Contingent Claims Analysis in Corporate Finance

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Abstract

The Contingent Claims Analysis (CCA) is a general approach to analyze the stakeholders of a corporation who have contingent claims on the future, uncertain cash-flows generated by the operations of the firms. The CCA allows valuing each stakeholder's claim and also to assess the risk incurred by the stakeholders. The CCA highlights the potential conflicts of interest among the various claimholders. In this paper we review applications of CCA including valuation of various forms of debt, rating, credit spread, probability of default and corporate events like dividends, employee stock options and M&A. The CCA framework is shown to be useful to address all these financial questions. In this paper the starting point is that the value and the risk of the firm's assets are given. The future distribution of the assets' rates of return is also known and given. The focus is on the liability side of the balance sheet, i.e., the funding sources of the activity of the firm, and more generally on the financial claims of the various claimholders of the firm.

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I. Introduction

The Contingent Claims Analysis (CCA) is a general approach to analyze the stakeholders of a corporation who have contingent claims on the future, uncertain cash-flows generated by the operations of the firms. The CCA allows valuing each stakeholder's claim and also to assess the risk incurred by the stakeholders. The CCA can also help to better understand the potential conflicts of interest among the various claimholders, and hence to better address corporate governance issues.

In this paper the starting point is that the value and the risk of the firm's assets are given. The future distribution of the assets rates of return is also known and given. Usually, for computational convenience, it is assumed that the value of the firm's assets is lognormally distributed, and, therefore, the assets rates of return are normally distributed, in continuous time. The fundamental assumption is that the assets can be analyzed using the CAPM model of Sharpe (1964) and Lintner (1965a and 1965b), as it is well documented in Fama and Miller (1972)'s book. This framework can be extended with the APT and the multi-index analysis. The focus is on the liability side of the balance sheet, i.e., the funding sources of the activity of the firm, and more generally on the financial claims of the various claimholders of the firm.

The starting point is to recognize that a firm, like a coin, has two sides, i.e., the asset side and the liability side. They are identical in overall value and risk. The balance sheet of the firm is well balanced in economic terms, i.e., at each instant the value of the assets is equal to the value of equity and the liabilities. There cannot be any gap between the two sides of the balance sheet. Hence, the value of the firm and its risk, are fully divided among the claimholders. The driving force of corporate events is usually on the assets' side while the liability side reflects the changes and adjusts itself. Both changes in the value or in the riskiness of the assets have an impact on the liabilities. The CCA provides a theoretical framework that dynamically connects the two sides of the balance sheet. Many questions, like analysis of various forms of debt, dividends, effect of corporate tax codes, or M&A as well as many other corporate events can be analyzed in the framework of CCA.

The firm is like a pizza with each claimholder having a claim on the pizza that is fully consumed by all the stakeholders. The conflicts of interest are on who gets its slice first, and how big is the size of the slice. The stakeholders favor a larger size of the pizza, i.e., the size of the firm, as long as it increases their share. Many corporate decisions have an impact on the distribution of the value among stakeholders and not only on the size of the whole firm.

When we analyze the value of the stakeholders' specific claims, we take as given all the factors that drive the value of the firm. This is in the spirit of Modigliani and Miller (M&M) (1958) analysis of the firm, in an economy with no corporate taxes. They show that under assumptions of perfect capital markets (PCM) the value of the firm is not a function of its capital structure. Actually, the CCA premise is that the value of the assets of the firm is fully distributed among all the claimholders. Merton (1977b) proves that the M&M propositions also hold for the case of bond default.

In this approach, if we add corporate taxes, the government, as a tax collector, is also a stakeholder and the tax claim should be endogenized; it implies that the assets of the firm should be considered on a before tax basis. Not only is the CCA consistent with M&M as it asserts that the pizza is fully eaten, and equity holders are entitled to get the residual value as long as it is non-negative. Again, the firm is like a coin that has two identical sides in terms of overall value and risk. Hence, there is no need to prove the M&M theorems.

Another important distinction between the two sides of the balance sheet (defined in economic rather than accounting terms) is that while we commonly assume that the value of the assets follows a stationary lognormal process, the value of the claims of the stakeholders doesn't necessarily follow a stationary distribution. Therefore, asset pricing models like the CAPM cannot be used to price the non-stationary claims of the stakeholders as shown by Galai and Masulis (1976). In the simple case of a firm, financed with equity and a pure discount bond, as analyzed by Merton (1974), the option pricing model (OPM) can be used to price simultaneously equity and debt.

In the literature on valuation of liabilities many assumptions are made, either explicit or implicit. The first set of assumptions is related to the asset side, e.g., the nature of assets' value uncertain behavior, and, the use of proceeds if money is currently raised. The second set of assumptions relates to the equity and liability side of the balance sheet, e.g., the current capital structure and whether it is going to change. Authors, dealing with warrants valuation, for example, used different set of assumptions: some analyzed companies with equity and warrants only, and for such companies, whether dividends are expected to be paid out or not. And even then, we can ask whether dividends are fully expected or not, remembering that dividends, per se, reduce the asset size of the firm. Also, how the proceeds from the potential exercise of the warrants can affect the valuation models whether the proceeds are used for a scale expansion of the firm, to retire debt, or to pay dividends (or, any combination of the above).

One very important assumption concerns corporate taxes. Many papers (e.g., Galai and Masulis (1976), Merton (1974)) follow M&M (1958) paper and assume the corporate tax rate is zero. However, other authors (e.g., Brennan and Schwartz (1978), Leland (1994)) assume the existence of corporate taxation. And even in the tax case some assume it is a full offset tax while others assume only partial offsetting or, no offsetting. Also, some assume the coupon payment is fully deducted as an expense as compared to the effective yield, taking the discount or premium on the debt value into account.

II. Foundations: The Pricing of Debt and Equity

Black and Scholes (1973), Merton (1973, 1974) (hereafter referred to as BSM) introduced the CCA to the valuation of debt and equity. In their approach, also referred to as the "structural approach": (i) the value of liabilities is derived from assets, (ii) assets follow an exogenous stochastic process, (iii) liabilities have different priorities (i.e., senior and junior debt), and (iv) default is endogenously determined. A firm defaults at maturity of its single, pure discount debt when the value of its assets drops to a level below the face value of its liabilities. In a regime of limited liability, the firm's shareholders have the option to default on the firm's debt. Equity can then be viewed as a European call option on the firm's assets and risky debt as a default-free debt minus an implicit put option.

Merton (1974) initially considers a firm with the simplest capital structure consisting of equity, *S*, and a zero-coupon debt, *B*. This model assumes that the firm defaults only at the maturity date, *T*, of the firm's outstanding debt, when the market value of the firm's assets, V_T, falls below the face value of the debt, *F*. When it does occur, debt holders take over the firm's assets without incurring any distress cost and shareholders are wiped out, i.e., $S_T = 0$. If there is no default, i.e., $V_T \geq F$, debt holders redeem the face value of the debt. Equity can then be seen as a European call option on the firm's assets with a strike price equal to the face value of debt, *F*, expiring at the debt's maturity, *T*. Debt holders have a position equivalent to a riskless bond and a short position in a European put option, *P,* written on the assets of the firm. The strike price of *P* is equal to the face value of debt, *F*, and it has the same maturity, *T*, as the debt. Following BSM assumptions, the value of equity can be modeled as follows:

(1) $$S = VN(d_1) - Fe^{-rT}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V}{Fe^{-rT}}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

(2) $$P = -VN(-d_1) + Fe^{-rT}N(-d_2)$$

where σ is the standard deviation of the rate of return of the firm's assets, and *r* is the continuously compounded default-free interest rate, and $LR = \frac{Fe^{-r}}{V}$ is the quasi leverage ratio of the firm.[1] P denotes the value of the put option on the firm's value, and it fully values the credit risk of the corporate debt. See also Crouhy, Galai and Mark (1998).

Figure 1 shows the value of equity and debt at the maturity date of the zero-coupon debt. The value of S_T is equivalent to the payoff of a call option on the bank's assets with strike price of F, the face value of the zero-coupon debt and maturity T. The

[1] Usually the leverage ratio is defined in market value terms of $\frac{B}{V}$, where B is the current value of the debt and Fe^{-rT} is the present value of a default-free debt.

terminal value of debt is equivalent to a risk-free debt with a terminal value of F across all possible values of the bank's assets, V_T, and a short position in a put option on the bank's asset with a strike price of F and maturity T. Figure 2 shows the value of equity and debt before the maturity of debt.

The model illustrates that credit risk and its cost is a function of the riskiness of the assets of the firm, σ, the maturity of debt, T, and the risk-free rate, r. The higher r is, the less costly is the credit risk. The cost is a homogenous function of the quasi leverage ratio, LR.

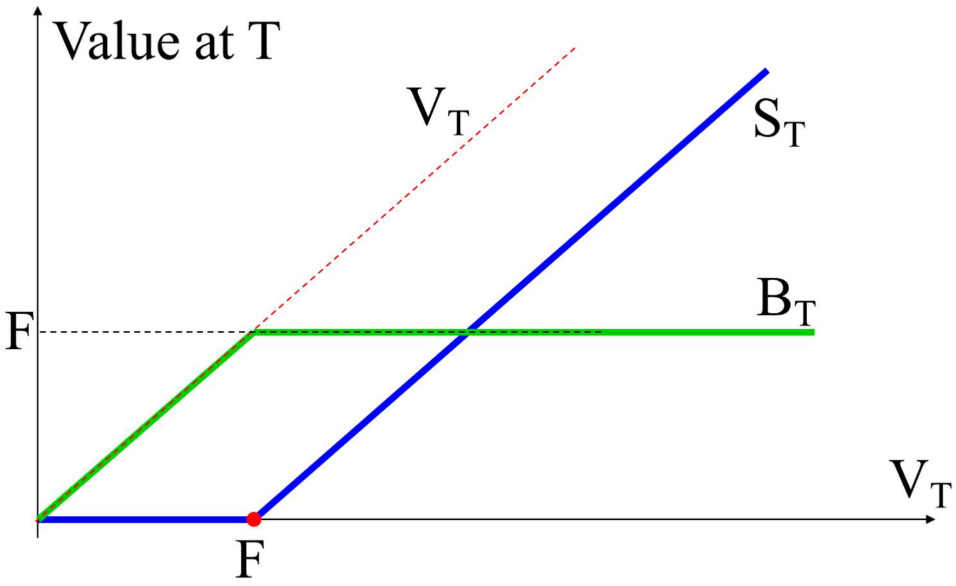

Figure 1. Value of equity and debt at the maturity date of the debt

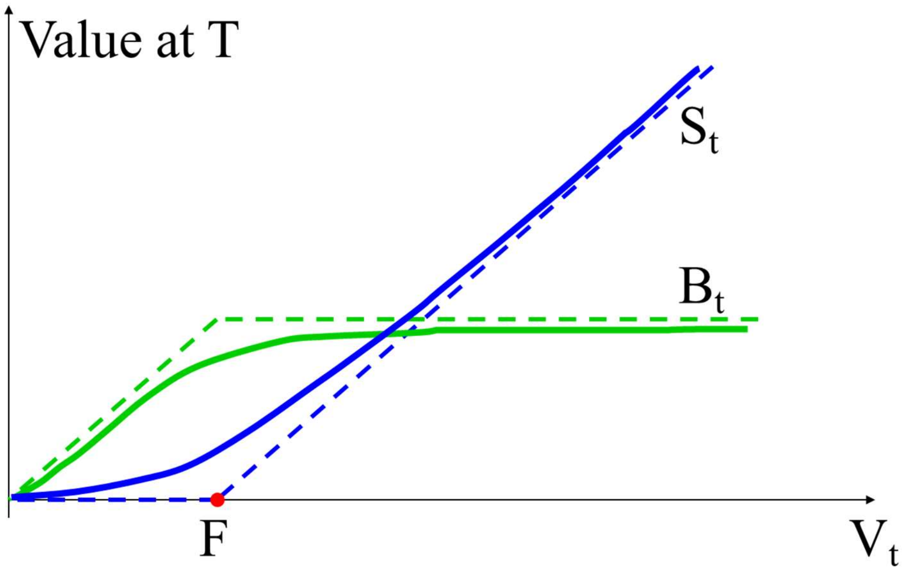

Figure 2. Value of equity and debt prior to the maturity of the debt (dashed lines depict the same values at maturity T).

In Merton's model, the value of the corporate debt is equal to the value of a risk free bond minus a put option, P, on the assets of the firm:

(3) $$B = Fe^{-rT} - P$$

The present value of credit risk, i.e., the spread between the risky debt and an otherwise identical risk-free debt is simply the value of this put option, from which it follows that:

$$B = Fe^{-yT}$$

with

$$y = r + \pi$$

being the yield to maturity on the corporate debt, where π is the credit spread for duration T:

(4) $$\pi = -\frac{1}{T} ln\left[N(d_2) + \frac{V}{Fe^{-rT}} N(-d_1)\right]$$

The credit spread can be computed as a function of the quasi leverage ratio, LR, the volatility of the underlying assets, σ, and the debt maturity, T.

Within the Black-Scholes framework, we can derive the credit spread, the "risk-neutral" default probability and the expected discounted recovery rate. All these values are endogenously determined and depend on both the firm's capital structure and the dynamics of its asset value.

Crouhy, Galai and Mark (1998) show that the value of the put option implicit in default can be decomposed into the expected shortfall, conditional on the firm being bankrupt at maturity and the expected probability of such an event. The risk-neutral probability of default in Merton's model is given by $N(-d_2)$ as defined in expression (1). Crouhy and Galai (2018) examine the relation between the risk-neutral and physical, observed probabilities of default, noting that the two can differ substantially. While the risk neutral probability is based on the riskless interest rate, r, the physical probability has the same structure as $N(-d_2)$ but is based on the expected rate of return the firm's assets.

Merton's basic model can be extended in many ways. For example, allowing for stochastic interest rates, a more complex capital structure with different levels of subordination, payout policy (dividends, coupons, etc.), different default triggers, the reorganization process in case of default, etc.

Black and Cox (1976) is the first significant extension of Merton's model. Rather than adhering to the strict European option scenario, they allow default to occur prior to the bond's maturity by introducing a "safety covenant" which allows debt holders to trigger default and take over the firm's assets when the asset value falls below a predetermined threshold known as a "default boundary" at any time before maturity.

The timing of default is no longer certain, and occurs the first time the asset value hits the default boundary. Equity is no longer a European call option on the firm's assets, but rather a "down-and-out" option on them. The introduction of safety covenants increases the probability of default and makes debt more expensive, which translates into lower credit spreads.

Galai and Masulis (1976) combine the CAPM and the option pricing model to derive the value of equity and its systematic risk. Based on equation (1) and the systematic risk of the firm's assets, β_V, it is shown that even when β_V is stationary, the systematic risk of both equity, β_s, and debt, β_B, will be non-stationary; the former will be higher and the later smaller than β_V. Corporate events, such as mergers, spinoffs and investments in new risky projects are also analyzed in this paper, showing how such corporate events can affect the value and riskiness of the firm equity.

Mason and Merton (1985) consider the application of CCA to capital budgeting and financing decision of large-scale projects. Traditional discounted cash flow methods underestimate the value of projects because, among other reasons, they ignore the value of "flexibility" options such as the option to expand or contract the scale of a project.

We discuss more elaborate extensions of Merton's model in the following sections of the paper.

III. Pricing of Warrants, Convertible Securities, Preferred Stocks and Employee Stock Options

The simple model of Merton of a firm with equity and pure discount debt was extended to include other contingent claims. One simple extension is to differentiate between senior and junior debt. This can be done in the standard framework as shown in Figure 3 below.

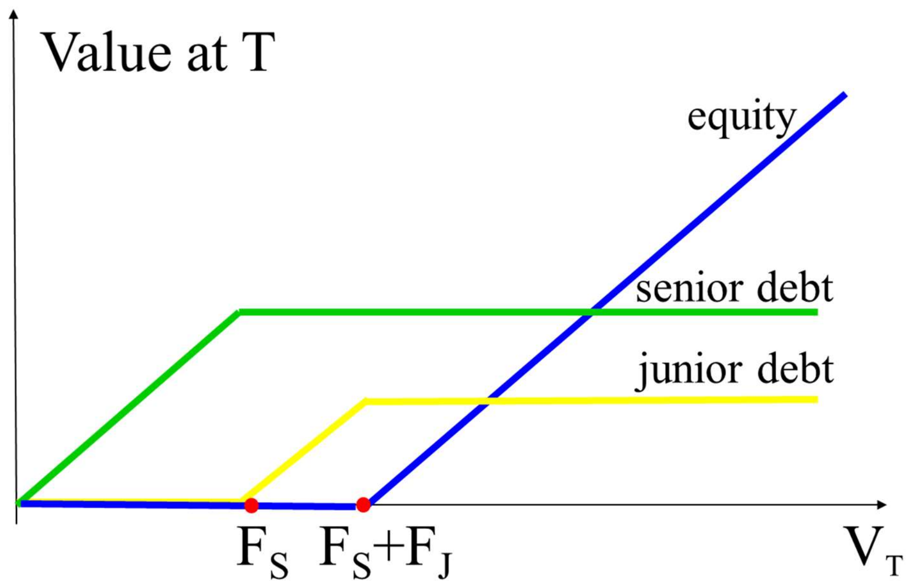

Figure 3. Senior and junior debts as contingent claims.

Another extension is to introduce warrants into the capital structure. Warrants are similar to call options with the difference that they are issued by the corporation. Once the warrants are exercised, there is an injection of new funds (i.e., the exercise price multiplied by the number of warrants) to the firm. At the same time the number of outstanding shares increases causing dilution of existing shareholders. The initial papers on pricing warrants assume that a firm has equity and warrants only in its capital structure. Warrants are similar to issuing equity and supplying loans to the warrant holders. It creates negative leverage.

Galai and Schneller (1978) propose the first pricing model for European warrants, which takes the potential dilution effect from warrant exercise explicitly into account in a firm's valuation. It is shown that the value of an option on a share in a firm with warrants is equal to the value of the warrant. Also, the value of the warrant on one share, W, is equal to the value of a call on one share in an identical firm with no warrants, C, adjusted by the dilution factor, q, where $W=C/(1+q)$.

Several extensions of this model have been proposed. Emmanuel (1983) focuses on the optimal exercise strategy for American warrants, given the fact that the exercise of some of the issued warrants has an impact on the value of the firm and the remaining warrants, since exercised warrant dilute the positions of the existing equity holders.

Constantinides (1984) analyzes optimal exercise strategies for both monopolist and competitive American warrant holders, also for the case of dividend distribution. He argues that, in the presence of a single large warrant holder and a competitive trader, the dominant player obtains a lower value than competitors because competitors free ride on the dominant player's strategy by delaying exercise. Spatt and Sterbenz (1988) elaborate on Constantinides and examine sequential exercise strategies by warrant holders and potential gains from hoarding warrants. They consider the impact of a firm's scale expansion on optimal warrant exercise strategy.

Crouhy and Galai (1994) analyze warrants and equity pricing in a firm financed by both equity and debt, and assume that the proceeds from warrant exercise are reinvested in the firm, in a scale expansion. The warrants are of the European type. The issuance of warrants affects the capital structure of the firm and its size when the warrants are exercised. The effect of a potential transfer of wealth from equity holders to debt holders at the time of exercise is analyzed. When warrants are exercised and the proceeds are reinvested in a scale expansion project, the probability of default may decrease. The authors conclude that debt is likely to appreciate in value at the expense of equity.

Finally, Simonato (2015) generalizes Crouhy and Galai (1994) by assuming that the maturity of the warrants differs from debt maturity. He also assumes pricing errors in the equity model. This may have a significant impact in the determination of the non-observable asset value of the firm that is simultaneously derived with the asset return volatility using numerical techniques. A calibration approach is proposed that produces stable results.

Ingersoll (1977a) is one of the pioneering works on the valuation of hybrid securities such as convertible bonds. He notes that there are similarities with the pricing of warrants, given shareholder dilution when bonds are converted to equity.

The dilution effect can be schematically illustrated by the following modification of the original model.

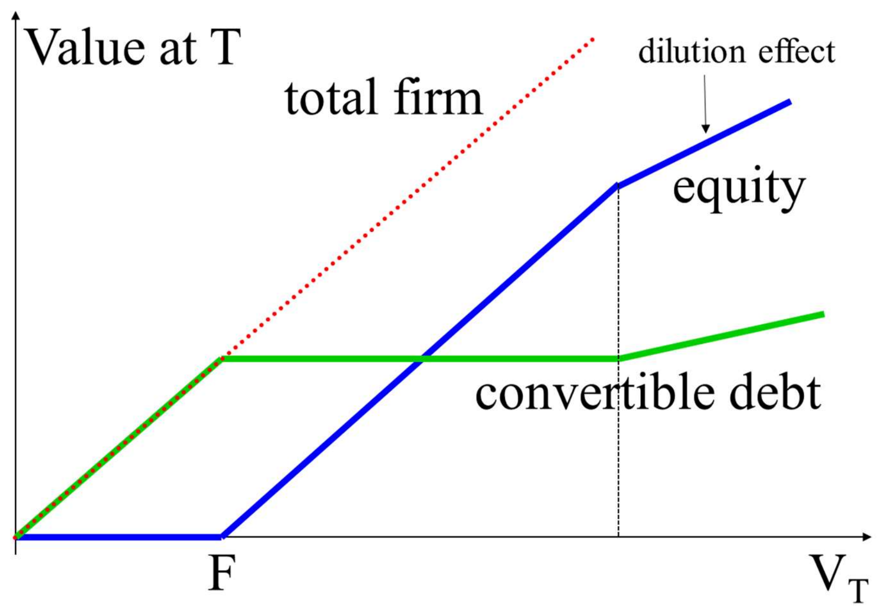

Figure 4. Convertible debt and the dilution effect

Ingersoll notes that the American feature of the conversion option is important and ignoring it can lead to undervaluation. Ingersoll develops a method for determining optimal call and conversion strategies and affirms the validity of the Black–Scholes model for pricing both callable and non-callable convertible bonds, as well as convertible preferred stock. One important assumption in Ingersoll model is that the convertible issue is the only senior debt of the firm. The call policies of convertible-issuing corporations systematically differ from the optimal policy suggested by CCA. Optimal call strategy for the issuer according to the CCA model is to call a convertible bond at the point when its conversion value equals the call price. Even after taking the call notice and underwriting costs into account, Ingersoll is still unable to reconcile observed behavior with the model prescription. He concludes that convertibles should sell at a premium over the model price.

Ingersoll examined the conversion policies of 179 issues between 1968 and 1975 and found that the call on all but nine were delayed “too long” with respect to model prediction. In his second paper (1977b) he relaxed some of the simplifying assumptions in an attempt to explain the deviations. For example, he takes into account the option of the debtholders to redeem the bond rather than convert it. Also he considers the effect of the “notice period”, between the date of announcement and the call date.

While Ingersoll concentrates on deriving closed-form solutions for the value of a callable convertible bond for several special cases, Brennan and Schwartz (1977a) offer a more general algorithm for determining the value of callable convertible bonds. They also assume a firm with equity and convertible debt only. Since they employ numerical methods to solve the differential equation, they can incorporate issues, such as discrete coupon payments on the bond, discrete dividend payments, American features of the conversion option, the right of the firm to call the bond, and the right of the bondholder to convert or redeem the bond.

In Brennan and Schwartz (1988) they extend the analysis of convertible bonds to the case of uncertain interest rates and also analyze a firm with equity, senior straight bond and a convertible bond.

Dorion, Francois, Grass and Jeanneret (2014) focus on the agency conflict between shareholders and bondholders. Since equity is essentially a call option on the firm’s assets and because option values increase with risk, shareholders have an incentive to transfer wealth away from bondholders by unexpectedly increasing risk once debt has been issued. This paper conforms with existing literature, which has shown that properly designed convertible debt can reduce this agency conflict, since conversion forces existing shareholders to share the firm’s upside potential with bondholders. The same point was also highlighted by Brennan and Schwartz (1988).

Another area where Contingent Claims Analysis became important in corporate decisions is related to stock based compensation. This type of compensation was in use for senior management and key personnel for a long time. However, in recent years the use of stock based compensation was extended to a much wider group of employees. It is widely accepted in High-Tech companies but recently it became popular in other types of businesses as well. Most of the companies in the S&P 500 offer stock options to their top employees (see Murphy (1999) and Hall and Murphy (2002)). The most popular forms are RSU (restricted stock units) and ESO (employee stock options).

Stock based compensation often combines financial and non-financial features. Typically, eligibility is established over time and is conditioned on such factors as staying at the company and often some company's performance conditions. This is called vesting.

Some restricted stocks are given as new shares, while others are in a form of a phantom providing monetary compensation that reflects the stock price or the stock price increase over a certain threshold.

Some corporate events can accelerate vesting, while other events (like leaving a company) can lead to a significant shortening of the remaining time to maturity. Typically, an employee leaving a company has a short period to exercise the vested options.

Some forms of stock based compensation are adjusted for dividends (typically by reducing the strike price). There are also more complex forms of ESO where the strike price is set not as a fixed value, but relative to a benchmark (market or competitors).

Many employees end up with a large exposure to their employer. They tend to exercise the options prior to maturity as soon as the stock price becomes high enough. This behavior is often suboptimal from a pure financial point of view but they are often dictated by liquidity needs or risk aversion and lack of marketability. This feature is often modeled as an exotic option with a barrier.

Till 2004 the accounting rules treated various forms of stock based compensation on the basis of the "intrinsic value". As a result, most companies reported them at zero value, while this form of compensation was very important for the employees and eventually for the shareholders. Since the new regulations (FAS 123R and IFRS 2) became mandatory, all public firms are required to measure and report the fair value of the stock based compensation.

Stock based compensation policies are typically approved at the level of shareholders (board or general meeting of shareholders) since they are paid not by the firm but by the shareholders directly in a form of potential future dilution. There are many features of stock based compensation that are different from standard financial contingent claims, like vesting, forfeitures, early voluntary exercise, performance criteria and more. As a result, the straightforward use of the Black-Scholes formula is in most cases inappropriate. The features described above make the stock based compensation strongly path dependent and often there is no simple method of valuing them. Advanced methods of valuation of contingent claims were developed to include these special features. The most popular methods are based on binomial trees, Monte Carlo simulations and advanced analytical formulas. In Cvitanic et al. (2008), an analytic option pricing formula is developed, which incorporates special features of employee stock options (ESO), like vesting, forfeitures and voluntary exercise. This analytic formula became particularly important after accounting standards adopted fair value reporting of stock based compensation.

IV. Valuation of Corporate Debt

Merton's primary insight (1974) is that corporate debt, in the form of a zero coupon debt, is equivalent to a riskless government debt with similar terms plus a short put option written on the value of the firm's assets with a strike price equal to the redemption value of the firm's debt. The put option reflects the economic value of default.

In Section II we already considered the first extension of Merton's model by Black and Cox (1976) who introduce a safety covenant which allows bond holders to trigger default before the debt maturity. In this section we consider additional extensions to the basic models of Section II.

IV.1 Valuation of corporate debt and default triggers

Geske (1977) extends Merton's model, who derives the value of a zero-coupon bond, and derives the value of a bond for risky coupon-paying debt, using the compound option pricing approach. The firm can default at any coupon paying date if the value of the firm's asset falls below the coupon at that date.

Leland (1994) extends Merton (1974) and Black and Cox (1976) to account for taxes, bankruptcy costs, continuous coupon payments and bond covenants. This model allows determining the optimal capital structure, the debt capacity, and credit spreads. Leland derives the optimal default boundary and the value maximizing optimal capital structure. Leland's endogenous default boundary is a generalization of Black-Cox (1976).

Mason and Bhattacharya (1981) extend the analysis of risky corporate bond pricing by introducing safety barriers based on debt covenants and also consider the case when the value of assets has jumps. They illustrate the effects on the pricing of bonds with the binomial example.

Leland and Toft (1996) expand Leland (1994) by seeking an endogenous solution for capital structure and debt maturity. Optimal debt maturity is determined by striking a balance between the tax advantage of long-term debt and the disadvantages engendered by bankruptcy and agency costs. These deviations from the assumptions of M&M lead to determining an optimal capital structure.

Franks and Torous (1989) document the fact that in many bankruptcy settlements, junior claimholders receive more than expected under the pure absolute priority rule. Mella-Barral and Perraudin (1997) investigate strategic debt service, whereby equity holders under distress renegotiate the terms of the debt. They introduce another variable to the complex stakeholders' relationship, i.e., the bargaining power of shareholders. The major result is that strategic debt service can account for the premium on risky corporate debt.

Francois and Morellec (2004) extend Leland and Toft (1976) model to allow for a delay between the default trigger and the liquidation of the firm's assets. They examine whether the so-called exclusivity period defined in Chapter 11 of the U.S. Bankruptcy Code impacts asset prices, the choice of the capital structure and the decision to default. Liquidation occurs when the total time that the firm's assets spend under the distress threshold exceeds a predetermined grace period. They then look at the ex-ante pricing of corporate liabilities. Again, the model deviates from Merton's assumption of a clear fixed date on which bankruptcy can occur. It considers a more realistic case of a dynamic process where the firm enters a bankruptcy state from which it can "escape" due to better performance later on, while still under the bankruptcy procedure.

Moraux (2002) complements Francois and Morellec (2004) and introduces "memory" for the cumulative time the firm is in bankruptcy procedure. Galai, Raviv and Wiener (2007) extend the two previous models by taking into account the effect the severity of the distress event has on the decision to liquidate the firm's assets, in addition to the cumulative time the firm is in distress. The intensity of distress affects the valuation of debt. But there is no closed-form solution for this more complex formulation.

IV.2 Taxation and Accounting

Brennan and Schwartz (1978) employ the CCA to relate the value of a levered firm to the value of an unlevered firm, the amount of the debt, and the time-to-maturity of this debt. They show conflicting effects stemming from an increase in leverage that increases tax savings as long as the firm is alive, but also increases the probability of default. An optimal capital structure can be derived through the analysis of the impact the trade-off between tax savings and default risk has on the firm value.

Galai (1998) introduces the government, in its role as a tax collector, as another stakeholder in a firm with equity and pure discount bond. Under the assumption of a full loss offset tax, it can be shown that the government owns a futures contract on the corporation. By allowing for the adjustment of interest rates to account for default risk it is shown that in this case, there is no optimal capital structure. He shows that the M&M propositions can be rederived in this extended framework from the pre-tax scenario. The implied cost of capital of the government claim can be derived. When taxes are introduced, the value of debt is affected by the nature of the tax rules (e.g., whether it is full-loss offset tax or partial one) and the priority rules of taxes and senior debt.

Barth, Landsman and Rendelman (1998) adopt the CCA approach to assess the fair value of corporate bonds, taking into account different features of the bonds including conversion, call, put and sinking fund features as required under FASB accounting standards. Because of all the dependencies among the valuations of these features, finding the fair value of each component is complicated and depends on the order of the valuation procedure. Still, they find the CCA to be consistent with the

FASB requirements in corporate bonds fair value estimation.

IV.3 Stochastic interest rates

Shimko, Tejima and van Deventer (1993) extend Merton's (1974) model for risky debt valuation to the case of stochastic interest rates. Through their model, they examine the combined effect of term structure and credit variables on debt pricing.

Longstaff and Schwartz (1995) extend the Black and Cox (1976) model by incorporating both default risk and interest rate risk into the analysis and by allowing for deviations from strict absolute priority, which empirical evidence indicates is rarely adhered to by distressed corporations. They derive closed-form solutions for risky coupon bonds and risky floating-rate debt.

Briys and de Varenne (1997) extend the previous model to allow for a variable default barrier, that is the discounted value of a fixed quantity. When this barrier is crossed, bondholders receive an exogenously specified fraction of the remaining assets which, unlike in other models, cannot exceed the value of the firm upon default.

Duffie and Lando (2001) extend the previous literature to the case of incomplete secondary-market information regarding the credit quality of the firm's debt. They solve for optimal capital structure and default policy, and derive the conditional distribution of the firm's assets, given incomplete accounting information, along with the associated probability of default, default arrival intensity and credit spreads.

IV.4 Moody's KMV model

The first commercial product based on Merton model was proposed by KMV, in 1989. The initials KMV stand for the first letter of the last names of Stephen Kealhofer, John McQuown and Oldrich Vasicek, who founded the KMV Corporation. (Kealhofer and Vasicek are former academics from U.C. Berkeley.) In 2002 Moody's Corporation acquired KMV and it has since been referred to as Moody's KMV.

Moody's Analytics develops the EDF, or Expected Default Frequency, model to measure default probabilities for publicly traded companies over a 12 month horizon

While Merton provides an analytic model, the KMV model is more heuristic and empirically based. The papers by Vasicek (1984), Sundaram (2001), Kealhofer (2003) and Crosbie and Bohn (2003) discuss several issues related to the practical implementation of Merton's model, as reflected in the KMV approach.

The KMV approach is based on estimating an index called the distance to default (DD) of the firm (see Figure 5). DD is the number of standard deviations between the mean value of the distribution of the asset E(V) at the one year horizon, and a threshold, the default point (DPT). DPT is set somewhat arbitrarily at the par value of current liabilities, including short-term debt, to be serviced over the time horizon, say one year, plus half the long-term debt (where the debt value is taken from the pro-format balance sheet). Empirically it was observed that default occurs when the asset value falls in between total debt and short-term debt:

(5) $$DD = \frac{E(V) - DP}{\sigma}$$

where:

STD = short-term debt

LTD = long-term debt

DPT = default point = STD + ½ LTD = distance between the mean value of the asset value in one year, E(V) and the default point, expressed per unit of standard deviation of asset returns, σ.

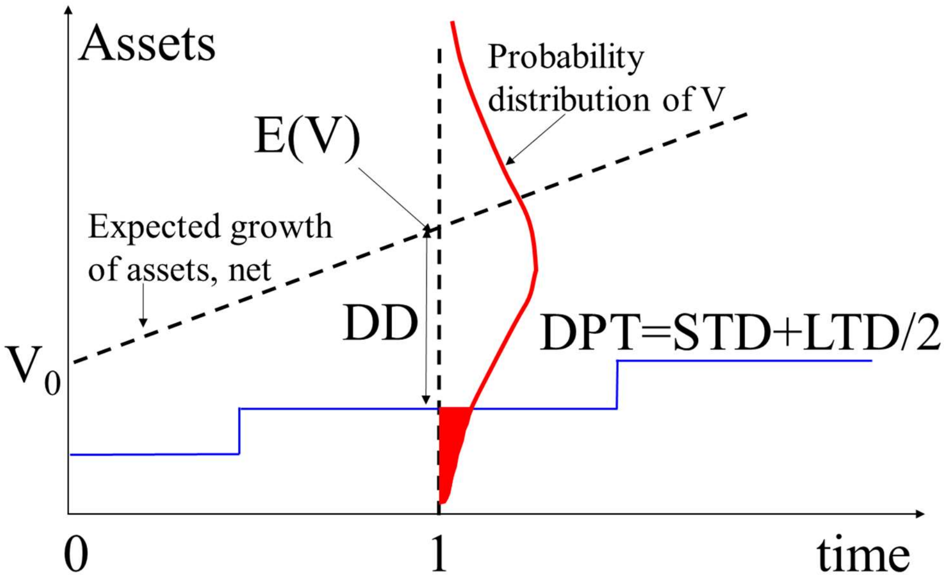

Figure 5. Distance to Default (DD)

The next step is to map the distance to default, DD, to the probability of default for a given time horizon, say one year. This is an empirical approach based on thousands of observations of companies that both defaulted and remained alive in the past. The proxy for the default probability of default over, say a one-year horizon, called Expected Default Frequency or EDF, is the proportion of firms with a given DD at one point in time that defaulted within one year.

The EDF is a forward looking estimate of default probabilities and anticipate quicker the deterioration of the credit standing of companies than traditional ratings that are more stalled since they are based on accounting figures disclosed with delay.

V. Applying CCA to the Valuation of Various Securities and Contracts Issued by Corporations

Firms acquire knowhow and other intellectual property for which they pay royalties that may take various forms. Contingent claims can also apply to corporate governance, concerning alternative ways to share voting power in major corporate decision-making. This section highlights several cases in which the use of CCA goes beyond the assessment of credit spreads and the valuation of equity and debt.

Jones and Mason (1980) introduce loan guarantees by the government and analyze the effect of the guarantees on the value of non-callable coupon debt, junior and senior non-callable debt and callable coupon debt. They provide numerical results under two regimes: full and partial government guarantees, and show the value of the unguaranteed debt, the value of the guarantee and the sum of the two components.

Cooper and Mello (1991) show that in perfect markets, swaps with default risk generally leads to a transfer of wealth from the shareholders to the debtholders. This result poses the question why shareholders accept debt to be swapped if it has a negative impact on their wealth? Agency considerations are put forward. Swaps allow the borrower to benefit of private information regarding future creditworthiness, since the swap spread incorporates a premium to compensate the swap counterparty against default risk.

Galai and Ilan (1995) apply the CCA to the economic evaluation of remuneration from patents and licensing activity. The licensors become stakeholders in the firm, and the value of their claims can be derived under different licensing schemes. They derive a fair transaction price for the technology transfer between licensors and the licensees based on Merton's (1977a) approach to the valuation of deposit insurance (see Section VIII). The elasticity of demand for the licensed product will play a major role in determining the royalty scheme.

Das (1995) models the value of derivative instruments when the underlying asset is credit risk. Such derivatives were initially proposed in 1992 by the International Swaps and Derivatives Association (ISDA). He proposes a way to strip credit risk from a bond's total risk by employing a stochastic exercise price for the credit risk option, and shows that the value of the credit risk option is the expected forward value of a put option on a risky bond with a credit level-adjusted exercise price.

Galai and Wiener (2008) propose a new approach to the dynamic representation of various groups of stakeholders on corporate boards and general meetings. This approach is based on the CCA, dividing voting rights proportionally to the marginal value of the corresponding contingent claim, i.e., the "delta" ratio of each stakeholder. Of course all the deltas must sum to 100%. Such an approach can mitigate some of the potential conflicts of interest among the stakeholders and in particular, between shareholders and bondholders.

Goshen and Wiener (2004) develop an option-like valuation of contractual freeze-out clauses — an important legal right given to majority shareholders to compel minority shareholders to liquidate their shares under certain circumstances. Minority shareholders have an offsetting right to demand an entire fairness test. The model shows how to incorporate these features into equity pricing.

VI. Dividends as Contingent Claims

This section addresses how dividends can be treated as contingent claims on the corporation. Dividends are a component in equity valuation, and as such, constitute a claim on a firm's future cash flow. Dividends are also considered in several papers appearing in other sections (see, for example, Black and Cox (1976) in Section II).

Garbade (2001) devotes his book on corporate securities pricing to analyzing how various sets of assumptions affect value for different stakeholders. The first part of his book maintains the assumptions of the perfect priority rule, i.e., that all the firm's net income is paid to the original debt holders, and only after paying the debt in full, distributed to other claim holders. Garbade articulates the valuation of debt and equity for a firm that pays periodic coupons to debt holders as well as cash dividends to shareholders. Initially, the dividend is a function of the value of the firm, but other forms of dividend distributions are considered.

Galai and Wiener (2019) show how, in a Merton-type model with bankruptcy, the dividend policy impacts the values of equity and debt as well as credit risk. When dividends are paid from assets (including cash), there is a potential for a conflict of interest between shareholders and debtholders. The model presented allows for a quantitative setting of restrictions on dividends and gives a useful tool to support dividend payments or preclude a distribution when such could otherwise jeopardize the firm. The implications of the CCA approach is compared to the implications of the Signaling Approach, and the Smoothing Approach. It is shown that the Miller-Modigliani irrelevance of dividends theorem must rely on more assumptions than in the original paper.

Galai and Wiener (2018) highlight potential conflicts of interest between equity holders and debt holders surrounding dividend payments. If dividends are withheld, even when the company has the resources, the value of debt may increase, diverting value away from equity. If too much is paid in dividends, there may be a shift of value from debt to equity. Beyond the protection provided by debt covenants, expectations determine the outcome. These arguments are presented in a simple binomial, two-period framework.

The effect of a dividend on the probability of default can be illustrated by the following figure.

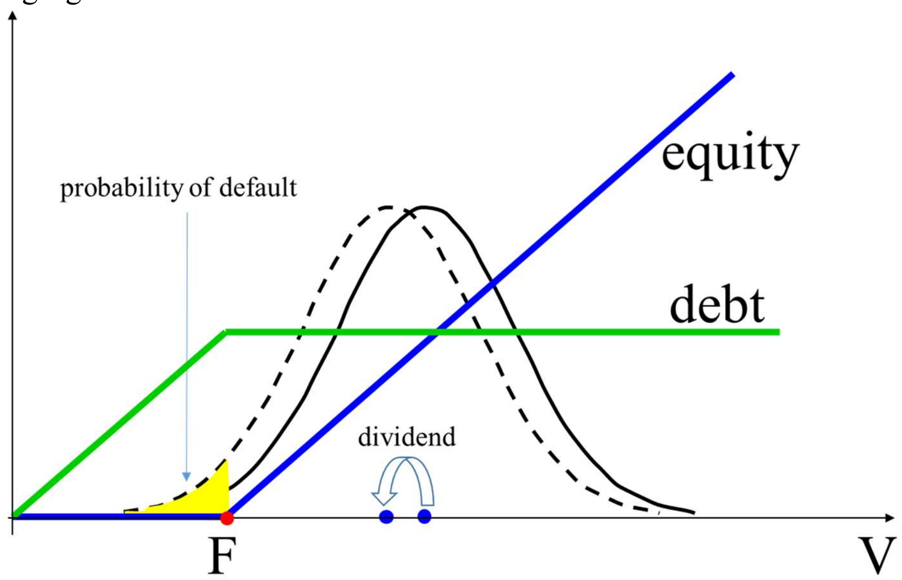

Figure 6. Dividend and its impact on the probability of default

As shown in Figure 6, dividends reduce the assets of the firm and shift the whole distribution function of V. As a result, the probability of default increases if the rest of parameters are the same.

VII. Empirical Studies

In this section we present empirical papers, which directly or indirectly test CCA models, particularly for the pricing of corporate bonds and yield spreads. The major empirical question is whether the contingent claims approach, which accounts for asset volatility, values and the probability of default, can better explain the yield observed in the market place, when compared with more traditional regression models for bond pricing.

Jones et al. (1984) test the predictive power of the CCA model on a large data set, which includes both investment grade and lower grade corporate bonds. They find that, for non-investment grade bonds, the CCA model has incremental explanatory power over the naıve model. They also suggest that the introduction of stochastic interest rates as well as taxes would improve the model's performance.

Odgen (1987) tests the CCA model on newly issued bonds of firms with very simple capital structures. He finds that two measures of default risk, the firm's standard deviation and its leverage ratio, explain 78% of the bond's rating.

Franks and Torous (1989) note that one cannot ignore the empirical significance of bankruptcy protection, such as under Chapter 11 of the U.S. Bankruptcy Code, which is assumed away in Merton's (1974) model. Disregard for protection under the law can lead to a downward bias of the premium on risky debt.

Sarig and Warga (1989) use pure discount bonds to show that their time profile estimates of default risk premia resemble the time profile obtained from Merton's (1974) model.

Acharya et al. (2000) present a cash-flow based model of corporate debt valuation that incorporates two novel features. First, they allow for the separation and optimal determination of the firm's debt-service and dividend policies. Second, they consider the possibility of raising resources through the costly issuance of new equity. Their numerical analysis indicates that endogenizing dividend policy and allowing for equity-issuance costs can substantially enhance the model, while ignoring these factors could introduce nontrivial biases into the valuation.

Gemmill (2002) argues that empirical tests of theoretical models are hampered by the complexity of real-world bonds — which typically include coupons, calls and sinking funds — and by increasingly complicated and changing corporate capital structures. In this paper, he utilizes a new database of zero-coupon bonds issued by closed-end funds in the UK, which have very simple capital structures and for which the asset and liability values are transparent. He finds that, on average, model spreads and market spreads are of similar magnitude. Similar to previous research, market spreads are high (relative to model spreads) for low-risk bonds and bonds nearing maturity. On the whole, the results are surprisingly supportive of Merton's model and suggest that it is important to allow for expected changes in leverage when computing credit spreads.

Ericsson and Reneby (2005) tackle the issue of estimating corporate asset value and riskiness, the two parameters that are unobservable but are essential in the practical implementation of structural models. They perform a simulation to evaluate an applicable maximum likelihood method and find strong support for their proposed approach.

Bruche (2007) describes how all available price data (equity prices, bond prices, and possibly credit derivative prices) can be used in the estimation of structural models, and illustrates that using bond price data in addition to equity price data greatly improves estimates. The issue of possible noisy data and/or model error is also discussed.

Davydenko and Strebulaev (2007) relate empirical credit spreads to various measures of capital structure complexity and shareholder characteristics which influence strategic debt service and distress bargaining. They empirically demonstrate how the negative effect of strategic default on debt prices more than offsets gains from a reduction in liquidation costs in renegotiations.

Brigo, Morini and Tarenghi (2011) develop structural first passage models with time-varying volatility to analyze the credit risk of corporate debt. They apply the models to the precise calibration of Lehman Brother's CDS data immediately prior to its default. They consider alternative deterministic and stochastic default barriers.

Ericsson, Reneby and Wang (2015) use a structural approach to explain the price of default protection for a sample of US corporations. Their major conclusion is that the previously documented underestimation of bond spreads may be sensitive to the choice of the risk-free rate, since when using credit default swap premia (rather than the Treasury Yield Curve) they do not find any systematic underestimation.

VIII. Banking Models

As we have witnessed in the previous sections, the CCA framework is a valuable analytic tool with which to examine contingent claims within a general corporate setting. Its application to specific issues encountered in banking can be equally beneficial. Banks (and insurance companies) are special and differ from other corporations because they are heavily regulated. Whether designed to prevent or address financial crises, banking regulation introduces new stakeholders into the mix.

Bank are also heavily levered. While a non- financial corporation is usually financed by 40-60% equity, in banks equity very often is less than 15% of its total financial resources. In highly levered firms, the contingent claim approach is essential. In banks we have depositors as claim holders as well as long-term debtholders. We can also analyze the deposit insurance as a stakeholder of the bank.

The guarantee of bank deposits is a key element in the regulatory arsenal. Bank deposits are exposed to the credit risk of the bank, i.e., the risk that the bank defaults, so that depositors cannot access their money. The objective of deposit insurance is to foster stability in the banking system and prevent bank runs when trust in a bank has evaporated, as it was the case with Northern Rock, a U.K. bank, in September 2007 at the outset of the GFC (Great Financial Crisis). The outflow of deposits stopped only when the U.K. Government stepped in and fully insured bank deposits. Bank deposits are insured by a government owned corporation, such as the Federal Deposit Insurance Company (FDIC) in the U.S. These deposit insurance agencies usually charge the same flat premium to all the banks under their purview, as opposed to a risk-adjusted premium which depends on the default risk of each insured bank.

Deposit insurance may generate incentives among banks to increase the riskiness of their assets. In order to contain excessive risk taking by banks the regulator imposes regulatory measures such as: capital adequacy, reserve and liquidity requirements, interest-rate ceilings. These measures are taken to reduce the effective insurance cost. CCA is the appropriate framework to price the fair equilibrium value of deposit insurance and analyze the impact of regulation.

Merton (1977a), in his seminal paper, notes that issuing a guarantee imposes a liability on the guarantor. Guarantees, such as deposit insurance and loan guarantees, are liabilities or costs to the guarantor. Like any insurance contract, a guarantee can be viewed as a put option on the bank's assets. He uses CCA to price guarantees and deposit insurance, and show that the cost of loan guarantees can be substantial. In Merton's simulation it varies between 1 basis point (bp) and 3% depending on the bank leverage and the cumulative variance of the asset returns over the time to the next audit.[2]

Ronn and Verma (1986) investigate the feasibility of estimating risk-adjusted premiums using market data for equity prices and book values for debts, in order to estimate the asset value and the variance of the asset returns. They adapt Merton's model to account for the fact that the FDIC forces bank closure only when the value of the bank's assets falls below a fraction of total debt, after trying to rescue the troubled bank through a capital infusion.[3] The authors derive the fair deposit insurance premium taking into consideration the extent of the bail-out effort. In their model the FDIC doesn't liquidate the bank's assets when the net worth of the bank becomes negative. Instead, it injects funds to avoid closure as long the bank shows a reasonable chance of recovery. There is a threshold, ρB , where B is the total debt of the bank and $\rho \leq 1$, such as if the net worth falls in between this threshold and B, the insuring agency injects up to $(1-\rho)B$ to make the value of the assets equal to B, while the shareholders retain their ownership of the bank. Only when the value of the bank's assets falls below the threshold the bank's assets are liquidated and the shareholders are wiped out.

Crouhy and Galai (1991) use the CCA to assess the bank's stakeholders exposure to three types of risks: interest rate risk, financial risk due to bank leverage and default risk. They derive the equilibrium price of equity, and the fair interest rate that depositors should require to be fully compensated for the risks they face, when there is no deposit insurance. They also derive the equilibrium deposit insurance premium as a function of the capital ratio, and investigate the trade-off between regulatory measures, such as capital adequacy requirements and interest rate ceilings.

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[2] Since deposit have no specific maturity, it is assumed that deposits are rolled over after each bank audit.

[3] This is the FDIC policy of DAPA (Direct Assistance and Purchase Assumption).

They show that when both are imposed, one of them is ineffective.

In their model the bank balance sheet is:

Assets		Liabilities and equity	
Risky assets	$(1-\beta)$	Deposits	$(1-\alpha)$
Default-free asset	β	Equity	α
Total	1	Total	1

where α and $(1-\alpha)$ are the proportions of funding through equity and deposit respectively. β and $(1-\beta)$ are the proportions of assets in default-free and risky financial instruments (e.g. a loan portfolio), respectively.

They derive the value of equity and deposits and show that the value of equity increases with the volatility of the returns of the risky assets. Then, when deposits are insured there is an incentive for the bank to increase unexpectedly the riskiness of the assets. When deposits are not insured the authors derive the required rate of interest that fairly compensates the deposit holders for the risk of default of the bank. The credit spread can be substantial and depends on the capital to asset ratio and the volatility of the returns of the risky assets.

The imposition of a minimum capital requirement limits the risk of bankruptcy. To this minimum capital requirement corresponds a required interest rate on uninsured deposits. If in addition to the imposition of a minimum capital requirement a ceiling on interest rates offered to depositors is imposed, one of the two constraints is ineffective.

The imposition of a non-earning reserve requirement reduces the risk of default but introduces a tax-like burden imposed by the regulator on the bank's claimholders. It generates the incentive for shareholders to unexpectedly increase the riskiness of the bank's assets. Finally the authors derive the fair deposit insurance premium as a function of the capital to asset ratio, α.

Dermine and Lajeri (2001) extend Merton's (1977a) model of deposit insurance by explicitly linking corporate loans on a bank's balance sheet to the balance sheet of borrowing firms. The stochastic process followed by the assets on the bank's balance sheet doesn't follow anymore an exogenous lognormal process. The bank's credit risk exposure depends specifically on the borrower's default risk. The valuation of deposit insurance is similar to a put option, as discussed in the literature, except that the underlying bank's assets are loans whose default risk depends on the capital structure of the borrower.

IX. Contingent Convertibles Issued by Banks (CoCos)

In the wake of the global financial crisis (GFC) of 2008, new capital instruments called contingent capital or contingent convertibles (CoCos) have emerged to reduce the likelihood that banks experience financial distress, and to mitigate the risk of bank failures that warrant government bailouts. These instruments are convertible bonds, which convert automatically to common shares, or are subject to a write-down, if a bank's equity or its equity/debt ratio falls below a certain threshold. It is a mechanism for automatic capital restructuring which reduces or extinguishes debt, and replaces it with equity, facilitating regulatory compliance and enhancing bank stability.

CoCos are a response of the regulator to the fact that during the GFC bank hybrid tier 1 capital, and tier 2 capital, failed to perform their role of loss-absorbing instruments. Banks kept paying interest on these instruments to avoid being shut out of the market in the future.

Pennacchi (2010) considers a bank with assets that follow a mix jump-diffusion process, and three sources of funding: equity, short-term deposits with instantaneous maturity, and contingent capital. The bank is assumed to gradually adjust the amount of deposits raised so that the leverage ratio is mean-reverting, with a given target asset-to-deposit ratio. Part of the deposits are FDIC insured and are paid the risk-free interest rate. Other deposits are uninsured and are paid the risk-free interest rate plus a fair credit risk premium that compensates depositors for the risk they incur. Interest rates are assumed to be stochastic.

Default occurs, and the bank is closed, when the value of the bank's assets falls below the promised value of total deposits. Note that, with this setup, only Poisson jumps can cause bank failure losses to uninsured depositors. Various conversion triggers are considered, in particular when the market value of capital (equity plus contingent capital)[4] falls below some pre-specified threshold.

Pennacchi adopts the CCA to determine the fair credit spreads for uninsured deposits and the yields that investors in contingent capital instruments should require to be fairly compensated for the risk they undertake for various contractual terms. As expected both the credit spread on deposits and the contingent capital bond's yield spread are inversely related to the capital-to-deposit ratio. When the bank is slower to adjust deposits in order to move the capital ratio toward its target, the contingent capital bond's yield spread is higher when the bank is undercapitalized, and lower when the bank is over capitalized.

Glasserman and Nouri (2012) consider the case where the capital-ratio trigger is based on book values to be consistent with existing regulatory capital requirements for banks. Also, when the capital-ratio hits the threshold only part of the contingent debt is converted into equity in order to maintain the required capital ratio. This is different from the usual assumption where the convertible debt is converted in full as soon as the threshold is hit. Just enough conversion takes place to maintain the minimum required capital ratio. When the contingent capital is fully exhausted then the bank is liquidated.

The authors find that the fair yield for contingent capital, in their model, is sensitive to the size of the convertible tranche, the volatility of the bank's asset return and the recovery rate in the event of liquidation.

De Spiegeleer and Schoutens (2012) consider CoCos as a derivative with a trigger expressed as a barrier on the equity price. They show that the CoCos are equivalent to a long position in a corporate bond, plus a knock-in forward together with a short position in a binary down-and-in option, corresponding to the foregone coupons once the conversion trigger occurs. Then, using Black–Scholes modeling, they are able to derive closed-form solutions for the price of CoCos with various terms.

Sundaresan and Wang (2015) also consider CoCos bearing a market-based trigger for mandatory conversion. They show that when market-triggered CoCos are used, the potential for price uncertainty, market manipulation and inefficient capital

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[4] The threshold expressed in terms of the sum of the values of equity and contingent capital avoids the multiple equilibrium problem when the conversion trigger is set at a level of the stock price (Sundaresan and Wang(2015)).

allocation may arise. To obtain a unique equilibrium, the conversion rule must ensure that, at the trigger price, conversion does not alter the value of the equity on which the trigger is placed. This condition excludes conversion with punitive dilutive conversion ratios that penalize equity holders for excessive risk taking, and encourage them to maintain higher capital ratios. Multiplicity, or absence of equilibrium, arise because the stakeholders are not given the option to choose a conversion policy in their best interest. Sundaresan and Wang propose modifying the terms of the contingent capital contract to eliminate the possibility of value transfer, and, in so doing, ensure a unique equilibrium.

X. International and Sovereign Debt

Sovereign debt and sovereign risk can be analyzed in the context of the overall balance sheet of the government and monetary authorities. One major difference between the balance sheet of a corporate or a bank, and the balance sheet of a government is that a large part of the assets of the government are related to future revenues and expenses, such as taxes and defense, education, welfare expenditures, etc. In addition, governments extend implicit guarantees to too-big-to-fail banks and other financial institutions that can be substantial as experienced after the collapse of Lehman Brothers in September 2008. These future cashflows are highly uncertain and depend on future interest rates, exchange rates, and on the state of the local as well as the world economies. Emerging market economies can also be subject to large capital flows that expose them to capital account crises and financial distress such as the Asian crisis in 1997. It is therefore a major challenge to estimate the market value and volatility of sovereign assets.

Based on an IMF working paper initially published in 2005, Gapen, Gray, Lim and Xiao (2008) apply CCA to the combined balance sheet of the government and monetary authorities to analyze sovereign debt and sovereign risk. The stylized accounting balance sheet for the sovereign has the following structure:

Assets	**Liabilities**
Foreign Reserves	Guarantees
Net Fiscal Asset	Foreign-Currency Debt
Other Public Assets	Local-Currency Debt

Base Money

Since the marked-to-market value of the assets is not observable they estimate sovereign asset value and volatility indirectly from the observable values of the liability side of the balance sheet. In order to apply the CCA they rearrange the balance sheet by subtracting the guarantees to the too-big-to-fail financial institutions from both the asset and liability sides. Liabilities then consist of foreign-currency denominated debt plus local-currency debt and base money whose values are publicly available. Foreign currency debt is assumed to be senior to domestic currency liabilities. Sovereign default in this setting occurs when there is a default on foreign-currency debt. It is assumed to happen when the asset value falls below the distress barrier, i.e., the promised payments to foreign lenders. Then, according to Merton (1974) domestic-currency liabilities can be modeled as a call option on sovereign assets, while the value of foreign currency debt can be modeled as a default-free debt (e.g., the distress barrier) minus the expected loss given default. Then, standard option pricing techniques can be applied to derive the implicit value for sovereign assets and its volatility.

Gapen et al. extend the Merton/KMV framework to develop a set of key credit risk indicators to measure sovereign balance sheet risk, including the distance to distress (in the KMV sense), probability of default, credit spreads, and the market value of risky foreign currency-denominated debt. Application to 12 emerging market economies shows the risk indicators to be robust when compared to market credit spreads on foreign currency debt.[5]

Gray, Merton and Bodie (2007) extend the Gapen et al. model to explore actual and potential applications of the CCA framework for investment management. These include: (i) valuing, investing and trading sovereign securities including Sovereign Capital Structure Arbitrage trading strategies; (ii) designing and managing sovereign wealth funds; and (iii) designing and valuing new sovereign risk transfer instruments and contracts.

In Belev and diBartolomeo (2015) the sovereign balance sheet is the same as in Gapen et al. (2008) and Gray et al. (2007) but they estimate directly all the components of sovereign revenues and expenses instead of using the implied values from the option features of the domestic and foreign debt. In their approach they trade model risk for

[5] Note that credit spreads are not used as inputs in the model.

statistical estimation risk. They apply their theoretical analysis to European countries, the U.S. and Japan and find consistency with observed variables and real-world events.

Gray, Merton and Bodie (2008) build on the previous CCA-based framework to develop early warning indicators of financial crisis and apply them to the U.S. subprime mortgage crisis of 2007–2008. They show how the CCA framework can trace the amplification of risk from household mortgages via structured products to off-balance sheet entities, and ultimately, to banks by using a system of interlinked risk-adjusted balance sheets. They show how the increase in CDS spreads for financial institutions, with and without subprime exposure, has been driven by changes in asset volatility, balance sheet leverage and, especially, changes in the market price of risk as experienced in March 2008 and July/August 2008. Option negative skew is shown to be a useful early warning signal of regime change. They also show how CCA can measure and analyze the contingent liabilities undertaken by governments and central banks such as the implicit financial guarantees to Freddie Mac and Fannie Mae. They propose new metrics for measuring financial vulnerability to help analyze systemic risk and show how these risk indicators can be incorporated in monetary policy models to better evaluate the relationships between financial stability, monetary policy and economic growth.

As in the previous article, Gray, Merton and Bodie (2011), view the sectors of the economy as interconnected portfolios of assets, liabilities and guarantees. By linking the balance sheets of various economic sectors, i.e., corporate, financial, household, and sovereign, they analyze different channels of risk transmission. The implicit put options in risky debt, guarantees, and other contingent liabilities allow for risk to be transmitted between sectors. The implicit put options of interlinked sectors is the source of highly nonlinear risk transmission. Through the CCA framework, they derive a forward-looking set of market-based indicators to measure the vulnerability of various economic sectors and to quantify the impact of asset–liability mismatches within and across institutions.

Galai and Wiener (2012) use the CCA framework to show that in a multi-currency environment, a firm wishing to minimize the probability of insolvency (and thus financing costs) may select to finance activities with a currency that is highly correlated with the rate of return on the firm’s assets. In that case the foreign exposure plays the role of a natural hedge. When the asset return is negative, the domestic currency on average appreciates against the foreign currency which partially compensates for the loss in asset value. Their model applies directly to banks that lend in local currency but obtains financing in foreign currency either from abroad or from local depositors.

XI. Conclusion

Classical financial decision-making is based on the assumptions of a pure equity firm operating is a perfect capital market, in an economy with no taxes. Once we allow for additional sources of funding for the firm, and an economy with taxation, the basic pricing models such as the CAPM are not suitable anymore to analyze the value of the liability instruments of the firm, as well as, the firm's equity. Leverage introduces non-stationarity in the distributions of returns of both equity and debt, and it is usually material for high leverage when the probability of default is non-negligible. Of course, as shown in many of the CCA papers, the probability of default is a complex function of the firm's leverage as well as the duration of debt, the riskiness of the assets, debt covenants, the level of the risk free term structure and more.

This review paper highlights the importance of the CCA in analyzing the values of corporate liabilities, the credit risk of various debt issues, and the probability of default of the firm. The analysis is performed under many different assumptions concerning the capital structure of the firm, and various market conditions and regulations, with and without taxes. The CCA is the only valid way to analyze the funding side of the corporation.

We also cover the applications of this approach to analyze the funding of banks and other financial firms that are subject to special regulations. Banks are also unique since most of the funding comes from short-term deposits that can be withdrawn instantly. Over the last decade banks started to use special innovative funding instruments, mainly the contingent convertible bonds (CoCos) which, under specific conditions imposed by the regulator, can be assimilated to Tier 1 or Tier 2 regulatory capital.

Commercial applications of the CCA to forecast probabilities of default of listed companies are proposed by commercial firms such as Moody’s KMV. Also, practical applications of the CCA have been extended to analyze sovereign debt. This research is relatively new and not yet applied in practice, though it has great potential to improve decision-making by central banks and governments.

The CCA is essential for risk management, first by the corporations themselves, but mostly by investors and financial institutions, providing funds to these corporations. Of course, it can be applied more accurately for traded companies, especially if debt is also traded. Nevertheless, it may be more complicated in practice to use it for private companies.

In recognition of the importance of CCA we edited 4 volumes containing 70 papers, most of them published before in leading professional journal. (See Crouhy, Galai and Wiener (2019)). We expect that future teaching of corporate finance as well as professional applications will highlight the contingent claim approach and, by better understanding the economic relationships among stakeholders it will lead to improved corporate governance.

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