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The value of Value-at-Risk: A theoretical approach to the pricing and performance of risk measurement systems

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ABSTRACT

Risk-based capital adequacy requirements are the main tool employed by government regulators to assure bank stability. This approach allows banks to choose from a number of alternative methods for calculating the required capital. Many systems for measuring risk differ significantly in cost, precision, and in the potential “capital savings”. We develop a statistical model for evaluating risk measurement systems and optimizing the selection process. The model is based on queuing theory. The selection of the optimal system is a function of available capital, the volume and the character of bank activity. While the most precise system may lower a bank’s minimal capital reserve requirements, it is not necessarily the optimal system once total costs are evaluated.

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1. Introduction

Regulators in many countries use capital adequacy requirements to control the stability of banks by restricting their exposure to risk. The initial Basel 1988 Capital Accord was based on a fixed ratio of risk-weighted assets (typically 8%) that should be financed by equity or equity-equivalent instruments. This accord has been adopted in most countries (Annual Report of BIS) and is designed to address credit risk. The original Capital Accord has been revised a few times since then. The revised capital adequacy framework is based on a more flexible approach to risk management in banking. As stated in the annual report of BIS, “While there is a continued focus on internationally active banks, the underlying

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principles should be suitable for application to banks of varying levels of complexity and sophistication in all countries”.

The Basel 1988 Accord reflects credit risk and is applied somewhat arbitrarily to the whole portfolio with allowances for risk-weighting the asset categories. The Basel 1998 Accord recognizes a part of the portfolio as a trading book. A different approach is used for measuring the risk of this part, which is comprised of publicly traded assets. Internal models based on either standard or VaR based evaluation are allowed. The Basel 2 approach takes this one step further by allowing internal models for measuring (non-traded) credit risk. In the last few years BIS conducted a series of studies (QIS) in order to gauge the impact of the new approach on minimal capital requirements (MRC). We focus in this paper on the advantages of sophisticated models of risk measurement and specifically on the capital savings reflected in reducing MRC.

Minimal capital requirements are contingent primarily on risk assessment, which includes both market and credit risk measurement. In many countries the first part – market risk assessment – is already implemented. This means that banks employ internal models to quantify their market risk. The current regulatory framework gives banks greater discretion by allowing them to choose between the standard incremental risk and better tailored VaR-based approaches to risk management (see Jackson, Maude, and Perraudin (1997)).

VaR for capital requirements is measured as the lower 1% quantile of the Profit & Loss distribution over a 10-business day horizon. Models employed to calculate VaR vary between institutions both in terms of their sophistication and the risk factors used. This measure became very popular during the last decade; particularly in view of the “capital savings” it affords financial institutions. In general, adoption of the VaR approach has enabled banks to meet capital adequacy requirements with capital reserves of less than the standard 8%. However, the measure itself has several problems, such as the lack of sub-additivity.¹ Moreover, the methods used for calculating VaR are based on different assumptions, and often produce results with low precision. Banks may pay dearly for over-simplification and risk incurring regulatory surcharges for inaccurate internal models. In spite of these shortcomings, there remains a general consensus that the correct approach to capital adequacy is based on probability distribution rather than an arbitrary rule of a thumb, such as the flat 8% of assets mandated in the original 1988 Basel Accord. The goal of this paper is not to compare the different approaches of risk measurement, but rather to price the added value to a bank from using an internal model for risk measurement. Our model quantifies the benefit of lowering the minimal capital requirements for a given level of risk against the cost of developing and employing a risk measurement system. There are many ways to implement VaR. They vary in cost and in their degree of precision. Similarly, the capital savings derived from employing VaR vary across institutions and the specific models employed. An optimal choice regarding risk measurement systems, therefore, is contingent on the capitalization of a bank, and the type of primary activity in which the bank engages. In this paper we propose a model for optimizing the selection of a risk measurement system.

The list of available software for measuring risk is long and constantly growing (see for example Kates (2000) for a comparison of 50 different systems). These programs calculate the required capital according to the standard model or according to the P & L distribution. The implementation of an effective risk management system should improve the bank's performance, i.e. allow for better risk diversification and more precise hedging.

From the standpoint of the bank, an optimal decision weighs the costs required in developing or purchasing such a system against performance benefits. A different approach based on the optimization of the capital structure of a financial firm, is described in Shephard-Walwyn and Litterman (1998). Many ready-to-use systems are currently available: CARMA, RiskWatch, RiskMetrics, Four Fifteen, Outlook, TARGA, Kamakura and Panorama, to mention a few. The prices of the software vary from a few thousand dollars a year for lower-end products to millions of dollars a year for the upper end. In addition to the initial investment, the costs of implementation, updates, databases, salaries and other expenses can contribute significantly to the total cost of ownership (see Spain (2000)).

¹ An alternative measure of risk which avoids this problem can be based on the expected loss among the lowest quantile, see Artzner, Delbaen, Eber, and Heath (1999), also Artzner (1999).

Risk measurement systems differ in their approaches and underlying assumptions. While some are based on historical simulations, others use the variance-covariance method (based on normality and linearity assumptions). More sophisticated systems employ Monte Carlo methods for risk measurement. Some research has been done in comparing different systems and approaches (see for example Marshall and Siegel (1997), and Vlaar (2000)). Because software packages change and are constantly updated they vary greatly in their specifications and cannot be easily compared. However, a significant correlation has been observed between the price and the precision of these systems, where generally the more expensive systems are also the more sophisticated and precise.

Not all banks are necessarily interested in the most sophisticated system. Many, particularly small and medium-sized banks shy away from active risk management and incorporate risk measurement systems solely to satisfy regulatory requirements. A bank employing risk measurement systems primarily for reporting purposes will likely seek a less expensive solution that probably renders less precise results. Imprecision may lead to an overestimation of risk that will be “corrected” by maintaining a higher level of capital reserves. Higher *de facto* capital requirements may become binding in the sense that after committing to a certain system the bank will need additional capital reserves to extend business.

The selection of a risk measurement system carries implications that extend beyond the realm of portfolio. Even for banks that opt not to engage in active risk management, the selection of a risk measurement system can have a significant impact on business strategy and financial structure. In a perfect market, capital requirements should not constitute a significant restriction (as predicted by the Modigliani–Miller Propositions) to business activity. However, in reality, issuing new capital is costly and is often interpreted by the market as a negative signal. Accordingly, banks should prefer developing a more precise risk measurement system to raising new capital. In any case, an optimal decision must take these considerations into account. An optimal risk measurement system must not only accurately capture the risk profile of the bank’s asset portfolio, but enable the bank to optimize its performance as well.

In recent years we have witnessed the growing role of securitization as part of risk management practices as well as an extensive use of various off balance sheet activities. These steps are taken in order to reduce capital requirements by mitigating some types of risks and transferring them to other market participants. However as the financial crisis of 2008 clearly demonstrated not every type of hedging eliminates risks completely. Often hedging only reduces some components of risk, typically leaving basis risks (and other types of risks) with the originating bank. Here again a more advanced risk management system can provide an advantage and capital savings by using more precise methods of risk measurement.

Other recent developments are various types of hybrid instruments used for raising capital, from subordinated debt with coupon payments subject to regulatory requirements to CoCo (Contingent Convertible) bonds. Regulators are still considering how to treat these forms of capital. All this together with the continuing consolidation of banks creates an incentive to make a strategic selection of the most appropriate risk measurement system that matches the specific needs of each bank and is able to deal with various types of risk, as opposed to using old and oversimplified methods similar to the Cooke (1998–2006) and McDonough (from 2007) ratios.

Our goal is to develop a simple and intuitive model that can be used by banks in their decision to choose the most appropriate risk measurement system. The model takes into account such parameters as the percentage of traded assets, the available capital and typical activity of a bank.

2. The proposed model

Consider a bank that has some capital C . Regulators require the bank to develop a risk management system. They set a minimal capital requirement, which in the simplest case takes a form of a fixed percentage of assets. The old rule was to use 8% or more² of the risk-weighted assets. The modern

² In England for example FAS sets capital requirement for each bank separately and for most banks the requirement is more than 8.5%.

approach allows the distinguishing between credit risk applied mainly to the banking book of non-traded assets and market risk applied mainly to the book of traded assets. For the purpose of the model, the capital adequacy requirements for credit risk follow the old rule.³ Capital requirements for traded assets on the other hand are quite different and can reduce the required capital significantly (see Crouhy, Galai, and Mark (2000)). Moreover there are several alternative ways of calculating market risk. The underlying assumption of the model is that the maximum allowable risk exposure is determined by the bank's capital level and its risk measurement system, i.e. the method by which risk is calculated by the bank. Denote by A – the total assets of a bank, by α – the percentage of traded assets that qualify for market risk assessment and by C the minimal capital needed to satisfy the capital adequacy requirements. Accordingly the minimal required capital can be stated as:

$$C = 0.08 \cdot (1 - \alpha) \cdot A + k \cdot \alpha \cdot A$$

In the formula above 8% is required for all non-traded assets,⁴ and k is the percentage required for traded assets. The percentage k varies for different risk measurement methods and we will use it for optimizing the choice of a risk measurement system.⁵ This formula can be inverted to show the relationship between maximum risk exposure (assets), the level of available capital and the risk measurement system:

$$A_{\max} = \frac{C}{0.08(1 - \alpha) + \alpha k}. \quad (1)$$

The parameter k is contingent on the type of risk measurement system, and we assume that it is defined by the cost of the system $k(p)$. Cost – p is measured as a rent, which includes all expenses related to the risk measurement system, which means that for each short time period dt there is a total cost of $p \cdot dt$. We assume that for any cost p there exists a corresponding system.⁶ A bigger p corresponds to a better system and a lower $k(p)$. Basic (and cheap) risk measurement approaches fail to capture certain types of more subtle risks. As a result there is the so-called model risk that forces banks that use simple methods to limit themselves voluntarily (or with regulatory guidance) to a lower risk to capital ratio. Effectively this leads to a higher required capital for banks that choose a cheaper risk measurement system. The specific form of k as a function of p should be estimated in each case. In general it is bounded between $k(0) = 8\%$ (no risk measurement system – standard approach) and some value $k(\infty)$ for the best possible system which can predict the actual risk with infinite precision.

In Fig. 1 we show the maximum assets A_{\max} as a function of k for various values of α . When $k = 8\%$ (standard model) the maximum risk exposure is equal for all α . For more precise risk measurement systems, however, the higher the percentage of traded assets the larger the volume of capital savings improvement, i.e. more risk can be undertaken for the same amount of capital.

This model can be viewed as describing the choice between tools of various precisions (and costs). Paying less for a tool yields a less precise result that may not pass the backtest applied by regulators to compare the risk forecasted by the bank to actual past performance. To compensate for this imprecision, the bank will be required to set aside additional reserves. Hence, a less expensive (and less precise) system will effectively require more capital.

Let us assume that projects (for simplicity all identical in size) are submitted to a bank randomly according to a Poisson process with density λ . One can think about a project as a short term automatically renewed deposit, such as NOW accounts (negotiable orders of withdrawal) in the United States.⁷ This means that there is a probability $\lambda \cdot dt$ that during the next short time period dt one new

³ Except for the Basel 2000 proposal, which allows for internal rating models.

⁴ Typically some weighting is used, but we ignore this for simplicity.

⁵ From our experience based on consulting projects with several banks the reasonable values of k vary between 1.5% and 8% and depend on the types of financial instruments in the trading books. A higher percentage of government bonds leads to a lower k .

⁶ A choice among a continuous spectrum of models is not a realistic one for a bank's management which typically has to decide between a limited number of systems, but this question is important for the companies developing these systems as they wish to serve their clients in the optimal manner.

⁷ The analysis can be extended for other types of projects as well.

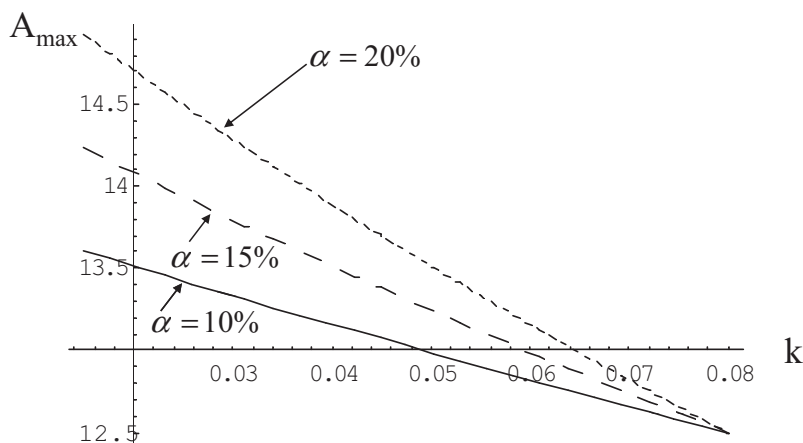


Fig. 1. Maximal risk per unit of capital as a function of k for different proportions of qualifying traded assets.

project will arrive. With probability $(1 - \lambda) \cdot dt$ there will be no new projects and the probability that more than one new project will be submitted is of the second order in dt (and thus negligible). A new project is undertaken if it does not conflict with current capital adequacy requirements. This means that if there are currently less than A_{\max} active projects a new project is accepted, otherwise it is lost to competitors.⁸ We do not allow the bank to raise new capital, since that typically entails significant time, effort, and cost.

Every project has a random life. Let us assume that the termination (e.g. withdrawal) of any project is described by a Poisson process with parameter μ .⁹ This means that with the probability $\mu \cdot dt$ an existing project will be terminated during dt , and the probability that this specific project will continue is $(1 - \mu) \cdot dt$.

When a deposit is alive, the funds are used by the bank for investments, and we assume that each investment has the same spread δ (the difference between the interest paid and interest earned). This means that a bank with n existing projects has an income stream of $\delta \cdot n \cdot dt$ dollars during a short time interval dt . The payment for the risk measurement system during the same time interval is $-p \cdot dt$ regardless of the number of existing projects. Thus the total infinitesimal profit of the bank is $(\delta n - p) \cdot dt$.

Denote the maximum number of projects a bank can have by s after some risk measurement system is implemented. $s = A_{\max}(C, p)$ according to Eq. (1). For the purpose of our discussion $k(p)$ and $s(p)$ correspond to a risk-measuring system at cost p . Then there are $s + 1$ possible states, one with no active projects, one with one active project only, and so on up to s active projects as shown in Fig. 2. According to our model there are certain probabilities for the bank to pass from one possible state to another. Since time is continuous in our model there is a finite probability of a transition from one state to a neighboring state and the probability of a bigger jump is negligible.

This model is well studied in the theory of queues, see for example Kleinrock (1975), Wolff (1989), and Panico (1969). This model¹⁰ assumes a unique stationary distribution for each state. Intuitively this happens when at each state and its neighbor the probability of moving from one to another is

⁸ In reality the bank can invest the money into safe asset, which often does not require capital, but in this case the project has a very small spread, equivalently to a lost project.

⁹ Later on we relax this assumption.

¹⁰ Known as M/G/s/s or M/M/s/s – Poisson arrival, Poisson/general departure with s servers and no queue.

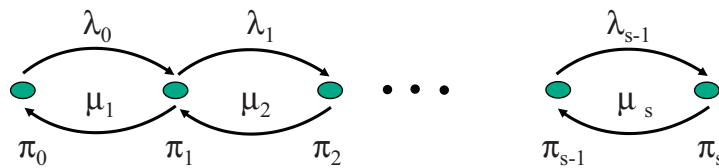


Fig. 2. Transition chain. Arrival of a new project increases the total risk by one unit until maximum risk exposure is attained. Termination of an existing project has the opposite effect.

equal to the probability of an inverse move. Denote by π_n the probability of a bank to be in state n ($0 \leq n \leq s$), see Fig. 2. Then the following equations must hold for a stationary distribution:

$$\begin{aligned} \pi_0 \lambda_0 &= \pi_1 \mu_1 \\ \pi_1 \lambda_1 &= \pi_2 \mu_2 \\ &\vdots \\ \pi_{s-1} \lambda_{s-1} &= \pi_s \mu_s \end{aligned}$$

The implication of each of these equations is that the probability of arriving and leaving each state is equal. The distribution π will not be stationary if in some states the rate of arrival differs from the rate of departure. Using backward induction we attain the following for any n ($0 \leq n \leq s$):

$$\pi_n = \frac{\lambda_{n-1}}{\mu_n} \pi_{n-1} = \dots = \frac{\lambda_0 \lambda_1 \dots \lambda_{n-1}}{\mu_1 \mu_2 \dots \mu_n} \pi_0$$

The additional condition $\sum_{n=0}^s \pi_n = 1$, allows to determine each π_n explicitly:

$$\pi_0 + \pi_1 + \dots + \pi_s = 1$$

$$\pi_0 \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2} + \dots + \frac{\lambda_0 \lambda_1 \dots \lambda_{s-1}}{\mu_1 \mu_2 \dots \mu_s} \right) = 1$$

$$\pi_0 = \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2} + \dots + \frac{\lambda_0 \lambda_1 \dots \lambda_{s-1}}{\mu_1 \mu_2 \dots \mu_s} \right)^{-1}$$

To solve this equation note that the arrival rate for a new project is constant λ , and thus for the bank the arrival rate does not depend on the state $\lambda_0 = \lambda_1 = \dots = \lambda_{s-1} = \lambda$. The termination rate for a specific project is also constant – μ , however the rate of terminations for a given bank is proportional to the number of open projects. The greater the number of open projects, the higher the probability that one of them will terminate during dt , thus $\mu_1 = \mu, \mu_2 = 2\mu, \dots, \mu_s = s\mu$.¹¹ This leads us to a simple formula:

$$\pi_n = \frac{(\lambda^n)/(n! \mu^n)}{\sum_{i=0}^s (\lambda^i)/(i! \mu^i)} = \frac{\tau^n/n!}{\sum_{i=0}^s \tau^i/i!}, \quad \text{where } \tau = \frac{\lambda}{\mu}, \tag{2}$$

this formula is well-known and called Erlang B, see Kleinrock (1975), vol. 1, p. 106.

Processes of this type are modeled in queuing theory and many interesting properties of this process are known. For example, the probability of losing new projects due to capital adequacy requirements is equal to π_s , the probability that there are exactly s (the maximum amount) projects currently active at the bank. For large values of s formula (2) is computationally inconvenient, since it is a long series of small terms. An analytic version of this formula can be derived using the Euler gamma function:

$$\Gamma(1 + s) = \int_0^\infty x^s e^{-x} dx,$$

¹¹ We consider these events as independent thus ignoring bank runs.

and the so-called incomplete gamma function, defined as:

$$\Gamma(1 + s, \tau) = \int_{\tau}^{\infty} x^s e^{-x} dx.$$

Both functions are well known and computationally expedient. We employ the following formula:

$$\sum_{i=0}^s \frac{\tau^i}{i!} = \frac{e^{\tau} \Gamma(1 + s, \tau)}{\Gamma(1 + s)},$$

to derive a computationally efficient form:

$$\pi_n = \frac{e^{-\tau} \tau^n \Gamma(1 + s)}{\Gamma(1 + n) \Gamma(1 + s, \tau)} \tag{3}$$

Another method of simplifying this expression by using cumulative normal distribution is shown in [Appendix A](#).

In deriving these equations we have assumed that the termination of projects follows a Poisson process. This assumption can be easily relaxed. Due to the PASTA property, see [Wolff \(1989\)](#), the same stationary distribution is obtained for any type of random termination process with the same mean life of a single project ($1/\mu$ in our case). This means that we do not need the assumption that the termination of projects is described by a Poisson distribution.

It is also well known that the average time such a system spends in each state n is equal to the probability of this state π_n . This immediately implies that the probability of losing a new project is equal to the portion of time when the bank cannot accept any new projects, which is equal to probability π_s . This is the probability of loss stemming from capital adequacy requirements.

At each moment in time with probability π_n there are exactly n open projects. The expected cash-flow for the bank is then $(\pi_1 \cdot \delta + \dots + \pi_s \cdot s \cdot \delta - p) \cdot dt$ reflecting both the income from the spread and the cost of the risk measurement system. In a realistic situation a bank has to decide which risk measurement system to select. Typically there are many possible choices. We assume that the set of available risk measurement systems is continuous and for any p there exists a system with price p and an appropriate maximal risk exposure $s(p)$.

The optimization problem now is to maximize the expected profit, where¹²:

$$P = E(\text{profit}) = (\pi_1 \delta + \dots + \pi_{s(p)} s(p) \delta - p)$$

Then the expected profit is:

$$P = \delta \frac{\sum_{i=1}^{s(p)} \tau^i / ((i - 1)!) }{\sum_{i=0}^{s(p)} \tau^i / i!} - p = \delta \tau \frac{\sum_{i=0}^{s(p)-1} \tau^i / i! }{\sum_{i=0}^{s(p)} \tau^i / i!} - p \tag{4}$$

or using formula (3), it can be written as:

$$P = \delta \tau \frac{s \Gamma(s, \tau)}{\Gamma(1 + s, \tau)} - p \tag{5}$$

note that s (the maximum risk) depends on p (the quality of a risk measurement system). The optimal choice of a risk management system is equivalent to maximizing this expression with respect to p . Let's differentiate with respect to p and equate the derivative to zero:

$$\frac{dP}{dp} = \delta \tau \frac{\partial}{\partial s} \left(\frac{s \Gamma(s, \tau)}{\Gamma(1 + s, \tau)} \right) \frac{ds(p)}{dp} - 1 = 0 \tag{6}$$

There is an analytic expression of this derivative using hypergeometric functions (except for the ds/dp part of course), but it does not help in understanding the efficient choice. Instead we provide

¹² The following formula gives the expected density of the profit stream. The expected profit is the integral of this density over time.

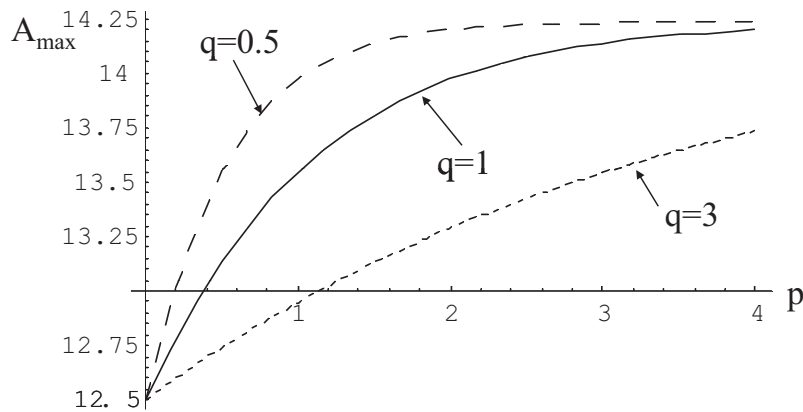


Fig. 3. Maximum risk per unit of capital as a function of the quality (cost) of a risk measuring system for different values of the scaling factor q , assuming $\alpha = 15\%$ of total assets.

a simple example. Note that Eq. (6) is contingent on the following parameters: profitability – δ , the frequency of new project arrival relative to the termination of existing ones – τ , the capital level of the bank – C , the percentage of banking assets invested in traded securities – α , and the maximal risk exposure as a function of the price – $s(p)$.¹³

3. Example

In this section we construct a simple numerical example of a bank seeking an optimal risk measurement system. First we present the basic version and then demonstrate how it depends on the assumptions.

Define the function k describing the capital charge as a function of rent in the following form:

$$k(p) = 0.015 + (0.08 - 0.015)e^{-p/q} \tag{7}$$

This formula corresponds to the example when the capital adequacy for a trading book is between 1.5% for an ideal (infinitely expensive) system and 8% for those banks that use the standard approach. The lowest possible capital requirement is an important parameter of the model. In practice it should be based on trial runs of the risk measurement system on sample portfolios that should reflect the typical investment policy of a bank. From our experience (building and validating risk measurement systems for several banks) the value of 1.5% seems reasonable but it should be calibrated to each specific market environment as well as the scaling factor q . Under these assumptions each unit of capital can be leveraged by a factor that varies from 12.5 to 67. The maximum admissible risk $s(p)$ as a function of p is presented in Fig. 3.

The same maximum risk exposure as a function of α when $q = 1$, is shown in Fig. 4. This figure demonstrates that the maximum risk assumption is higher for banks with larger trading books, since traded assets typically receive more precise risk assessments based on observed price behavior.

The unit of measurement in this example is \$1M. Consider a bank with \$200M capital and assume that on average 200 new projects are submitted each day. Let us assume that the size of each project is \$20,000 (or 0.02 units). One day is $1/365$ of a year, thus $\lambda \cdot dt = 200$, and $\lambda = 73,000$. The average life of a project is 2 years, thus $\mu = 0.5$.¹⁴

¹³ The stationary distribution is appropriate when considering a long-term profit–cost relationship. However at the moment of making a decision on a risk measurement system a bank is at some specific state with respect to its ability to accept new projects. This state should impact the optimal selection. Obviously the best strategy is a dynamic one, in which the bank switches from one system to another as soon as it approaches the limits of its current system. However in reality this process is long and costly and we ignore it as well as a possibility to raise a new capital.

¹⁴ This example uses the typical time scale for projects arrival/termination – of the order of days or weeks, while the process of raising new capital or replacing an existing system is typically of the order of months or years. This justifies our approach to a continuous scale of arrival/departure, while the capital and risk measurement system are fixed.

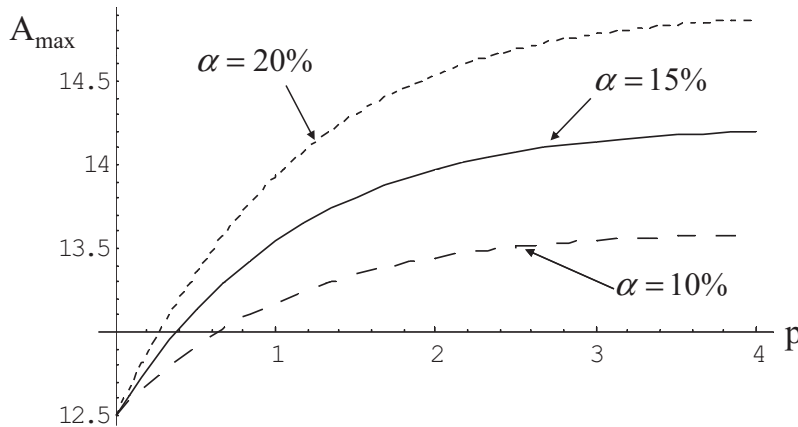


Fig. 4. Maximal risk per unit of capital as a function of the quality (cost) of a risk measuring system for different proportions of qualifying (traded) assets $\alpha = 10\%$, 15% , 20% , assuming scaling factor $q = 1$.

Let us assume that the spread is 1.25%, this spread already takes into account all fixed costs excluding the cost of the risk measurement system. Accordingly, δ can be defined as $\delta = 0.0125 \times \$20,000 / \$1M = 2.5 \times 10^{-6}$, and $\tau = \lambda / \mu = 146,000$. In addition, we assume that 15% of the bank's assets qualify for market risk assessment and that the current technology is characterized by $q = 1$. The profit function (5) becomes:

$$P = \delta \tau \frac{s \Gamma(s, \tau)}{\Gamma(1 + s, \tau)} - p,$$

where the maximum risk exposure is determined by the choice of risk measurement system and available capital. The profit as a function of p is presented in Fig. 5.

Maximum profit corresponds to the optimal risk measurement system with an annual cost of \$1.5M. Using this system will provide a 16.5% return on capital as compared to 15.6% when employing a standard approach at no cost with a lower level of allowable risk assumption. Under these assumptions, the probability of losing a new project when adopting the standard approach to risk management is 14.4%. With the optimal risk measurement system this probability is reduced to 5.3%. One can see that in this case there a significant improvement in bank performance that can be attained by adopting an optimal risk measurement system. Note that optimal in this case does not mean procuring the most expensive system available.

A sensitivity analysis of our model is presented in Tables 1–3. The upper half of each table describes our assumptions and the lower half demonstrates the results. The basic case appears in column A. The changes in the assumptions relative to the basic case are highlighted in bold in each column.

We did not incorporate variations of the daily arrival rate into our analysis, since a reasonable change of plus or minus 10% does not significantly affect the results. In Table 1 we present the results

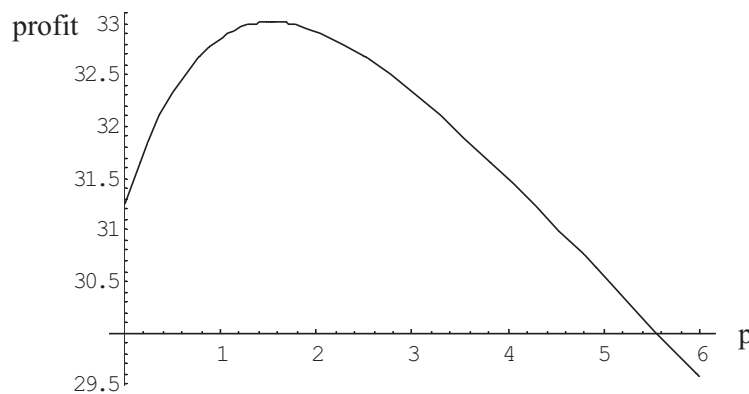


Fig. 5. Bank profits as a function of cost p . $C = \$200M$, arrival rate 200/day, size \$20K, average life 2 yr, spread 1.25%, $q = 1$.

Table 1

The optimal risk measuring system as a function of the major parameters of a bank, when $q = 1$. The basic case is presented in column A. ROC stands for return on capital. The probability of loss is the probability that a new project will be lost due to a lack of capital in a steady state, % of qualified is the percentage of traded assets. The two bottom rows show the ROC and the probability that a new project will be lost (Prob. loss) when using the standard approach ($p = 0, k = 8\%$).

Case	A	B	C	D	E	F	G	H	I	J	K	L
Assumptions												
Regulatory capital	\$200M	\$1B	\$250M	\$150M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M
Arrival rate	200/d	1000/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d
Project size	\$20K	\$20K	\$20K	\$20K	\$10K	\$30K	\$20K	\$20K	\$20K	\$20K	\$20K	\$20K
Average life	2 y.	2 y.	2 y.	2 y.	2 y.	2 y.	2.5 y.	1.5 y.	2 y.	2 y.	2 y.	2 y.
Spread	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.0%	1.5%	1.25%	1.25%
% of qualified	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	20%	10%
Results												
Optimal p	\$1.5M	\$3.2M	\$0M	\$1.2M	\$0M	\$1.5M	\$1.5M	\$0M	\$1.3M	\$1.7M	\$1.8M	\$1.0M
Profit	\$33M	\$174M	\$37M	\$24M	\$18M	\$33M	\$33M	\$27M	\$26M	\$40M	\$34M	\$32M
ROC	16.5%	17.4%	14.6%	16.3%	9.1%	16.5%	16.5%	13.7%	13.1%	20.0%	17.2%	16.0%
Probability of loss	5%	3%	0%	30%	0%	37%	24%	0%	6%	5%	1%	10%
Average assets	\$2.76B	\$14.2B	\$2.92B	\$2.05B	\$1.46B	\$2.76B	\$2.76B	\$2.19B	\$2.74B	2.78B	2.89B	2.64B
Capital charge k	2.9%	1.8%	8%	3.4%	8%	2.9%	2.9%	8%	3.3%	2.7%	2.5%	3.8%
ROC – standard	15.6%	15.6%	14.6%	15.6%	9.1%	15.6%	15.6%	13.7%	12.5%	18.7%	15.6%	15.6%
Prob. loss – standard	14%	14%	0%	36%	0%	43%	31.5%	0%	14%	14%	14%	14%

Table 2
The optimal risk measuring system as a function of the major parameters of a bank, when $q = 3$.

Case	A	B	C	D	E	F	G	H	I	J	K	L
Assumptions												
Regulatory capital	\$200M	\$1B	\$250M	\$150M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M
Arrival rate	200/d	1000/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d
Project size	\$20K	\$20K	\$20K	\$20K	\$10K	\$30K	\$20K	\$20K	\$20K	\$20K	\$20K	\$20K
Average life	2y.	2y.	2y.	2y.	2y.	2y.	2.5y.	1.5y.	2y.	2y.	2y.	2y.
Spread	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.0%	1.5%	1.25%	1.25%
% of qualified	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	20%	10%
Results												
Optimal p	\$0.9M	\$6.2M	\$0M	\$0M	\$0M	\$0.9M	\$0.9M	\$0M	\$0.1M	\$1.6M	\$2.1M	\$0M
Profit	\$31M	\$169M	\$36M	\$23M	\$18M	\$31M	\$31M	\$27M	\$25M	\$38M	\$32M	\$31M
ROC	15.7%	16.9%	14.6%	15.6%	9.1%	15.7%	15.7%	13.7%	12.5%	18.9%	16%	15.6%
Probability of loss	12%	4%	0%	36%	0%	41%	29%	0%	14%	10%	7%	14%
Average assets	\$2.58B	\$14.0B	\$2.92B	\$1.87B	\$1.46B	\$2.58B	\$2.58B	\$2.19B	\$2.51B	\$2.63B	\$2.72B	\$2.50B
Capital charge k	6.3%	2.3%	8%	8%	8%	6.3%	6.3%	8%	7.9%	5.4%	4.7%	8%
ROC – standard	15.6%	15.6%	14.6%	15.6%	9.1%	15.6%	15.6%	13.7%	12.5%	18.7%	15.6%	15.6%
Prob. loss – standard	14%	14%	0%	36%	0%	43%	31.5%	0%	14%	14%	14%	14%

Table 3
The optimal risk measuring system as a function of the major parameters of a bank, when $q = 0.5$.

Case	A	B	C	D	E	F	G	H	I	J	K	L
Assumptions												
Regulatory capital	\$200M	\$1B	\$250M	\$150M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M	\$200M
Arrival rate	200/d	1000/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d	200/d
Project size	\$20K	\$20K	\$20K	\$20K	\$10K	\$30K	\$20K	\$20K	\$20K	\$20K	\$20K	\$20K
Average life	2y.	2y.	2y.	2y.	2y.	2y.	2.5y.	1.5y.	2y.	2y.	2y.	2y.
Spread	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.0%	1.5%	1.25%	1.25%
% of qualified	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	20%	10%
Results												
Optimal p	\$1.1M	\$1.9M	\$0M	\$1.0M	\$0M	\$1.1M	\$1.1M	\$0M	\$1.0M	\$1.2M	\$1.1M	\$0.9M
Profit	\$34M	\$175M	\$37M	\$25M	\$18M	\$34M	\$34M	\$27M	\$27M	\$41M	\$35M	\$33M
ROC	17.0%	17.5%	14.6%	16.8%	9%	17.0%	17.0%	13.7%	13.5%	20%	17.7%	16.3%
Probability of loss	4%	3%	0%	28%	0%	36%	23%	0%	4%	4%	0.2%	8%
Average assets	\$2.81B	\$14.2B	\$2.92B	\$2.09B	\$1.46B	\$2.81B	\$2.81B	\$2.19B	\$2.80B	\$2.81B	\$2.91B	\$2.68B
Capital charge k	2.2%	1.6%	8%	2.4%	8%	2.2%	2.2%	8%	2.4%	2.1%	2.2%	2.6%
ROC – standard	15.6%	15.6%	14.6%	15.6%	9%	15.6%	15.6%	13.7%	12.5%	18.7%	15.6%	15.6%
Prob. loss – standard	14%	14%	0%	36%	0%	43%	32%	0%	14%	14%	14%	14%

for $q = 1$, while Tables 2 and 3 show the corresponding results for other values of q as well. The optimal p is the optimal annual cost (in dollars) of a risk measurement system. The return on capital (ROC) is given in two cases – when the optimal system is selected and under the standard model (no system, but higher capital adequacy requirements). The probability of loss is the probability that the bank must reject or defer a new project due to the capital adequacy requirement. We also provide the probability of loss in the case of applying the standard approach (prob. loss – standard) to risk measurement. In addition we provide profit, average assets, and the capital charge k for the optimal choice of the risk measurement system.

As demonstrated in Table 1, the lower the profitability of the bank, the lower the added value of a risk measurement system. In fact, the standard approach ($p = 0$) is optimal in three cases: banks with excess capital, banks that undertake very small projects and banks that undertake very short-term projects. The more profitable the banking business, the more advantageous a precise risk measurement system is. Another important parameter is q , which signifies the efficiency of a risk measurement system relative to its cost. A lower coefficient q corresponds to the case in which one can procure a very efficient system inexpensively. Tables 2 and 3 present results for $q = 3$ and $q = 0.5$.

Column B in Table 1 shows that when consolidating the 5 banks presented in the standard case one can improve performance by using a more expensive (efficient) risk measurement system. When the capital and the arrival rate both increase 5 times it make sense to switch from a system that costs \$1.5M to a system that costs \$3.2M (annually). This leads to an increase of return on capital from 16.5% to 17.4%. Column C shows that when there is too much capital relative to existing opportunities, then it is optimal to use the standard approach, not spending money on risk measurement and keeping the capital charge at 8%. Column D shows that when there is too little capital, a risk measurement system will have only a limited impact on performance. Columns E and H (and also C, D, E, H in Table 2 and C, E, H in Table 3) show that when the projects are small or short lived the optimal decision is to use the standard approach ($p = 0, k = 8\%$).

A sophisticated risk measurement system is very useful for bigger banks with bigger and longer projects (columns B, F, G). Also banks that can put a higher percentage of assets in the trading books (column K versus column L) enjoy better performance by using more precise (and also expensive) risk measurement systems. Higher spreads also lead to a need in a more precise risk measurement system as shown in columns I and J this effect is stronger when the price scaling factor q is bigger (Table 3 versus Tables 2 and 1).

In our opinion these examples support the basic intuition that bigger banks operating in a more dynamic environment will use more advanced risk measurement systems to optimize their performance.

4. Conclusions

The current regulatory environment in financial markets around the world encourages the adoption of active risk management. Since Basel 1998 and Basel 2000, regulatory requirements in a growing number of countries allow banks to depart from fixed capital adequacy rates and develop internal models better tailored to the task of assessing asset risk. Sophisticated risk measurement systems enable financial institutions to reduce minimal capital requirements by using more precise models. To take advantage of this opportunity, banks have to discover the optimal tools for measuring risk. Each bank must factor in the level of precision with which it chooses to measure risk. The selection of an optimal model affects not only the level of capital required to meet regulatory requirements, but can have a significant impact on both the character and scope of a bank's business activity. Opportunity costs can be avoided and profitability increased through the selection of the proper risk measurement system.

In this paper, we provide a framework for the optimal selection of a risk measurement system under the assumption that a higher degree of accuracy corresponds to a higher cost to the bank. We limit our analysis to the simplest use of such a system – the reduction of the required capital, however the same approach can be extended to incorporate additional benefits to active risk management.¹⁵ This

¹⁵ In this case we will add a function describing the benefits of an active risk management $b(p)$ to the infinitesimal profit $(\delta n - p + b(p)) \cdot dt$ and perform the same optimization procedure.

model reflects the reality that many small to medium sized banks are interested in risk measurement models mainly due to the pressure of regulators through capital requirements. The suggested model is simple, intuitive and flexible.

The results of the model demonstrate that the most accurate (and therefore expensive) systems are appropriate for bigger banks with low capitalization, which operate in an unstable environment. These banks face radical and dynamic variations between high and low levels of activity. The higher the capital cushion the lower the cost of an optimal risk measurement system. Similarly, a stable business environment (many small independent projects) decreases the optimal cost, while a more volatile environment (fewer, short-term larger projects) requires a more sophisticated system at a higher cost. An extension of this model might incorporate an analysis of various types of activity, such as loans and deposits with a discount window. Due to the simplicity and ease of use of this model, further variations can be introduced into the general framework presented here.

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Appendix A. Appendix

Another way to simplify the expression for π_n is by approximating the Poisson distribution by a normal one. Following (2) we have:

$$\pi_n = \frac{\tau^n/n!}{\sum_{i=0}^s \tau^i/i!} = \frac{(\tau^n/n!)e^{-\tau}}{\sum_{i=0}^s (\tau^i/i!)e^{-\tau}} = \frac{\text{Prob}(X_\tau = n)}{\text{Prob}(X_\tau \leq s)},$$

here X_τ is a random variable with a Poisson distribution with parameter τ . Note that for the numerator there is a simple analytical formula, however the denominator is the sum of a very large number of small terms, since τ is typically big (see example in Section 3). Thus it might be useful to use the standard approximation of the Poisson distribution by the normal distribution (this approximation is appropriate for large values of τ):

$$\frac{X_\tau - \tau}{\sqrt{\tau}} \sim \text{Norm}(0, 1)$$

Hence $\text{Prob}(X_\tau \leq s) = \text{Prob}(((X_\tau - \tau)/\sqrt{\tau}) \leq ((s - \tau)/\sqrt{\tau})) \sim N((s - \tau)/\sqrt{\tau})$, where N is the cumulative normal distribution. This approximation gives reasonable results for big values of τ . In the example above τ was 219,000 which is big enough to make the approximation error negligible.

For the probability of loss π_s there is another analytic formula, derived in Jagerman (1974) (see Theorem 3). This probability can be expressed as an integral:

$$\pi_s = \frac{\tau^s/s!}{\sum_{i=0}^s \tau^i/i!} = \left(\tau \int_0^\infty e^{-\tau x} (1+x)^s dx \right)^{-1}.$$

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