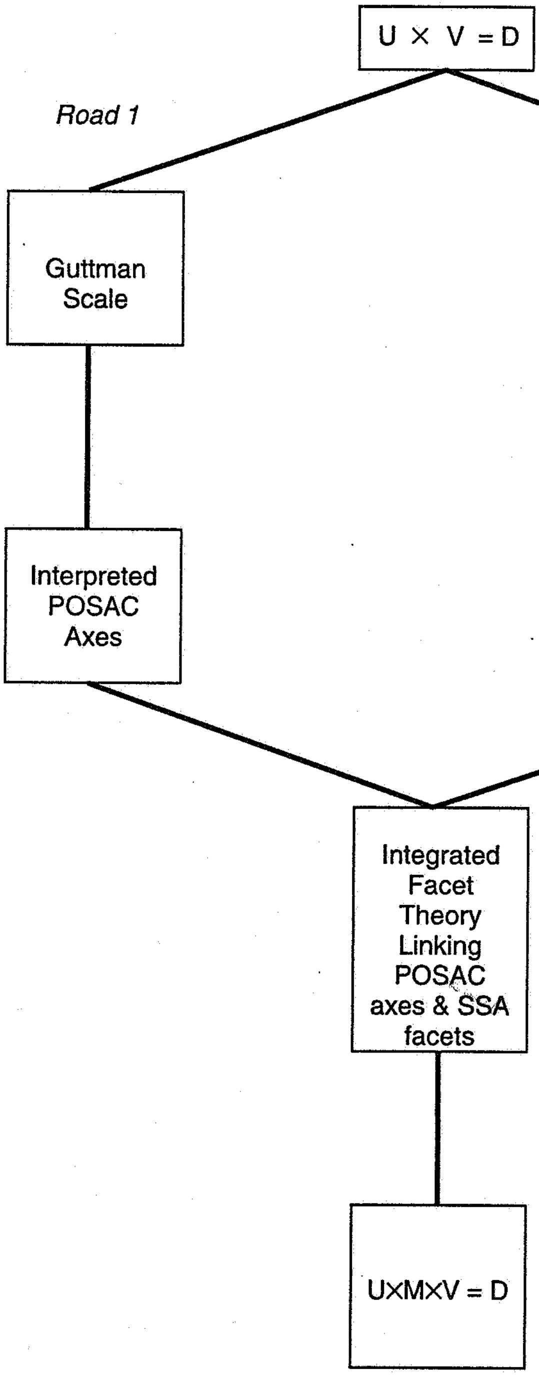
$$U \times M \times V = D \qquad (9)$$

which *is* algebraically feasible:

$$\begin{array}{ccccccccccc} & \text{p axes} & & & \text{q facets} & & & \text{n variables} & & & \text{n variables} \\ \text{N Persons} & \boxed{U} & \times & \text{P Axes} & \boxed{M} & \times & \text{q facets} & \boxed{V} & = & \text{N persons} & \boxed{D} \end{array}$$

A major objective of Multiple Scaling Theory is indeed the study of M, which relates structural components of POSAC space with those of SSA in its LSA version, as explained above. This paradigm has direct implications for practical issues in Facet Theory research; for example, the choice of variables for POSAC on the basis of Faceted SSA, and how to interpret POSAC scales.

The Evolution of Facet Theory Prediction Paradigm



Facet Theory: Challenges and Prospects

Complex Systems

Facet Theory has been regarded as scaffolding for the construction of general scientific theories. Its application to the study of general action system theory has proved useful in validating the recursive "functioning mode facet" {expressive, adaptive, integrative, conservative} in diverse research domains (Shye, 1985b, 1989). Faceted Action System Theory has been found useful in organizing complex domains such as human Quality of Life by Cairns (1990), by Wozner (1991) among others; in Quality of Work Life, by Elizur and his colleagues in international collaborative studies (e. g., Elizur & Shye, 1990); and by Canter in crime research. Recent applications of this theory to organizational justice have been reported by Mevorach-Levy and Shye

(1996); to psychotherapy by Wiener (1997); and to decision making in social work by Davidson-Arad (1997). Most often, the systemic functioning-mode facet has been found to play an angular role, in accordance with theoretical predictions. But much more remains to be done in this direction. For example, the refinement of systemic modal facet structure and its representation in content spaces of higher dimensionalities.

Algebraic Representation of Content Spaces

For testing regional hypotheses in general spaces, progress must be made in overcoming the problem of algebraic facet representation. The problem is that while the geometric representation is sufficient for examining partition patterns in 2-dimensional spaces, it is inadequate for spaces of higher dimensionalities. Still missing is an algebraic formulation of the partitionability statistic; that is, algebraic criteria for the various partition patterns in n-space. The simplest problem of this kind is to find the defining algebraic characteristics of the generalized 2-dimensional simplex.

The Problem of the Range

Multiple scaling by POSAC assumes the existence of an underlying measurement space from which responses to be recorded (ranges of observed variables) are, in effect, a sample. A satisfactory theory and technique for effective sampling from this space is still missing. Thus, the specification of response categories is often arbitrary, with possible implications on the stability of the POSAC solution and scale interpretation. Designing variables that are intrinsically dichotomous, if possible, may be one way to deal with this question.

Generalizing E*, POSAC/SSA link

While the general algebraic foundations for multiple scaling by POSAC have been laid, detailed results (including the definition of E* and theorems linking the content and measurement spaces) and applications have been mostly limited to 2-d measurement spaces. Although satisfactory algorithms for the n-dimensional SA-POSAC have recently been devised and tested, further mathematical research in this field is needed. Also, greater rigor is needed in the statistical analysis of logical relations, which is at the core of multiple scaling of empirical noisy data.

The Rigor of Space Partitioning

Statisticians as well as research scientists often question the validity of lawfulness claimed in the content space. Criticism is directed both at the free-hand line drawing, which turns and zigzags so as to accommodate preconceived notions, and at the flexibility exercised by researchers in their re-interpretation of deviant variables, so as to prove their theory.

Objective and optimal space partitioning by the FSSA (Shye, 1991; Borg & Shye, 1995) and FSSAWIN programs have done much to alleviate the first problem, although the algorithm still has to be generalized to partitioning hyperplanes in spaces of dimensionalities higher than two. As for the second problem, procedures for testing the definitional reliability of items are being developed and need to be perfected. Definitional reliability testing is illustrated by Wiener (1997) in validating modes of psychotherapy and by Goldzweig (1997) in validating the distinction between creativity and inductive ability (Klauer, 1990; Shye & Klauer, 1991).

Criticism of Facet Theory is often voiced by applied scientists who, finding the notion of concept space too abstract, question the value of studying its structure. Some social scientists prefer to adhere to techniques such as factor analysis which treat observed variables as definitive or exhaustive (rather than as representative or sampling) of the studied phenomena. Others are content to employ the less formal research methods of narration and clinical observations.

Misgivings about Facet Theory are sometimes voiced also by statisticians who have been trained to worry about inferences from a sample of observed subjects to the entire population. For them, the dual task (required by the multivariate nature of behavioral concepts) of inference from a sample of variables to the entire universe of variables may seem occult and intangible. (In the natural sciences concepts are usually univariate, so that observed variables typically embody the studied phenomena themselves, rather than "sample" them.) Some believe that Facet Theory should systematically address questions of statistical inference (both from the sample of subjects and from the sample of variables), e. g., how likely is an observed partition pattern to have occurred by chance. On this issue, Facet Theory practitioners have tended to rely on their intuition and on experimental replications.

The State of Facet Theory

Aspects of Facet Theory have made their way into the current mainstream of social sciences and psychology. Examples are the cartesian design of questionnaires and the notion of facet (e. g., in Guilford's cube of intelligence) as well as the widespread use of SSA (MDS) and of POSAC, recently included in commercial statistical software packages (e. g., SPSS and SYSTAT). The use of these techniques, however, is often selective and partial, without regard to their comprehensive rationale as explicated by Facet Theory. Published examples of complete facet design, regional hypotheses, and multiple scaling, while not widespread yet, are steadily increasing.

Résumé

La théorie des facettes est une stratégie qui intègre l'analyse formelles des contenus de recherche (en utilisant comme outil le cadrage de la phrase) avec l'analyse des données intrinsèques (des procédures multivarées telles que l'analyse à facettes du plus petit espace: Faceted Smallest Space Analysis: FSSA) et le Scaling Multiple par l'analyse d'un scalogramme d'ordre spatial par coordonnées de base (Partial Order Scalogram Analysis by Base Coordinates: POSAC). L'intégration de la conception du contenu et de l'analyse des données est présentée comme étant essentielle pour la construction d'une théorie et une mensuration significative dans les sciences du comportement, qui étudient typiquement des systèmes complexes avec beaucoup, et éventuellement un nombre infini de variables. Les phrases cadrées permettent une sélection équilibrée des variables d'observation à partir de l'univers du contenu étudié.

En se focalisant sur des variables observées (les colonnes de la matrice des données) le FSSA crée une représentation géométrique de l'espace conceptuel et la répartition dans les régions permettent ainsi une investigation systématique du concept de structure. En se focalisant sur les individus observés (les rangées de matrice des données), POSAC fournit des échelles de mesure optimales pour l'évaluation des individus. Les modèles traditionnels de prédiction supposent l'existence de facteurs (les variables sous-jacentes) par lesquels à la fois les variables et les individus peuvent être évalués, ce qui détermine des observations empiriques. La théorie moderne des facettes montre que deux sortes de facteurs sont en jeu: les facettes (des comportementages) de l'espace du concept pour évaluer les variables et les échelles (coordonnées) de l'espace de mesure pour évaluer les individus, les deux types de facteurs orientent vers deux voies pour atteindre le but de la prédiction. Les résultats mathématiques dans le Scaling Multiple et particulièrement l'introduction de concept de similarité structurelle récemment défini contribue à la convergence des deux voies.

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